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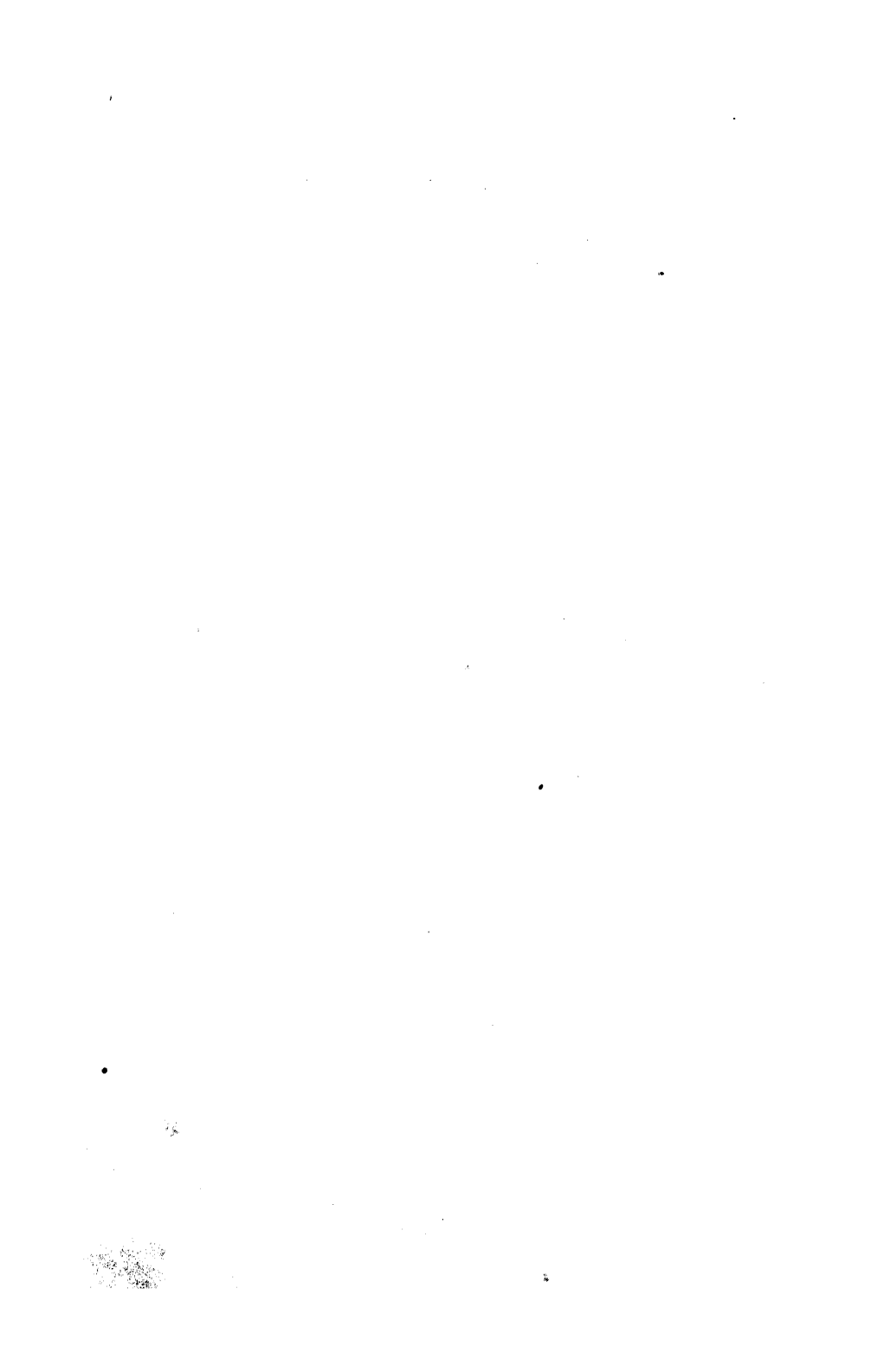
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**PLANE AND GEODETIC SURVEYING
FOR ENGINEERS**



PLANE AND GEODETIC SURVEYING

FOR ENGINEERS

BY

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VOLUME ONE
PLANE SURVEYING

FIFTH EDITION REVISED AND ENLARGED

BY

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(*Surveyor-General, Gold Coast, 1926-1938*)



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PREFACE TO THE FIFTH EDITION

IN the Preface to the last edition of this book it was stated that, for various reasons, including the fact that little in the way of addition or revision then appeared to be necessary, only minor corrections and alterations were made in the text and any new matter that was found necessary was added in the form of Appendixes. As the original type of the book was no longer in existence, that edition was printed from plates, and not from movable type, and, as it would have added greatly to the cost of the book and hence to its selling price, if it had to be completely re-set in type, it was decided to use the existing plates as far as possible for the revision for the fifth edition. This set certain limitations on the amount of revision which could conveniently be included in the main text, and for this reason much, though by no means all, of the new matter has again been relegated to Appendixes. Since the last edition was published there have been a number of new developments in surveying instruments and the main accounts of these are gathered together in Appendix I, though others have been fitted into the text where the deletion of some out-of-date material, etc., made this possible and appropriate.

In selecting new instruments for description, I have chosen those which appear to embody some new or specially interesting features in design or construction, or else are typical of a class. In some cases, also, descriptions of instruments which are no longer being manufactured have been retained, in some instances because the instrument in question embodies some fundamental or specially interesting feature and in others because, although it is now being replaced by a new model, large numbers of the existing model are still in use and are likely to be for some considerable time.

In Chapter X I have made a number of changes in the sections dealing with transition curves, particularly in the case of transition curves on roads. It has been possible to include much of the new matter in the main text but additional notes are contained in Appendix IV. The changes are due mainly to the fact that there has been a tendency in recent years in this country to prefer the Euler Spiral to the Lemniscate as a transition curve for roads, while abroad the Spiral appears to be far more generally favoured, and hence more space has been devoted to the Spiral than was given to it in the last edition.

I am indebted to a number of correspondents for calling my attention to errors in the last edition and for suggestions for the

new one. In particular, however, I would like to express my thanks to the Astronomer Royal for providing me with the latest data regarding the magnetic declination and to Mr. A. G. Tyson, M.Eng., A.M.I.C.E., M.I.H.E., not only for permission to reproduce Tables I and II, Appendix IV, which are part of much more extensive Tables designed and computed by him though not yet (1957) published, but also for many useful suggestions and criticisms regarding the sections on transition curves. Thanks are also due to Messrs. Cooke, Troughton & Simms, Ltd.; Messrs. Hilger & Watts, Ltd.; Messrs. Wild & Co.; Messrs. Zeiss & Co.; and Messrs. Kern & Co., manufacturers of surveying instruments, for providing me with information regarding some of their latest instruments and for allowing me to reproduce illustrations from publications issued by them.

J. CLENDINNING.

ANGMERING-ON-SEA,

SUSSEX.

14th September, 1957.

PREFACE TO THE THIRD EDITION

IN undertaking the revision of this book for a third edition, I have added certain matter, some of which, perhaps, is of greater interest and importance to engineers and surveyors working abroad or in the Colonies than it is to those whose work is confined to home practice. In Great Britain, the existence of the Ordnance Survey often simplifies the work of the private engineer or surveyor very considerably and makes it unnecessary for him to aim at the same degree of accuracy that is sometimes required abroad. Consequently, I have added a new chapter on linear measurements, in which work with the long steel band or tape, an article that is much more extensively used abroad than it is at home, is dealt with in considerable detail, and I have also made fairly considerable additions to the matter dealing with the theodolite traverse, for which purpose I have divided the original chapter on traversing into two, one dealing with the field work and the other with the office computations. Hitherto, most engineers and surveyors have regarded the limit of the standard of accuracy attainable by simple theodolite traversing as lying somewhere between $1/2000$ and $1/5000$, but, with the more extensive use of modern small theodolites with micrometer readings, such as the small Tavistock theodolite, and with reasonably careful taping with the long steel band, an accuracy of anywhere between $1/10,000$ and $1/30,000$ is now easily attainable in ordinary engineering and cadastral work. This makes it possible to substitute traversing for triangulation in cases where accuracy is necessary but where triangulation is difficult or unduly expensive.

In the chapters dealing with linear measurements and theodolite traversing I have used some of the results of the theory of errors, although a formal treatment of the theoretical aspects of this subject is reserved for Chapter IV of Vol. II. This is because it is of the utmost importance that the surveyor should have some idea of the different sources of error inherent in his work, the probable magnitude of these errors, and the manner in which they are propagated. Otherwise, it is not possible for him to do his work in the most economical manner or to select the best methods. This applies particularly to traversing, and hence, and mainly for purposes of reference, I have thought it advisable to include with the description of certain operations some discussion of the resulting effect of those errors that are likely to affect them.

Among other additions are a short description of road transition

curves, following the discussion of transition curves on railways already included in the second edition, and a brief description of echo sounding, with special reference to its advantages and possibilities as applied to engineering problems. Road transition curves are becoming of increasing importance in the lay-out of modern roads, and echo sounding is a fairly recent development which is likely to replace the older methods of sounding in modern hydrographical surveying.

In conclusion, I should like to express my thanks to the following :— The Astronomer Royal, for providing me with data regarding the present values of the magnetic elements ; The Director General, Ordnance Survey, for permission to include a short summary of the Ordnance Survey methods of detail survey, and Colonel G. Cheetham, D.S.O., M.C., R.E., Ordnance Survey, for looking over my draft on this subject and providing me with other information ; Messrs. Carl Zeiss (London), Ltd., for lending the block of Fig. 74 ; Mr. A. D. Simms, of Messrs. Cooke, Troughton & Simms, for providing me with special information regarding instruments made by his firm ; Messrs. E. R. Watts & Sons for providing details regarding the Connolly Standard Compass manufactured by them ; Messrs. Henry Hughes & Son, Ltd., for lending the blocks used in printing Figs. 369 and 370 and for giving me information concerning echo sounding apparatus of their manufacture, and Dr. E. B. Worthington, of the Freshwater Biological Association, for allowing the use of a photograph, reproduced as Fig. 371, of an echo sounding record obtained during the course of an echo sounding survey of the English Lakes.

J. CLENDINNING.

ANGMERING-ON-SEA,
SUSSEX.

21st June 1939.

PREFACE TO THE FIRST EDITION

THIS textbook is designed to form a complete treatise on plane surveying with such parts of geodetic work as are of interest to the civil engineer. The author would emphasise at the outset that he does not claim that a knowledge of geodesy is a very essential part of the equipment of the engineer. The execution of surveys of such extent and character as to necessitate the general methods of geodetic surveying and levelling does, however, occasionally fall within his province. For this reason, a knowledge of its principles is required in the examinations of the Institution of Civil Engineers and of universities and colleges.

In a general text on surveying there is little room for originality, except in treatment. Although this work is intended to serve as a reference book for practising engineers and surveyors, the chief aim has been to cover ground suitable for a degree course, and, while it is hoped that the book will prove of value to those pursuing a college course, the needs of the self-taught student have been specially kept in view. In consequence, many explanatory notes and practical hints have been inserted, particularly with reference to the more common surveying operations. The latter are not meant to take the place of practice in the field, which, needless to say, is an essential part of a training in surveying, but are intended as a guide to the reader with limited opportunities for field practice, and are mainly suggested by the author's experience of the initial mistakes of young engineers in practice and of students undergoing field training in camp.

The subject-matter is presented in two volumes. The first covers in ten chapters the more common surveying operations of the engineer, and the second deals with astronomical and geodetic work and the methods employed in large surveys generally.

In the arrangement of the present volume it has been thought desirable, for convenience of reference, to group descriptions of the more commonly used instruments and their adjustments to form Chapter I. It is hoped that the detailed method of treating the subject of adjustments will afford a sound understanding of the geometrical principles in each case. In Chapters II to VI the subjects of Chain Surveying, Theodolite and Compass Traversing, Ordinary Levelling, Plane Table Surveying, and Contouring are described as applied to cadastral and engineering surveys. Chapter VII deals with the office work of computing areas and volumes, the latter with particular reference to the measurement of earthwork

quantities. The practice of setting out works is treated in Chapter VIII. The setting out of railways is the only branch of this subject meriting detailed description, and problems in connection with the setting out of simple, compound, reverse, and transition curves are treated. The principles and practice of Tacheometry are given in Chapter IX, and Hydrographical Surveying, including Marine Surveying and Stream Measurement, is dealt with in Chapter X. Owing to the number of texts available on mine surveying, no special reference has been made to that subject.

Sets of illustrative numerical examples and answers are given for practice. For permission to reproduce questions set by the Institution of Civil Engineers, the University of London, and the Royal Technical College, Glasgow, the author desires to express his thanks to the authorities concerned.

Lists of references have been inserted, after the appropriate chapters, on such subjects as readers might wish to pursue further. The author would gratefully acknowledge the assistance he has derived from these and other books and papers on Surveying.

D. C.

PREFACE TO THE SECOND EDITION

IN the preparation of this edition it has been found desirable to enlarge the text and to re-write a considerable part of the book. The number of illustrations has also been increased. The eight years which have elapsed since the issue of the First Edition have witnessed extensive developments in the design and manufacture of surveying instruments, and an endeavour has been made to bring the book up to date in respect of these. For permission to reproduce questions set by the Institution of Civil Engineers, the University of London, the Royal Technical College, Glasgow, and Trinity College, Dublin, the author desires again to express his indebtedness. He would also express his thanks to Messrs. C. F. Casella and Co., Ltd., Cooke, Troughton and Simms, Ltd., George Russell and Co., Ltd., E. R. Watts and Son, Ltd., Henry Wild Surveying Instruments Supply Co. Ltd., and Carl Zeiss for placing at his disposal information regarding their recent instruments. He gratefully acknowledges his further indebtedness to Messrs. Cooke, Troughton and Simms for permission to reproduce Figs. 71, 72, and 301, to Messrs. C. F. Casella and Co. and Henry Wild Co. for Fig. 69, and to Messrs. Carl Zeiss for Fig. 102.

D. C.

TRINITY COLLEGE,
DUBLIN, 1931.

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PLANE AND GEODETIC SURVEYING FOR ENGINEERS

PLANE SURVEYING

INTRODUCTION

SURVEYING is the art of making such measurements of the relative positions of points on the surface of the earth that, on drawing them down to scale, natural and artificial features may be exhibited in their correct horizontal or vertical relationship.

Less comprehensively, the term, "surveying," may be limited to operations directed to the representation of ground features in plan. Methods whereby relative altitudes are ascertained are distinguished as "levelling," the results being shown either in vertical section or conventionally in plan.

Plane and Geodetic Surveying.—A plan is a projection upon a horizontal plane, and in its construction all linear and angular measurements drawn down must be horizontal dimensions. It is impossible to give a complete representation of distances following the undulations of the ground other than by a scale model. Now a horizontal plane is one normal to the direction of gravity as indicated by a plumb line, but, on account of the spheroidal form of the earth, the directions of plumb lines suspended at different parts of a survey are not, in normal circumstances, strictly parallel, and the horizontal plane at one point does not precisely coincide with that through any other point. No reference is here made to surface irregularities, the effect being due to the curvature of the earth's mean surface, which is taken to be that corresponding to mean sea level supposed extended round the globe.

In surveys of small extent the effect of curvature is quite negligible, and it is justifiable to assume that the mean surface of the earth is a horizontal plane within the area covered. Surveying methods based on this supposition are comprised under the head of *Plane Surveying*. The assumption becomes invalid in the accurate survey of an area of such extent that it forms an appreciable part of the earth's surface. Allowance must then be made for the effect of curvature, and the operations belong to *Geodetic Surveying*.

No definite limit can be assigned for the area up to which a survey may be treated as plane, since the degree of accuracy required forms the controlling factor. The sum of the interior angles of a geometrical figure laid out on the surface of the earth differs from that of the corresponding plane figure only to the extent of one second for about every 76 square miles of area, so that, unless extreme accuracy is required, plane surveying is applicable to areas of several hundreds of square miles.

Plane Surveying.—Plane surveying is of wide scope and utility, and its methods are employed in the vast majority of surveys undertaken for various purposes, such as engineering, architectural, legal, commercial, scientific, geographical, exploratory, military, and navigational. As applied to civil engineering, all surveying methods are utilised in the various surveys required for the location and construction of the different classes of works within the province of the engineer. These surveys embrace rapid reconnaissances of an exploratory character undertaken to facilitate the selection of an approximate site for the work. These are followed by more detailed surveys of the selected region, in which a much greater degree of accuracy is sought, and from which the best location is ascertained. The obtaining of various data required in the design of the proposed works forms part of the preliminary operations, and may involve surveying methods of a specialised character. Previous to and during construction, the surveyor's duties also include the routine of setting out the lines and levels of the works and the measurement of areas and volumes.

Geodetic Surveying.—Geodetic surveys are usually of a national character, and are undertaken as a basis for the production of accurate maps of wide areas, as well as for the furtherance of the science of geodesy, which treats of the size and form of the earth. The most refined instruments and methods of observation are employed, and the operations are directed to the determination of the absolute positions on the earth's surface of a series of points which serve as controls for all other surveys.

Use of Geodetic Data.—As a general rule, in countries where a geodetic survey exists, much useful information concerning points established by it may be obtained, either free or on payment of a small fee, on application to the government survey authority. This information may include descriptions to enable the points to be found on the ground, co-ordinate values for each point, and, in many cases also, the height above sea level. Also, it often happens that the true bearings between fixed points, not too far apart, are available and can be used either to orientate the survey or to provide checks on the bearings. If extensive survey operations in any area are contemplated, it is always well, even when there is no legal obligation or reason to do so, to ascertain if information of this kind can be obtained, because, not only may it save the

engineer or surveyor a great deal of work in providing his own control points, but it may also enable him to obtain most useful checks at various stages of his work.

Another advantage of using points established by the government survey is that it enables local surveys to be laid down and plotted, without any great difficulty, on the official printed maps and plans. Indeed, in many countries the law insists that certain classes of surveys must be tied in to the "government datum points." This is particularly the case where surveys of property, in which the resulting plans are to be used as part of legal documents conveying title to land, are involved, and the laws of some countries insist that plans of such things as the underground workings in mines should show the relation of the survey to some point fixed, or established, by the official survey department.

In Great Britain, there are certain cases where special maps or plans, based on the Ordnance Survey sheets, have to be prepared and submitted to the authorities concerned, while the larger scale plans indicate the positions and heights of bench marks, this information being of great value to civil engineers and others concerned with levelling operations.

~~REVISED EDITION~~

Field and Office Work.—A record of measurements made on the ground is usually, in plane surveys at least, of little or no service until the dimensions are laid down to scale on a drawing. The usual stages in the production of the finished drawing may be summarised as :

Field Work : (a) A preliminary examination, or reconnaissance, of the ground to discover how best to arrange the work :

(b) The making of the necessary measurements.

(c) The recording of the results in a systematic form.

Office Work : (a) The making of any calculations necessary to transform the field measurements into a form suitable for plotting.

(b) The plotting or drawing down of those dimensions.

(c) The inking-in and finishing of the drawing.

(d) The calculation of quantities to be shown on the drawing for special purposes, such as areas of land, etc.

Principles of Surveying.—The fundamental principles upon which various survey methods are based are themselves of a very simple nature, and may be stated here in a few lines.

It is always practicable to select two points in the field and to measure the distance between them. These can be represented on paper by two points placed in a convenient position on the sheet and at a distance apart depending upon the scale to which it is proposed to plot the survey. From these initial points others can be located by two suitable measurements in the field and laid down in their relative positions on the sheet by means of appropriate

drawing instruments. Points so obtained serve in turn to fix the positions of others.

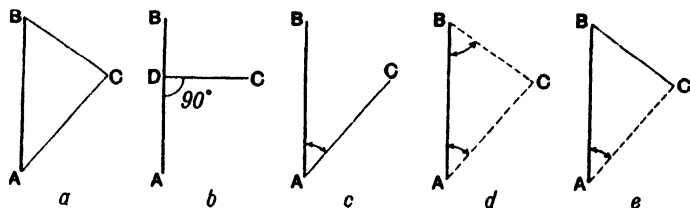


FIG 1.

The more direct methods of locating a point C, with respect to two given points A and B, are illustrated in Fig. 1.

(a) Distances AC and BC are measured, and C is plotted as the intersection point of arcs with centres A and B and radii scaling the measured distances. This system is employed in linear or chain surveying (Chap. III).

(b) Perpendicular CD and the position on AB of its foot D are measured, and C is plotted by the use of a set-square. This method, termed *offsetting*, is used, in combination with other surveying systems, for locating subsidiary points not required for extending the survey but for defining details.

(c) Distance AC and angle BAC are measured, and C is plotted either by means of a protractor or trigonometrically. The traverse method of surveying (Chap. IV) is founded on this principle.

(d) Angles BAC and ABC are measured, and C is plotted by solution of the triangle or by use of a protractor. Distance AB being known, C is located without further linear measurement, and, in consequence, the method, known as *triangulation*, is applicable to the most extended surveys, in which it is desirable to reduce linear work to a minimum.

(e) Angle BAC and distance BC are measured, and C is plotted trigonometrically or by protracting the angle and swinging an arc from B. This method is of minor utility, and is required only in exceptional cases.

The same methods could be employed in measuring relative altitudes as well as positions in plan, but for this purpose methods (b), (c), and (d) only are of practical utility. If the diagrams are regarded as elevations, with AB horizontal, ordinary spirit levelling (Chap. VI) is illustrated by (b). The elevation of C relatively to that of A is obtained by establishing instrumentally a horizontal line AB through A, such that AB and C are in the same vertical plane, and measuring the vertical distance CD. Trigonometrical levelling (Vol. II, Chap. V) is exemplified in (c) and (d).

Several of the above systems, both for horizontal and vertical location, may be employed in the same survey, and various types

of instruments are available for the angular and linear measurements. A survey may therefore be executed in several ways by different combinations of instruments and methods, and some parts of the work may require different treatment from others. The principal factors to be considered are the purpose of the survey and the degree of accuracy required, the nature of the country, the extent of the survey, and the time available for both field and office work. To select the methods best suited to a particular case demands on the part of the surveyor a degree of skill which can be acquired only by experience.

Working from the Whole to the Part.—In most types of survey the ruling principle is to work from the whole to the part. Thus, in fairly extensive surveys, such as those of a large estate or of a town, the first thing to be done is to establish a system of control points. The positions of these points are fixed with a fairly high standard of accuracy, but, between them, the work may be done by less accurate and, consequently, by less expensive methods. In a town survey, for instance, the "primary horizontal control" will consist either of triangulation or of a traverse surrounding the whole town. If triangulation is adopted as the control, the larger "main" or "primary" triangles will be surveyed with the greatest care, but these will be "broken down" by smaller "minor" or "secondary" triangles, which, in general, will be measured by less rigid methods and with less elaborate instruments than those used in the survey of the larger triangles. Similarly, if the control consists solely of points established by traverses, other traverses, which will usually run along the main streets, will be used to connect points on the outer surround. Some of these radial traverses will probably be measured with the same degree of accuracy as the main outer surround traverse so as to stiffen it and provide a traverse "network" rigidly held together. Between these radial traverses, however, other less important ones, surveyed by less precise methods, will be established, and, as a general rule, the survey of detail—that is of the positions and shapes of houses, streets, etc.—will be done by still less elaborate methods, using the minor traverses as a base from which to work. The idea of working in this way is to prevent the accumulation of error, which, in some cases, tends to magnify itself very quickly. If the reverse process is adopted and the survey is made to expand outwards, it will generally be found that minor errors become so magnified in the process of expansion as to become uncontrollable at the finish. On the other hand, if an accurate basic control is established in the first place, not only are large errors prevented and minor ones controlled and localised, but it will be found that the detail begins to fall almost automatically into its proper place, like filling in the smaller pieces in a jig-saw puzzle.

These remarks regarding "working from the whole to the part"

apply also to such operations as levelling. Thus, in contouring on a fairly large scale, it will generally be found advisable to establish a system of bench marks, using an accurate high-class surveyor's level for the purpose. The actual survey of the contours or form lines can then be made by using an Abney level or Indian clinometer, or other similar instrument of minor accuracy, or even, when the scale of the plan or map is very small and the "contour interval" large, by aneroid barometer or by hypsometer.

Nature of the Subject.—By virtue of the simplicity of the underlying principles of plane surveying, there is little of theory to be studied, and a training in the subject must be chiefly directed towards a thorough working knowledge of field methods and the associated instruments, as well as of office routine. Success in the field is the outcome not only of skill in solving the larger problems connected with the general organisation of surveys, but also of attention to the methodical performance of the numerous details of field work. Frequent practice in the field under expert guidance saves the beginner much memorising of these details, makes for skill and speed in manipulating instruments, and promotes systematic habits of work. Numerous minor problems requiring special treatment are likely to be encountered in field work, and to the beginner these are apt to assume a more difficult aspect on the ground than they do on paper. A little experience is necessary before one can entirely overcome the distractions of field conditions, especially when these are aggravated by physical fatigue and bad weather.

Even in favourable circumstances there are many ways in which errors may enter into the work, and it is important to realise that *absolute precision can never be attained*. Any desired degree of refinement of practical utility may, however, be secured by the use of suitable instruments and methods of observation. The surveyor must keep in view the uses to which his results will be put, and must select those methods which will yield sufficient accuracy for the purpose. Much time may be wasted in needless refinements, and the necessary judgment must be cultivated by a study of the nature and relative importance of the various sources of error affecting different surveying operations.

Errors.—While a knowledge of the theory of errors as dealt with in Vol. II, Chap. IV is not required in connection with small surveys, we shall find it convenient to use one or two fundamental relations in the chapters dealing with linear measurements and traversing. This matter is of practical importance because traversing involves a combination of linear and angular measures; and, unless it is possible to form some sort of estimate of the errors likely to arise from each type of measurement, the manner in which they are propagated, and their probable effect on the final result, it is not possible to choose the best or most economical methods of working. Consequently, although reference must be

made to Vol. II for an account of the principles and of the scientific basis of the theory of errors, those results that are of practical importance regarding the subject under discussion will be stated and used in Chapters II and IV of this volume, but, if necessary, the sections dealing with errors may be omitted on a first reading or left over until Chap. IV of the second volume has been studied in detail. Here, however, it may be well to describe the principal types of error and to give a very brief explanation of how they are propagated, and their total effect estimated.

The ordinary errors met with in all classes of survey work may be classified as mistakes, systematic or cumulative errors, and accidental or compensating errors.

Mistakes arise from inattention, inexperience, or carelessness. Since an undetected mistake may produce a very serious effect upon the final results, the surveyor must always arrange his work to be self-checking, or take such check measurements as will ensure the detection of mistakes. On discovering that a mistake has been made, it may be necessary to repeat the whole or part of the survey. It should, however, always be possible for the surveyor to guarantee that his completed work is free from mistakes.

Systematic Errors are those which are recognised as existing in the performance of any particular survey operation. Their character is understood, and their effects can be eliminated either by the exercise of suitable precautions or by the application of corrections to the results obtained. Such errors are of a constant character, and are regarded as positive or negative according as they make the result too great or too small. Their effect is therefore cumulative: thus, if, in making a measurement with a 50-ft. linen tape which is 1 in. too long, the tape is extended ten times, the error from this source will evidently be 10 in. The effects of constant errors may become very serious, and the precautions to be adopted against them in various field operations are treated in detail throughout the text.

Accidental Errors include all unavoidable errors which are present notwithstanding that every precaution may have been taken. They arise from various causes, such as want of perfection of human eyesight in observing and of touch in manipulating instruments, as well as from the lack of constancy in the conditions giving rise to systematic errors. Thus, in the example cited above the error of 1 in. in the tape may fluctuate a little on either side of that amount by reason of small variations in the pull to which it is subjected or even by changes in the humidity of the atmosphere. Such errors are usually of minor importance in surveying operations when compared with the systematic errors, because their chief characteristic is that they are variable in sign, plus errors tending to be as frequent as minus errors, so that they are of a compensating nature and thus tend to balance out in the final results. In consequence, it is needless to adopt elaborate precautions against the occurrence

of errors of this type while possibly overlooking the propagation of serious cumulative errors.

As regards the method of propagation of the different kinds of error, the effect of the cumulative errors is additive since each one tends always to be of one particular sign. The effect of the accidental errors is also additive in the sense that, in any particular observation, the total error is the sum of all the errors made during that observation. However, we do not know the signs of the individual errors, but we do know that, while one error may be positive, another one is equally likely to be either positive or negative. Accordingly, the ordinary addition law cannot be applied, and all that is possible is to form a general estimate of the probable effect of combining all errors, when each one is as likely to have a positive as it is to have a negative sign. In this case, the mathematical law of probability shows that the best way of combining errors of this kind is to take the square root of the sum of the squares of the individual probable errors. Mathematically, this can be expressed as :

$$r = \pm(\epsilon^2_1 + \epsilon^2_2 + \epsilon^2_3 + \dots + \epsilon^2_n)^{\frac{1}{2}}$$

where r is the probable error resulting from the combination of the probable errors $\pm \epsilon_1, \pm \epsilon_2, \pm \epsilon_3 \dots \pm \epsilon_n$, the plus or minus sign in all cases indicating the equal probability of either sign. In this way, the difficulty regarding the unknown signs of the individual errors is, to a large extent, overcome.

CHAPTER I

INSTRUMENTS—CONSTRUCTION AND ADJUSTMENT

BEFORE proceeding to consider the actual work of surveying it is desirable that a knowledge of the instruments employed should be obtained, and in this chapter are described the instruments in most regular use by the engineer and surveyor in the everyday work of chain and angular surveying and levelling. Those of a more specialised character are treated later under the branches of surveying with which they are related.

While the necessity for a thorough understanding of instruments is self-evident, it is not suggested that the surveyor need at first make an exhaustive study of the construction of all the fittings which go to make up his instruments. The more detailed his knowledge, however, the better qualified will he be to effect temporary repairs—an important matter when working far from headquarters. In general, these individual fittings can be treated in groups forming definite and essential parts of the instrument, and the fundamental principles underlying the arrangement, use, and adjustment of the instrument are to be studied by reference to the functions and relationships of those parts.

The Nature of Adjustments.—Adjustments are of two kinds—*Temporary* and *Permanent*. The former are those which have to be made at each setting up of the instrument, and as they form part of the regular routine in using the instrument, need no preliminary explanation. The so-called permanent adjustments, on the other hand, are directed to eliminating errors of workmanship or defects which may develop either through wear or accident, and in general consist in setting essential parts into their true positions relatively to each other.

The method of making these adjustments in any instrument is entirely dependent upon the arrangement of its essential parts and the geometrical relationships they are designed to bear to each other. The adjustment of an instrument is simply a practical problem in geometry. Unless so regarded, the operation can only be performed mechanically, and the methods will be much more readily forgotten.

Note.—The examination of an instrument from the geometrical standpoint should lead the student to a consideration of:

(1) The nature of the errors of measurement made by the instrument due to (a) erroneous relationships between adjustable parts, (b) defects in non-adjustable parts.

(2) The method of manipulation whereby these errors may be reduced to a minimum.

(3) The manner in which the instrument should be tested to discover whether it is in adjustment or not.

(4) The nature of the correction necessary to eliminate an error discovered in the test.

The Principle of Reversal.—In testing for instrumental errors, considerable use is made of the method of reversal. Such tests are usually directed to examining whether a certain part is truly parallel, or perpendicular, to another, and on reversing the one part relatively to the other, erroneous relationship between them is made evident.

To take the simple instance of examining the perpendicularity between two edges of a set-square, let the line BC (Fig. 2) be

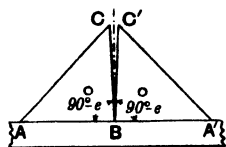


FIG. 2.

drawn with the set-square in the first position, and BC' after reversal. The reversal constitutes the test, for if C and C' coincide, the required condition that $ABC = 90^\circ$ obtains. If not, let e be the angular error. Then clearly $CBC' = 2e$, or the apparent error on reversal is **twice** the actual error, and therefore, if correction is possible, the error to be eliminated

is only half the amount of the apparent error. The example further illustrates how good results are obtainable from a defective instrument, viz. by reversing and taking the mean of the two erroneous results. As a second example of the effect of reversal, the level tube (page 35) should be studied.

Notes on Adjustments.—1. Most instruments have several relationships to be established by adjustment, and it is important that these should be performed in such sequence and in such a manner (a) that those first executed will not be disturbed in making subsequent adjustments, (b) that, in performing any adjustment, possible errors of the adjustments to follow will be balanced out, if they are such as to influence the accuracy of that under examination.

2. As it is sometimes troublesome to ensure that a particular adjustment will be quite independent of the others, it will to repeat the adjustments from first to last and so gradually perfect them all, especially if serious errors have been discovered.

3. In making an adjustment, it is difficult to eliminate an error completely at the first trial, and the test and correction should be repeated a number of times, using greater refinement as the error decreases.

4. Instability of the instrument makes it almost impossible to adjust it satisfactorily, and the instrument should therefore be placed upon firm ground. In adjustments involving the taking of test sights, parallax (page 28) must be carefully eliminated.

5. Adjusting screws must be left bearing firmly, so that they will not slacken on being accidentally touched, but at the same time they should never be forced. The nuts are commonly provided with holes into which an adjusting pin, called a tommy bar, may be fitted, and, being rotated in the manner of a capstan, are usually referred to as capstans. In using them, consideration must be given to the required direction of rotation.

6. The length of time an instrument will maintain its adjustments to the accuracy with which they are made depends both upon the instrument and the manner in which it is handled. Some adjustments are of much greater importance than others, and require more frequent examination. In precise work these should be tested every day. To obtain the best results in surveying, it is assumed that no instrument is free from error, and the routine in observing must be arranged to eliminate the effects of residual errors left after adjustment, latent errors which may have developed since adjustment, and inherent defects in the non-adjustable parts.

PARTS COMMON TO SEVERAL INSTRUMENTS

Before proceeding to a consideration of particular instruments, some features common to various instruments will be treated separately. Parts thus dealt with are the telescope, level tube, vernier, magnetic needle, and tripod stand.

THE TELESCOPE

In the majority of surveying instruments the several fittings may be broadly classed according to their functions as optical parts and measuring parts. In many instruments, such as the theodolite and the level, the optical fittings take the form of a telescope, the use of which is to assist the eye in obtaining distinct vision of distant objects, as well as to furnish a definite line of sight of which the direction and inclination may either be known or are to be measured. The provision of a telescope in such instruments is not absolutely essential, but the utility and accuracy of the instrument would be greatly impaired without it. Some instruments, however, such as the tachemeter, the photo-theodolite, and the sextant, depend for their action upon certain laws of optics, and in these the optical parts are essential and assume the character of measuring parts.

A knowledge of the principles underlying the optical arrangement of the telescope is helpful both in the everyday use and the adjustment of the instruments of which it forms a part, and the student should therefore have some acquaintance with the elements of the science of optics. To introduce a description of the surveying

telescope, the elementary principles relating to image formation by lenses will here be summarised.

Definitions.—A *Lens* may be defined as a portion of a transparent substance enclosed between two surfaces of revolution which have a common normal, termed the *Principal Axis* of the lens. The curved surfaces employed are of spherical form, but one of the surfaces may be plane.

Lenses are classed as convex or concave. The various forms are shown in Fig. 3, these being distinguished as : (a) double convex or biconvex ; (b) plano-convex ; (c) concavo-convex or positive meniscus ; (d) double concave or biconcave ; (e) plano-concave ; (f) convexo-concave or negative meniscus. In the case of a and d the curvature of the two surfaces may be equal or unequal.

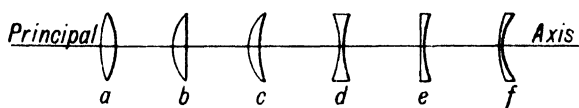


FIG. 3.

The *Optical Centre*, or simply the centre, of a lens is that point on the principal axis through which pass all rays having their directions parallel before and after refraction. A ray passing through this point undergoes a small lateral displacement, but, unless the lens is very thick, it is convenient to assume that the emergent ray is in the same straight line as the incident ray. The optical centre is so situated that its distances from the surfaces are directly proportional to their radii. In the case of double convex and concave lenses the centre lies within the thickness ; in plano-convex and plano-concave lenses it is situated on the curved surface, while in meniscus lenses it is outside the lens and on the same side as the surface of smaller radius.

A *Secondary Axis* is any straight line, other than the principal axis, passing through the centre of a lens.

Refraction through Lenses.—The nature of the refraction of rays traversing a convex lens is such that a beam of light is on emergence more convergent or less divergent than at incidence. Such lenses may be referred to as *converging lenses* in contradistinction to *concave* or *diverging lenses*, which have the opposite effect. The influence of the curvature of the surfaces is to produce different angles of deviation on the individual rays of a beam according to the position of their points of incidence, and such that if all the incident rays pass through a point on the principal axis, the refracted rays, produced if necessary, will pass more or less exactly through some other point on the axis.

In Fig. 4 an axial parallel beam of light is shown incident upon a convex lens, the source of the light being an infinitely distant point on the principal axis. The constituent rays after refraction converge to a point *F* on the principal axis. This point is termed the *Principal Focus* of the lens, and in the case of thin lenses its distance from *O*, the optical centre, is called the *Focal Length*, a quantity which will be designated by *f*. In the case of the concave

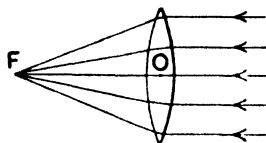


FIG. 4.

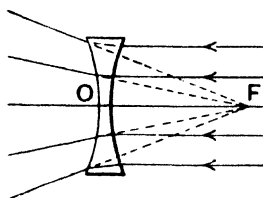


FIG. 5.

lens (Fig. 5) the parallel rays are, after refraction, made to diverge from the principal focus, which lies on the same side of the lens as the incident beam.

The principal focus may be regarded as the point at which is formed an image of the distant source of light. In the case of the convex lens, the refracted rays actually pass through *F*, and the image is real, so that it could be received on a screen. With a concave lens, however, the diverging rays have to be produced backwards to locate the point of divergence *F*. The image in this case has no real existence, and it is said to be a virtual image.

The *Power* of a lens is the reciprocal of its focal length, and is considered to be positive for a convex lens and negative for a concave. The unit of power is the diopter, which is the power of a lens having a focal length of one metre. Thus, a convex lens of 20 cm. focal length has a power of 5 diopters.

In the case of a lens formed of two or more individual thin lenses in contact, the power of the combination is the algebraic sum of the powers of the separate lenses. If the focal lengths of the several lenses in contact are *f'*, *f''*, etc., the relationship,

$$\frac{1}{f} = \frac{1}{f'} + \frac{1}{f''} + \dots,$$

enables the focal length *f* of the combination to be obtained. Thus, if a convex lens of $7\frac{1}{2}$ in. focal length is cemented to a concave lens of 30 in. focal length, the focal length of the combination is given by

$$\frac{1}{f} = \frac{1}{7\frac{1}{2}} - \frac{1}{30},$$

whence *f* = +10 in., the resulting lens being converging.

Conjugate Foci.—If rays of light proceeding from a point *P* on the principal axis are refracted through a lens, the emergent rays pass through another point *P'*, also on the principal axis (Fig. 6). The object point *P* and the image *P'* are reciprocal, and are termed conjugate foci. If the distances from *O* to *P* and *P'* are denoted by f_1 and f_2 respectively, it may be shown that

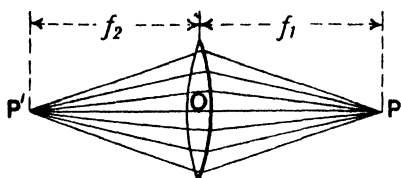


FIG. 6.

$$\frac{1}{f_1} + \frac{1}{f_2} = \frac{1}{f},$$

provided the following sign convention is observed. The focal length of a convex lens is positive, and that of a concave lens is negative. A distance f_1 from the optical centre of the lens to the source *P*, measured in the direction opposed to that of travel of the light, is regarded as positive; but if measured in the direction of the light, it is negative. A distance f_2 from the optical centre of the lens to the image *P'*, measured in the direction of travel of the light, is positive; if measured against the direction of the light, it is negative. The object distance f_1 is negative only in the case where the lens receives rays converging to a point *P* behind the lens, *P* belonging to a virtual object.

Notes.—(1) The above equation for conjugate focal distances is applicable only to cases in which the thickness of the lens is sufficiently small to be negligible in comparison with the conjugate distances, but it is sufficient for our requirements. The equation does not apply to conjugate focal distances along secondary axes.

(2) The reader may accustom himself to the sign convention by verifying the following positions of the image for the given positions of an object point in the cases of lenses of 10-in. focal length. When the object point is behind the lens, it is to be understood that the rays incident upon the lens converge towards that point, but do not actually pass through it.

Object Point at	Convex Lens. Image at	Concave Lens. Image at
Infinity in front	10 in. behind	10 in. in front.
20 in. „	20 in. „	6.7 in. „
10 in. „	Infinity	5 in. „
10 in. behind	5 in. behind	Infinity
20 in. „	6.7 in. „	20 in. in front
Infinity „	10 in. „	10 in. „

The Image of a Body.—Considered optically, a body is simply a collection of points. Cones of rays proceeding from those points, if incident on a perfect lens along any axis, are so refracted that they are brought to a focus, thereby forming an image of the point and in the aggregate an image of the body. To determine the position of the image of any point on the body, it is evidently sufficient to trace the paths of any two rays from it to their inter-

section after refraction. The two rays most easily located are those which pass through the optical centre and the principal focus respectively. These are used in the case of Fig. 7, which illustrates the formation by a convex lens of a real inverted image of an object situated at a distance from O greater than f . The distances f_1 and f_2 along the principal axis are again related by the equation for conjugate focal distances, and it is evident, from the similarity of triangles Oab and OAB, that the ratio of the size of the image to that of the object is f_2/f_1 .

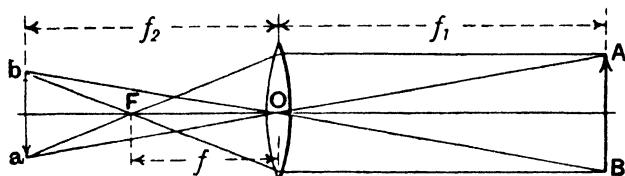


FIG. 7.

Defects of a Single Lens.—A simple lens is found to have several inherent imperfections affecting the formation of the image, and it is necessary to consider the means adopted to reduce these defects in order to understand the construction of the telescope. Some of the errors to which lenses are subject need not be emphasised as, although their correction is of considerable importance in the photographic lens, they are of little consequence in the surveying telescope, the aim of which is to afford a good image of the central portion only of the field of view.

The principal defects are: (a) Chromatic Aberration; (b) Spherical Aberration; (c) Coma; (d) Astigmatism; (e) Curvature of Field; (f) Distortion. The first two are by far the most important, since they affect the sharpness of the image at the principal axis. The others do not influence the image at the centre of the field, but are included on account of their possible effect in stadia tacheometry and in certain astronomical observations in which the image of a star is observed outside the principal axis of the lens system of the telescope.

Chromatic Aberration.—White light may be regarded as composed of a series of undulations, each of different wave length. Difference of wave length corresponds to difference of colour and also of refrangibility, and, as a consequence, a ray of white light after refraction at a single surface has its components separated from each other. The visible rays which have the shortest wave length and greatest refrangibility are those of a violet colour, and wave length increases, and refrangibility decreases, through the range of the colours of the spectrum—violet, blue, green, yellow, orange, and red.

Chromatic aberration is that defect of a lens whereby rays of white light proceeding from a point are each dispersed into their

components and conveyed to various foci, forming a blurred and coloured image. In Fig. 8, which illustrates the dispersion produced

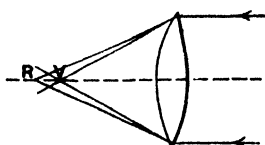


FIG. 8.

by a simple convex lens, it will be seen that a sharp image of the radiant point is nowhere formed. A screen placed at V receives an image surrounded by a halo of all the colours of the spectrum and bounded by a red fringe, and on removal of the screen to R the halo has a violet margin. If the rays of white light, instead of proceeding from a point, emanate from a white body, the middle portion of the image of the body is white, because the individual colours are recombined there, and the chromatic effects are seen only at the boundary of the image.

The circumstance that in different varieties of glass dispersive power bears no relation to refractive power renders the use of compound lenses available as a means of correcting the defect. Two kinds of glass are employed, crown and flint, both of which are manufactured in a considerable range of optical qualities. Flint glass has slightly the greater refracting power, but its dispersive power is roughly double that of crown. By combining a crown glass convex lens with a concave lens of flint glass, as in telescope objectives (Fig. 14), the proportions are arranged not only to yield the required focal length, but also so that the dispersion produced by the convex lens is neutralised at emergence from the concave.

Practically, however, complete achromatism cannot be achieved by the use of only two kinds of glass, since the ratio of their dispersive powers varies at different parts of the spectrum. A certain amount of residual colour, or secondary spectrum, is unavoidable, but does not have a serious effect on the sharpness of the image. A lens which is corrected for two colours of the spectrum is said to be achromatic, although it is only partly so.

In eyepieces the correction for chromatic aberration is commonly made without compound lenses by the use of two plano-convex glasses of such proportions, and at such a distance apart, that the dispersion produced at the first is eliminated by the second.

It can be shown that, if a combination of two thin lenses, of focal lengths f_1 and f_2 and separated by a distance d , is to be achromatic, d must satisfy the relation :

$$d = \frac{f_1 + f_2}{2}$$

As d is essentially a positive quantity, one or both of the lenses must be convergent.

Spherical Aberration.—Spherical aberration is a defect whereby the component rays of a beam proceeding from a point on the principal axis are not refracted to pass through a single point, but are differently focussed according to their positions of incidence

on the lens. It arises from the use of spherical surfaces, which are generally adopted because of the ease with which they may be accurately produced.

Fig. 9 illustrates spherical aberration in a convex lens, and shows that rays passing through the margin are brought to a focus nearer the lens than are the central rays, so that the image formed is not sharp. In this case the aberration is regarded as positive, that of a concave lens being of opposite effect or negative. The relative amount of error present in lenses of a given power is largely dependent upon the relationship between the curvatures of the two surfaces and upon whether that having the larger or the smaller curvature receives the incident rays. The surface of smaller radius should face the more nearly parallel rays. Thus, if parallel light is incident upon the curved surface of a plano-convex lens, the aberration produced is only about a quarter of what it would be if the plane surface faced the source of light.

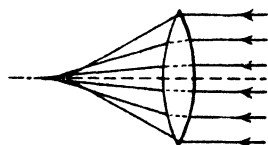


FIG. 9.

Spherical aberration may be reduced to any required extent by diminishing the aperture so as to cut out the marginal rays, but this has the serious disadvantage of diminishing the brightness of the image. The correction is actually effected by replacing the simple lens by a combination of two lenses such that the positive aberration of one is neutralised by the negative aberration of the other. The combination may consist of a convex lens in contact with a concave, as in telescope objectives, or of two plano-convex lenses placed at a definite distance apart, as in eyepieces. A lens or combination of lenses in which the defect is eliminated is said to be *aplanatic*.

The theoretical condition to be satisfied in order that a combination of two thin lenses, of focal lengths f_1 and f_2 , and separated by a distance d , should be aplanatic is given by :

$$d = f_2 - f_1.$$

Hence, if the combination is to be both achromatic and aplanatic, we must have $d = \frac{2}{3}f_2 = 2f_1$. These are the proportions used in the Huyghens' telescopic eyepiece. This eyepiece, however, is not generally used with telescopes for measuring instruments because, for one reason, although it corrects the image formed by the objective for both chromatic and spherical aberrations, it does not correct the image of the diaphragm, as this has to be put between the two lenses and is thus only viewed through one of them, with the consequence that its image is distorted. This would be a serious objection if stadia hairs or any sort of measuring scale formed part of the diaphragm.

Coma.—The spherical aberration described above refers to any image produced on the principal axis, and may be distinguished as *axial spherical aberration*. The remaining defects to be noticed result

from the spherical aberration of oblique pencils of light, the axes of which are secondary axes of the lens.

Rays emanating from a point on a secondary axis do not fall upon the lens in the symmetrical manner of an axial pencil. In consequence, the resulting image is neither sharp, nor has its confused outline a circular form as in axial aberration. *The image of a point source of light takes various forms, such as pear- and comet-shaped, with an axis of symmetry directed towards the principal axis of the lens. This effect of oblique spherical aberration is known as coma.*

The defect impairs the sharpness of an image away from the principal axis. It may be reduced considerably by a moderate decrease in the aperture of the lens, but in the surveying telescope the resulting loss of brightness would be more objectionable than imperfection of the image towards the margin of the field.

Astigmatism.—This effect of oblique spherical aberration is caused by the lens refracting the rays from an extra-axial point so that, instead of passing through a focal point, they pass through a focal line. The directions of the refracted rays are such that, on travelling onwards, they pass through a second focal line at right angles to the first and radial to the principal axis. Between these two lines the best image of the point is obtained as a circular disc.

If the object consists of a sheet of paper on which are ruled two lines, one radial and the other tangential, a satisfactory image of one of the lines will be received on a screen placed at one of the focal lines, but the image of the other object line will be confused. On moving the screen to the second focal line, the lack of definition applies to the image of the first object line. Astigmatism therefore prevents sharp definition in all directions simultaneously. Its correction in a system corrected for axial aberration necessitates the use of at least three component lenses. A lens combination corrected in this respect is said to be anastigmatic.

Curvature of Field.—Curvature is a further effect produced by the spherical aberration of oblique rays whereby the image of a plane surface normal to the principal axis is formed as a curved surface. When received on a plane screen, such an image is not equally sharp all over but, according to the position of the screen will be distinct at the centre or the margin or along an intermediate circle. The two sets of line foci, as well as the discs of least confusion, formed by a lens uncorrected for astigmatism really lie upon curved surfaces, but curvature may also be present in the anastigmatic lens.

If the object glass or eyepiece of a telescope possessed the defect in a marked degree, it would become necessary to adjust the focus to obtain a clear view of the different parts of the field. Curvature may be reduced by the use of a diaphragm or by means of two lenses placed at a suitable distance apart, as in the eyepiece of the surveying telescope.

Distortion.—Distortion is the defect, arising from the same cause

as the last, whereby straight lines on an object are not reproduced as straight lines in the image. It is always present in single lenses and in achromatic combinations of two lenses.

The error can be reduced by covering the margin of the lens by a

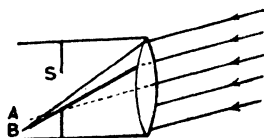


FIG. 10.

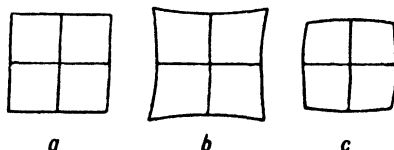


FIG. 11.

diaphragm placed on the surface of the glass, but it is accentuated if the stop is at some distance from the lens. The action of a stop set at a distance from the lens is illustrated in Fig. 10, which shows that rays passing through or near the centre of the lens may be intercepted by the stop *S* and prevented from contributing to the formation of the image at its true position *A*, while the marginal rays pass through the aperture of *S*, and, because of their greater refraction, cause the image of the point to be formed at *B*. If the object is rectilinear in outline (Fig. 11*a*), the sides of the image will in these circumstances be convex inwards, giving rise to what is termed pin-cushion distortion (*b*). The opposite effect is obtained when the stop is placed in front of the lens, and barrel distortion (*c*) is then produced. Since the defect does not influence the centre of the field, and is therefore of minor importance in the measuring telescope, the use of two compound lenses with a stop between is not warranted in the telescope objective as it is in the rectilinear photographic lens.

Types of Telescopes.—Considered in its simplest form, a refracting telescope consists optically of two lenses, the principal axes of which

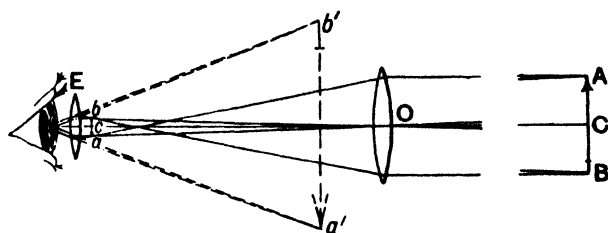


FIG. 12.

coincide to form the optical axis of the telescope. The lens nearer the body to be viewed is convex, and is termed the object glass or objective, the function of which is to collect a portion of the light emanating from the body. The rays transmitted by it, on passing onwards, suffer a second refraction at the other lens, called the

eyepiece or ocular, and are then suitably presented to the eye. According to the form of the eyepiece and its position relatively to the object glass, a telescope may belong to one or other of two main types.

In *Kepler's* or the *Astronomical* telescope (Fig. 12), rays from the object AB are, after refraction at the objective O, brought to a focus before they enter the eyepiece E, and, in consequence, a real inverted image ba is formed in front of the eyepiece. If this lens is so placed that ba is situated within its focal length, the rays after refraction at E appear to the eye to proceed from $b'a'$, a virtual image conjugate to ba . The object AB thus appears magnified, inverted, and placed at $b'a'$.

In *Galileo's* telescope (Fig. 13) the rays refracted by the objective O are intercepted by a concave eyepiece E before a real image is formed. On entering the eye, they therefore appear to diverge from the virtual image ab , which is magnified and erect.

Both these arrangements meet one of the requirements of a telescope for use in surveying, since they afford distinct vision of distant objects, and from this point of view Galileo's telescope has the advantage that the image is erect. But the telescope, as an adjunct to a measuring instrument, must also be capable of furnishing a definite line of sight, and it will be shown that only the Kepler telescope fulfils this condition.

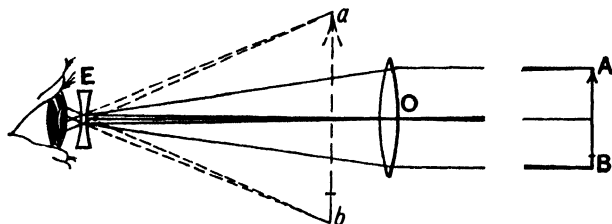


FIG. 13.

The Line of Sight and Line of Collimation.—By a line of sight of a telescope is meant any line passing through the optical centre of the objective, traversing the eyepiece, and entering the eye. The line of sight, or line of collimation, is one such line further and visibly defined as passing through a point marked by the intersection of cross-threads stretched in front of the eyepiece in a plane at right angles to the axis.

In Fig. 12, let it be supposed that these threads are situated in the plane of the image ba with their intersection at the point c on the optical axis. The line of sight cO meets the body in a certain point C , the real image of which is formed at c on ba , and therefore the intersection of the hairs coincides with the real image of that particular point on the object focussed which is situated on the line of collimation. Now, on viewing the image ba through the eyepiece, the

cross-hairs are simultaneously seen focussed in $b'a'$, and the coincidence of the plane of the hairs with that of the real image ensures that the observed position of the hairs relatively to points on the virtual image $b'a'$ is the same as on the real image. The hairs and the image ba are equally magnified, and distortion or other defect produced in the passage of the rays through the eyepiece affects both to the same degree. The observer consequently sees the intersection of the hairs apparently coinciding with that point on the object which lies in the line of sight.

The establishment of a telescopic line of sight therefore involves two essential conditions :

(a) A real image must be formed in front of the eyepiece.

(b) The plane of the image must coincide with that of the cross-hairs (see Parallax, page 28).

In Galileo's telescope no real image is produced, and it is therefore useless for quantitative observations. Cross-hairs are sometimes provided in such telescopes but they serve merely to indicate the centre of the field. Galileo's telescope may therefore be regarded as a viewing, as distinct from a measuring, instrument. It is adapted for field glasses, and is used in sextants to facilitate the sighting of distant points.

The Surveying Telescope.—The telescope of a surveying instrument is therefore of the Kepler type, and the construction of the external-focussing pattern is illustrated in Fig. 14. The body is formed of two tubes, one capable of sliding axially within the other by means of a rack gearing with a pinion attached to the milled focussing screw. It is essential that the movement should be strictly axial and free from shake.

The objective, the focal length of which serves to designate the size of the telescope, is invariably a compound lens, of which the outer component is a double convex lens of crown glass and

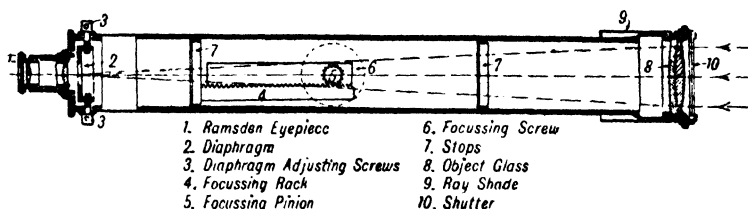


FIG. 14.—EXTERNAL FOCUSSING TELESCOPE.

the inner is of flint glass and convexo-concave. In the example illustrated, the objective cell is screwed to the inner tube, so that the focussing movement, required for placing the real image in the plane of the cross-hairs, is effected by movement of the objective relatively to the outer fixed tube carrying the cross-hairs and eyepiece ; but in cases where the objective is mounted on the outer tube the eye end moves in focussing.

In the majority of modern surveying telescopes focussing is effected without movement of either the objective or eyepiece by the introduction between them of a concave lens // (Fig. 15) which

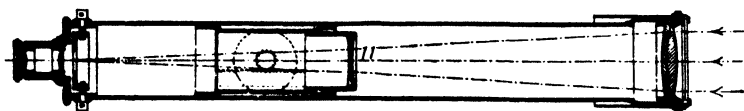


FIG. 15.—INTERNAL FOCUSING TELESCOPE.

is moved by the focussing screw to bring the image into the plane of the cross-hairs.

To examine the effect of the introduction of the internal focussing lens, let O and I (Fig. 16) be the optical centres of the object glass and internal lens, and let f and f' be their respective focal lengths. The distance x between the lens centres is variable, and for any observation the internal lens must be adjusted into such a position that the image of the distant object is formed in the plane of the cross-hairs at P_1 , which is at a constant distance l from O .

Considering the formation of the image of an object P distant f' from the object glass, the latter would itself form an image at P' , the position of which is obtained from the equation for conjugate focal distances,

$$\frac{1}{f_1} + \frac{1}{f_2} = \frac{1}{f}.$$

The rays after refraction by the object glass are, however, intercepted by the concave lens as they are converging towards P' , and

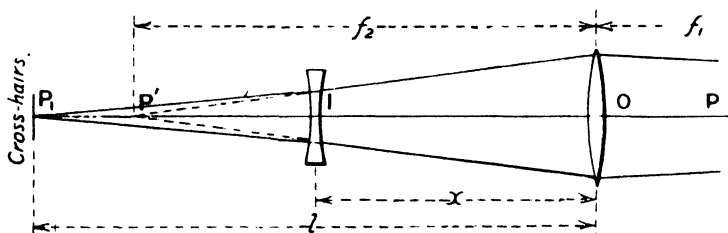


FIG. 16.

P' is the position of a virtual object, the image of which is formed by the internal lens at P_1 . The distances IP' and IP_1 are therefore conjugate, so that

$$\frac{1}{(f_2-x)} + \frac{1}{(l-x)} = \frac{1}{f'},$$

(f_2-x) and f' being negative, according to the sign convention of page 14.

These two conjugate focal equations show the required distance

x between the lenses, if l , f , and f' are fixed. If the object is infinitely distant, f_2 equals f , and x has its minimum value. The amount of movement of the internal lens towards the eyepiece required to focus a near object is greater for a low power focussing lens than for one of high power. The focussing lens may be a simple one, as shown, or it may consist of an achromatic combination.

A telescope with internal focussing is more likely to be proof against the entrance of water and dust than one with a draw tube. The chief merit of the system, however, lies in the avoidance of errors caused by change of position of the line of sight in focussing with an objective or eyepiece slide which has developed looseness through wear. If the movement of an internal focussing slide becomes non-axial, the resulting error in sighting is very much less. The disadvantage of internal focussing is the reduction in brilliancy of the image caused by the passage of the rays through the additional lens.

With the object of improving the distinctness of the image in both types of telescope, the interior of the body is painted dull black, to prevent reflection from the internal surfaces, and annular stops may be fitted to intercept rays not required in the formation of the image. In inferior instruments the front stop may be found placed near the objective for the purpose of diminishing spherical aberration, but the reduction of aperture thereby occasioned is a serious defect. To shield the objective from the rays of the sun, there is fitted a sun cap or shade capable of being extended when required. The shade is closed by a hinged shutter, which serves to protect the lens against rain and dust.

The end of the body remote from the objective is threaded to receive the tube in which the eyepiece is fitted to move axially for the purpose of focussing the cross-hairs (page 27). A cap to protect the eyepiece is useful, but is commonly omitted. For making solar observations, a sun glass is required: this consists of a piece of very dark glass mounted in a rim which can be fitted on the outside of the eyepiece. In front of the eyepiece is mounted the diaphragm, across which are stretched the cross-threads. The diaphragm and eyepiece call for further remark.

The Diaphragm and Reticule.—The diaphragm carrying the reticule, or graticule, of cross-lines is formed of a flanged brass ring held in place in the telescope tube by four capstan-headed screws, as shown in Fig. 17. These are screwed into the flange, but pass through slots in the telescope tube, so that, when slackened, they permit movement of the diaphragm horizontally and vertically as well as a small rotation about the axis of the telescope. The heads of the screws,

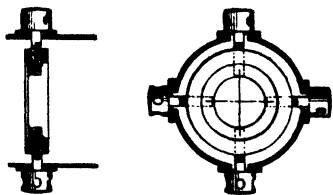


FIG. 17.—DIAPHRAGM.

instead of being exposed, may be protected by a cover to prevent their being turned accidentally.

In the interchangeable pattern of diaphragm, which is now generally fitted, the cross-hairs, instead of being mounted on the diaphragm ring itself, are carried by a cylindrical cell which fits into the ring (Fig. 19). The object of the design is to facilitate the rapid replacement of a defective reticule by the withdrawal of the cell and the insertion of a spare one in the manner described on page 26.

The cross-hairs, being viewed through the eyepiece, appear magnified, and it is therefore essential that they should be very fine. They should show as clean lines without fringes, and should be opaque. Various means are employed for providing suitable lines, and each has its merits and deficiencies.

Spider Webs.—These are most commonly used, and furnish a very fine, clean, and sufficiently opaque line. The most suitable webs are the darkest coloured ones of which the garden spider makes its nest. Although in moderate climates such webs should last for several years, excessive damp may cause them to develop slackness and sag, particularly if they have not been properly stretched when being mounted on the diaphragm. As most webs tend to lose their elasticity with age, extreme dryness may cause them to break by tension. Owing to their delicacy, spider webs must never be exposed outdoors by removal of the eyepiece for any purpose.

Silk Webs.—Single filaments of silk furnish somewhat similar, but coarser, lines. They are, however, sufficiently fine for use with low powers, and are rather less troublesome to insert than spider webs.

Lines on Glass.—Lines ruled upon a thin glass plate afford the least fragile reticule, and have the advantage that the relative position of the lines is permanent. The ruling can be performed so that the lines appear little thicker than webs. The plate, however absorbs a small amount of light, and is subject to troublesome dewing with a falling temperature.

Points.—The desirability of eliminating fragility without the necessity of inserting a glass diaphragm has led to the use of small pointers of platinum-iridium alloy, the fine ends of which fulfil the office of a hair. They are very durable, and are not liable to be damaged, but, if they happen to meet with injury, they cannot be repaired in the field. Points are quite convenient for ordinary theodolite work, but are useless for astronomical observations. In staff reading they involve a considerable waste of time owing to the necessity of bringing the extreme end of the horizontal point exactly to the edge of the graduations of the staff, an operation which demands the provision of a clamp and tangent screw on the instrument.

A few typical arrangements of lines and points are illustrated in Fig. 18.

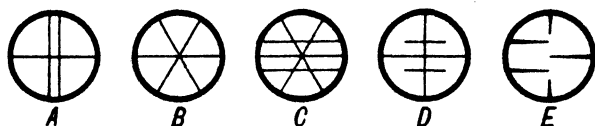


FIG. 18.

A shows a common system for levels. The horizontal hair defines the reading, and the vertical hairs enable the observer to detect lateral deviation of the staff from its required verticality. This type of diaphragm, with or without stadia hairs, is also now commonly used in micrometer or other theodolites where fine pointings on distant objects are necessary, the two vertical wires in this case being very close together so that the image of the object appears to be centrally between them. This arrangement of the hairs has the advantage that the object is clearly seen between them and, when it appears as a very fine line, does not appear to be covered by them. A very similar type of diaphragm is used in micrometer microscopes. (See Figs. 65 and 66.)

B is used in theodolites. The two inclined hairs are usually preferred to a single vertical hair for ease in bisecting the signal. In a variation of this pattern the diagonal hairs intersect a little above or below the horizontal hair, so that their intersection may be seen more distinctly.

C is the same with the addition of stadia hairs for tacheometry.

D is used both in levels and theodolites. The stadia lines do not extend right across the diaphragm, indicating that the lines are ruled on glass. Two closely spaced vertical lines or hairs are better than a single vertical line if the telescope is to be used for sighting plumb lines, since accurate pointings can be obtained by making the image of the plumb line bisect the narrow space between the lines.

E shows the usual arrangement of points for levels and theodolites, stadia points being fitted. The end of the horizontal point defines the line of sight, and is in line with the vertical points.

Replacing Cross-Hairs.—To replace broken hairs, first remove the diaphragm by loosening the capstan screws, and place it on a flat surface with that side uppermost which is provided with scratches for receiving the hairs. Clean off old cementing material adhering to these notches. Suitable webs should be selected from a spider's cocoon, failing which, if a spider can be found, procure a V-shaped piece of wire or stick, and get the spider on the end of one of the prongs. On shaking him off, he will spin a web, which is to be wound on the fork so that the separate strands are not too close. Immerse each web to be mounted for a few minutes in warm water, and

then gently remove the superfluous moisture with a colour brush. Stretch the web across the ring, using small weights to ensure tightness, and by means of a pin adjust it to lie exactly in the

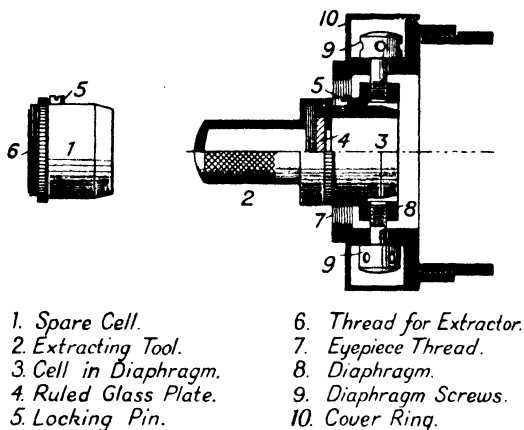


FIG. 19.—INTERCHANGEABLE DIAPHRAGM.

notches. Fix it by letting a drop of shellac fall on one end, and, when this has hardened, see that the web remains taut before similarly fixing the other end. If spider webs cannot be procured, floss silk forms a convenient substitute, as it is readily obtainable in reels. The operation of mounting is the same, except that the silk need not be moistened.

When the diaphragm is replaced in the telescope tube and screwed up, it is most unlikely that the intersection of the hairs will occupy the same position relatively to the optical axis as it did before. The position of the intersection of the hairs controls the position of the line of collimation in the telescope, and it is therefore essential to adjust the line of collimation by the methods to be described in connection with the adjustment of instruments.

The delay occasioned by the breakage of webs in the field becomes quite insignificant if the instrument is provided with an interchangeable diaphragm, a spare cell, fitted either with webs or a ruled glass plate, being carried. In order to extract a cell, the eyepiece is removed, and an extracting tool is screwed on to the projecting thread of the cell, which can then be withdrawn from the diaphragm carried by the capstan-headed screws. Fig. 19 shows the extracting tool in position. The spare cell to be inserted is screwed to the extractor, by which it is pushed into position. It is prevented by a pin from turning in the diaphragm ring while the tool is being unscrewed.

Interchangeable cells, as they leave the maker, are designed to have the intersection of the cross lines correctly centred, so that the

position of the line of collimation will be unaltered by the substitution of one cell for another. It is, however, wise to test the telescope after the operation, and it becomes absolutely necessary to do so if the spare cell has been re-webbed by the surveyor himself.

The Ramsden Eyepiece.—The type of eyepiece almost exclusively adopted in this country for surveying telescopes is the Ramsden or positive eyepiece (Fig. 20). It consists of two equal plano-convex lenses, called the field lens and eye lens respectively, placed with their curved surfaces facing each other at a distance between optical centres of two-thirds the focal length of either. The eyepiece does not invert, so that, on observing the real inverted image formed in the plane of the cross-hairs, the virtual image of the object sighted is also inverted. The Ramsden eyepiece, while not strictly achromatic, is very free from spherical aberration and curvature of field.

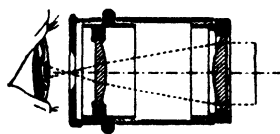


FIG. 20.
RAMSDEN EYEPIECE.

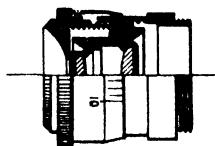


FIG. 21.
SCREW FOCUSING
RAMSDEN EYEPIECE.

The eyepiece of Fig. 20 is held by friction in a tube projecting from the end of the telescope, and the movement required for focussing the cross-hairs is effected by sliding the eyepiece out or in with a turning motion. Screw focussing eyepieces (Fig. 21) are regularly fitted in modern instruments, and give a much smoother movement. In this case, focussing is performed by turning the milled ring encircling the eyepiece. The ring carries a scale, which is read against a fixed index, so that the observer, having once ascertained the focus which suits his eyesight, can readily set the eyepiece for distinct vision of the hairs. In this eyepiece the cross-hairs and the image formed by the objective are in front of the field lens.

With the object of reducing chromatic aberration, modifications of the Ramsden eyepiece are sometimes employed. In the Kellner eyepiece the eye lens is a compound lens of crown and flint glass similar to the objective, and the field lens is a simple double convex glass. In the Steinheil eyepiece both lenses are compound.

The Erecting Eyepiece.—An erect image may be obtained by means of an eyepiece consisting of four lenses, which are arranged

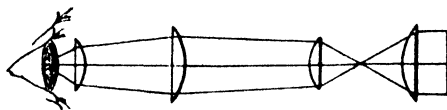
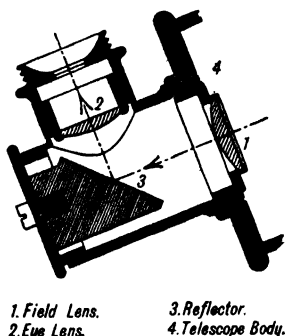


FIG. 22.—ERECTING EYEPIECE.

as in Fig. 22. Objections to the erecting eyepiece are the loss of light occasioned by the passage of the rays through the two additional lenses, and that for a given overall length of telescope the focal length of the objective must be smaller with an erecting than with a Ramsden eyepiece. Particularly on account of its better illumination, the Ramsden eyepiece is to be preferred, any inconvenience arising from the inversion of the image being negligible.

The Huygens Eyepiece.—This eyepiece is used only in the Galilean telescope in place of the concave eye lens shown in Fig. 13. It consists of two plano convex lenses with their convex faces towards the objective. The focal length of the field lens is three times that of the eye lens, and the distance between them is the difference of their focal lengths. The eyepiece must be placed to receive the rays from the objective before a real image is formed, and the virtual image presented to the eye is erect. As pointed out on page 17, Huyghens' eyepiece is not used in surveying instruments such as theodolites and levels.



1. Field Lens.
2. Eye Lens.
3. Reflector.
4. Telescope Body.

FIG. 23.
DIAGONAL EYEPIECE.

The Diagonal Eyepiece.—When the inclination of the telescope to the horizontal is great, it becomes inconvenient to bring the eye into position for sighting, and a diagonal eyepiece is required. That of Fig. 23 is of the Ramsden type. Between the two lenses there is fitted a reflecting prism or a glass or metal mirror placed at 45° to the telescope axis. This reflector turns the line of sight through 90° , so that the observer is enabled to look into the telescope in a direction at right angles to its axis. In consequence of the reflection, the image is presented right side up, but the right- and left-hand sides are still reversed. The diagonal eyepiece is a necessity for astronomical work, and often proves useful for terrestrial observations in precipitous country and in constructional work.

Parallax.—Parallax is a condition arising when the image formed by the objective is not situated in the plane of the cross-hairs. It is evidenced when movement of the observer's eye apparently produces movement of the image relatively to the hairs, the direction of the apparent movement of the image being the same as, or opposite to, that of the eye, according as the image is on the side of the hairs remote from or next to the eye. Unless parallax is eliminated, accurate sighting is impossible, as the line of sight may apparently be made to intersect different points according to the position of the eye.

Elimination of Parallax.—The two steps are: (1) to focus the

eyepiece for distinct vision of the cross-hairs ; (2) to bring the image of the object into their plane.

(1) Turn the focussing screw until no object can be distinguished in the field, or point the telescope to the sky. Adjust the eyepiece until the cross-hairs appear in sharp focus.

(2) Point the telescope towards the object, and, keeping the eye on the cross-hairs, turn the focussing screw until the image appears in sharp focus. The image and hairs should then be in the same plane, but the eye should be moved about to test whether any relative movement between them can be observed. If necessary, adjust the focussing screw until the apparent movement is eliminated.

Notes.—(1) To test the focussing of the eyepiece, turn aside and look at a distant point, or close the eyes for a moment. On again looking into the eyepiece, the hairs should at once appear perfectly distinct without the eye having to accommodate itself.

(2) The position of the eyepiece for distinct vision of the hairs depends only upon the individual observer's eyesight, and, when once found, should be permanently marked.

Qualities of the Telescope.—In addition to its important function of providing a definite line of collimation, the telescope is required to present a distinct view of distant objects, so that the observer may be able to direct the line of collimation with sufficient accuracy on to a distant mark or read its position on a graduated rod. The unassisted eye is unable to distinguish clearly between separate marks unless the angle subtended between them at the eye is from 1 to 2 minutes. Such capacity for distinguishing detail, or resolving power, as it is called, is quite insufficient for the majority of surveying operations, so that a telescope is required to assist vision.

The resolving power of the eye varies considerably in different individuals ; even in the normal eye it is not constant, but depends upon the various circumstances of the observation. Taking the case of viewing a graduated rod, the normal eye should, under average conditions, be able to distinguish clearly marks 0·01 ft. apart at a distance of 20 ft., the angle subtended at the eye being $\frac{1}{2000}$ radian. Now, if by means of a telescope it is desired to distinguish with equal clearness the same marks at a distance of, say, 500 ft., the angle subtended at the instrument is reduced to $\frac{1}{500}$ of its former value. The telescope is required to have a resolving power 25 times greater than that of the unassisted normal eye, and this ratio may be regarded as the resolving power or resolution of the telescope.

The performance of one surveying telescope may differ from that of another in several important respects, due to differences in manufacture and in the proportions between the parts. The most important qualities are : (1) Definition ; (2) Brightness of Image, or Illumination ; (3) Magnification ; (4) Size of Field.

Definition.—The quality of definition in a telescope is its capability of producing a sharp image. It depends upon the extent

to which spherical, chromatic, and other aberrations have been eliminated in the objective and eyepiece, as well as upon the accuracy with which all the lenses have been centered on one axis. Good definition, particularly in the axial region, is obviously a desirable attribute, and the resolving power of the telescope is chiefly dependent upon it.

Brightness of Image.—The quantity of light available for the formation of the image depends upon the amount admitted at the objective and upon the loss caused by the passage of the rays through the several lenses. The first depends upon the area of aperture of the objective, and therefore varies as the square of the diameter. Loss amounting to 15 to 25% of the light falling on the objective is occasioned by reflection and diffusion at the surfaces of the lenses and by absorption due to imperfect transparency of the glass.

In forming the magnified image, the light rays are spread out to a degree depending upon the magnification, and the intensity of illumination of the image is inversely proportional to the square of the linear magnification. The brightness of the image as judged by the observer is largely dependent upon the diameter of the pupil of the eye, and the best results are attained when the diameter of the beam emerging from the eyepiece—the exit pupil—is the same as that of the pupil of the eye.

Brightness of image is always desirable in the surveying telescope, especially for work in a bad light. In general, the brighter the image, the greater is the resolution.

Magnification.—Magnification is the ratio between the angle subtended at the eye by the virtual image and that subtended by the object. Varying position of the object causes a slight variation in this ratio, and the magnifying power of a telescope is measured as the ratio of the focal length of the objective to that of the eyepiece.

Since any focal length of eyepiece can be used in conjunction with a given objective, the magnifying power of a telescope can be increased indefinitely. Resolving power is promoted by increase of magnification, but the magnification should not be greater than is required to give a resolving power compatible with the sensitiveness of the instrument generally (page 92). The provision of a greater power than is strictly necessary is a disadvantage since it entails: (1) reduction of brilliancy of image; (2) waste of time in focussing; (3) reduction of the field of view. To obviate the loss of brilliancy accompanying large magnification, a larger object glass should be used for a high than for a low magnification. Although, within limits, the same resolving power may be secured by a high magnification, producing a dull image, as by a lower magnification and a bright image, the latter is usually preferable. The magnifying power of telescopes on surveying instruments for ordinary work varies from 10 to 30, and reaches 80 in precise instruments. Some-

times two eyepieces of different powers are supplied with an instrument.

Size of Field.—By the field of view is meant the whole extent visible at one time through the telescope. It is measured by angle COD=AOB (Fig. 24), the apical angle formed at the optical centre

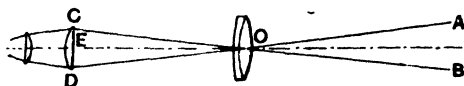


FIG. 24.

of the objective by the cone of rays refracted to enter the eye, and this angle is evidently that subtended at the optical centre of the objective by a diameter of the field lens of the eyepiece. Fig. 24 shows that the angle of field : (1) increases as the diameter of the field lens increases ; (2) decreases as the distance between the objective and eyepiece increases, so that it is smaller for short sights than long ones ; (3) is independent of the size of the objective. The size of field also decreases as the magnification increases, for if m = the magnification =

$$\begin{aligned} \frac{\text{focal length of objective}}{\text{focal length of eyepiece}} &= \frac{F}{f}, \text{ then} \\ \text{angle of field} &= 2 \tan^{-1} \frac{CE}{OE} = 2 \tan^{-1} \frac{CE}{F+f}, \text{ practically,} \\ &= 2 \tan^{-1} \frac{CE}{f \left(\frac{F}{f} + 1 \right)} = 2 \tan^{-1} \frac{CE}{f(m+1)}. \end{aligned}$$

A wide field is desirable on account of the facility with which the observer can discover distant objects prior to exact pointing of the telescope. In instruments having magnifying powers up to 30 a field of from 1° to 2° is suitable.

Testing the Telescope.—Looking to the importance of some of the properties of the telescope, it is desirable that the surveyor should be able to ascertain the optical qualities of his instruments. The tests described below are sufficient for this purpose.

Test for Chromatic Aberration.—Point the telescope on a brightly illuminated disc of white paper or the edge of a dark object appearing against the sky. The extent to which residual aberration is present is evidenced by the amount of tinge appearing at the edge.

Test for Spherical Aberration.—Prepare a disc of black paper the size of the aperture of the objective, and cut out a second disc of about half the diameter from its centre, thus forming a disc and a ring. Cover the margin of the objective with the ring, and focus the telescope on a page of print. Without altering the focus, substitute the disc for the ring, affixing it centrally on the objective,

and note whether the image produced by the rays now passing through the margin is as sharp as before. If a change of focus is required, the distance through which the focussing slide must be moved is a measure of the defect.

Test for Definition.—Focus on a well-illuminated page of print placed 30 to 40 ft. away. It should be possible to obtain a sharp image. If otherwise, the fault may be due to inaccurate curvature or poor finish of the lenses or bad centering of the system. To test for the latter, cut a small disc of white paper with a sharp knife, and place it against a dark surface at the former distance. Focus sharply, and then, by slightly moving the focussing screw, throw the image out of focus. The disc will appear to be surrounded by a haze, which should have a uniform breadth all round. Defective centering is indicated if the haze is broader on one side than the other.

Test for Aperture.—To ascertain whether part only of the full aperture of the objective is used, prepare a small strip of strong paper about $\frac{1}{8}$ in. wide. Moisten and affix it radially on the objective. On placing the eye a little distance back from the eyepiece, the image of the effective aperture is seen as a small bright circle, and the image of the strip of paper will be seen on it. Draw the strip towards the margin of the objective, and, when its image reaches the circumference of the bright circle, fix the strip on the lens. The end of the paper marks the edge of the effective aperture, the diameter of which can then be obtained. Instead of viewing the small image directly, a second eyepiece or magnifying glass may be held against the telescope to receive and magnify the image.

Test for Illumination.—The only simple test available is that of comparing the telescope being examined with another which is regarded as satisfactory in this respect. Set the instruments side by side, and point each telescope towards some well-defined object. Sight through them alternately, and judge the relative brightness of the images. Preferably make the comparison towards nightfall, and note through which telescope the object remains visible longest as the light wanes.

Test for Magnification.—Focus a boldly graduated staff held about 100 ft from the instrument. Sight the staff through the telescope with one eye, and at the same time observe it with the other eye unaided by looking along the outside of the tube. Bring the magnified and direct views alongside each other, and count the number of staff divisions as seen directly which appear to occupy the same length as one of the magnified divisions. This number represents the magnification.

Test for Angle of Field.—Focus the telescope on a levelling staff held vertically or horizontally at a distance of about 250 ft., or such smaller distance that neither end of the staff appears in the field. Note the two extreme readings which can be observed.

The length of staff intercepted within the field of view divided by its distance from the instrument is, with sufficient precision, the average value of the angle of field expressed in circular measure.

Test for Distortion.—Bring the image of any straight line towards the edge of the field, and observe whether it appears straight or not. Otherwise, draw a diagram of bold lines forming squares, and pin it up at a distance such that on being viewed through the telescope it fills the field. Focus, and note whether the squares near the margin appear free from distortion.

Care of the Telescope.—Dirty lenses, especially those of the eyepiece, are a source of impaired brilliancy and definition. Dust should be removed from the surfaces by means of a small camel-hair brush. The presence of the finest film of grease is, however, more objectionable than dust, and, as this may arise from perspiration from the hands, care should be exercised to avoid fingering lenses. Very dirty lenses should be wiped with a piece of clean, soft rag or chamois leather moistened with alcohol, using a clean part on every portion of the lens to avoid scratching. Lenses should be rubbed as little as possible, hence the importance of protecting them against dirt and moisture (page 133). If the objective is unscrewed for the purpose of dusting the inside surface, it must be returned exactly to its original position. Should a film settle between the two component lenses which does not evaporate, it is unwise to separate the lenses, and the defect is best treated by the maker. Dust which has settled on the cross-hairs may be removed by taking out the eyepiece and blowing very gently on the webs, but, unless the dust particles are on or near the line of sight, they should preferably be allowed to remain.

Both the main focussing and eyepiece slides should work freely, but with sufficient friction to stay in place whatever the inclination of the telescope. A screw is sometimes provided for adjusting the friction of the main slide; otherwise, a little tallow smeared on the tube will help to make a loose slide stick. When it is detected that slackness gives rise to collimation error (page 93), the telescope should be sent to the maker for repair. Grit adhering to the object glass slide may cause fretting of the brass tube, and, when this occurs, it must be attended to as soon as possible, as the defect rapidly gets worse. The injured portion should be smoothed off with the back of a penknife blade. The use of oil on slides is a fruitful source of such trouble, and should be avoided.

It sometimes happens that coloured rings are seen through the telescope. These may be due to chromatic aberration or the chromatic effects of spherical aberration, but, if they appear suddenly as definite rings, and particularly if they appear to move when pressure is applied to one of the lenses, they may be caused by two lenses, which should be in contact, having come apart slightly so that there is a minute air space, of variable thickness, between

them. In that event, the rings, generally known as "Newton's Rings," are due to the interference of waves of light. Pressure on the lenses in their mounts may be sufficient to cure this defect, but, if not, they should be properly cemented or bound together by an instrument repairer.

One very troublesome fault in optical instruments that often occurs in the tropics is "fungus." This is due to a micro-organism which attacks the surface of soft glass. It causes a sort of frosting effect, and in time it may become so bad that it is almost impossible to see through the telescope. The trouble, after it has become well established, can usually only be removed by an instrument maker as it generally occurs between two surfaces that are cemented together. Unfortunately, once it has attacked a lens, fungus is very liable to come back soon after it has been removed. Up to very recently, there was no real cure for it, but some makers now think that they have a means of overcoming it. If this were not so, this trouble might well be a great disadvantage as regards the use in the tropics of modern types of theodolites in which the graduated circles are made of glass, instead of silver, and in which there is a complicated optical system, with numerous glass surfaces, for reading the micrometers. For this reason, it is well to inform the maker if an instrument of this type is being bought for use in the tropics, and to ask his advice with regard to protection against fungus.

THE LEVEL TUBE

Since nearly all field measurements, angular or linear, are made in either the horizontal or the vertical plane, it is essential to have some ready method of determining the positions of those planes through any point, so that particular lines or planes in an instrument may be made to lie in them. The most convenient and accurate means available for this purpose is the level tube, the action of which depends upon the fact that the surface of a still liquid, being at every point normal to the direction of gravity, is a level surface.

Construction.—The level consists of a glass tube, carefully ground so that a longitudinal section of the inside surface is a circular arc. The area so ground may be the upper half only or the whole of the interior surface. In the latter case, the figure is barrel-shaped, the cross section being circular throughout. This is the better form, since, owing to its symmetry, the level can register correctly although it is turned about its longitudinal axis, and it must be used in instruments in which the level is required to function when turned upside down.

The tube is nearly filled with a liquid, such as alcohol, chloroform, or sulphuric or petroleum ether, the remaining space, forming the bubble, being occupied by air and spirit vapour. These liquids are considerably less viscous than water; ether has the lowest coefficient of viscosity, and is the most suitable for sensitive levels. All these

liquids possess the advantage of having a low freezing point, but their coefficients of expansion exceed that of water, the length of the bubble being considerably reduced in hot weather. For the purpose of estimating when the bubble is central, a number of equally spaced lines is etched on the tube on either side of the centre or sometimes on an attached scale. The length of the divisions is usually either 2 mm. or 0.1 in.

The level is usually mounted in a brass enclosing tube, which is connected to some part of the instrument in a manner permitting its adjustment relatively to the supporting piece. The tube may, however, be entirely unattached, in which case the bottom is made a plane surface. It is employed in this form in conjunction with the plane table and in certain artificial horizons.

Principle of the Level Tube.—The *axis* of a level tube is the trace in longitudinal section of the plane tangential to the interior upper surface at the mid-point or zero of the graduations. In section (Fig. 25) the trace AB of the level liquid surface forms a chord of the circular arc representing the surface of the tube, the curvature being exaggerated in the diagram. A tangent to this arc at a point midway between the ends of the bubble is parallel to the chord, and is therefore a horizontal line irrespective of the position of the bubble in the tube. If the tube is tilted so that this mid-point is coincident with C, the centre of the graduation, the tangent at C coincides with the axis of the tube, which is now horizontal.

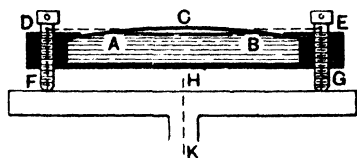


FIG. 25.

Let the tube be provided with two rigid supports in the same vertical plane with its axis, the lengths of which, measured from the axis, are DF and EG, and let these supports rest without attachment upon a plane surface FG. If it is desired that FG should be horizontal when the bubble is central, it is evidently necessary that $DF = EG$. In place of attempting to verify the equality of the supports by direct measurement, a test by reversal may be made by lifting the tube, turning it end for end, and replacing it in the reversed position. If the bubble preserves the same position as it occupied before reversal, line FG is horizontal, and $DF = EG$.

If not, let DF be the shorter, and let e denote the resulting angular error of parallelism between DE and FG. DE (Fig. 26) being levelled, DEFG represents the conditions before reversal, FG making an angle e with the horizontal. On reversing, the axis has the position D'E', and is now inclined at $2e$ to the horizontal. The apparent error is consequently twice the actual, and, in adjusting the supports to equality, it is therefore necessary to eliminate

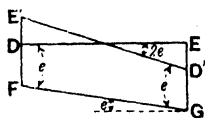


FIG. 26.

only half the apparent error by manipulating adjusting screws to bring the bubble half-way back to the centre. The adjusted tube can then be used to examine whether points on which it is supported are at the same level. In levelling a plane surface, the test must be made for two non-parallel positions of the tube, preferably at right angles to each other.

The bubble tube provides means for securing verticality as well as horizontality. Let the supports of the tube of Fig. 25 be now



FIG. 27.

screwed into the plate FG, which can be rotated about an axis HK, not necessarily in the plane of the tube axis. Reversal is now

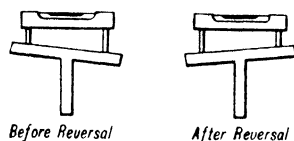


FIG. 28.

effected by rotating the plate and attached tube about HK through 180° . If the position of the bubble is unaltered by the reversal, HK is truly vertical. If not (Fig 27), by the same reasoning as before, only half the apparent error falls to be eliminated by adjusting the lengths of the sup-

ports. The adjustment of HK to verticality is secured when the bubble maintains one position during rotation.

It is frequently overlooked that this method of reversal does not test the horizontality of the plate FG, since, as in Fig. 28, FG may not be perpendicular to HK. The horizontality of a plate can be ensured only if the bubble is not displaced on removing the tube and replacing it end for end in two non-parallel positions.

✓ **Sensitiveness.**—By sensitiveness in a level tube is meant its capability of exhibiting small deviations from the horizontal. This quality depends primarily upon the radius of curvature of the internal surface—the larger the radius, the greater the sensitiveness. Sensitiveness is also promoted by increase in the diameter of the tube and in the length of the vapour bubble and by decrease of viscosity and surface tension of the liquid. A very smooth finish to the internal surface of the tube also adds appreciably to its sensitiveness.

Sensitiveness is sometimes designated in terms of the radius of curvature of the tube, but the better way is to state the angle through which the axis must be tilted to cause the bubble to travel a specified distance, usually that between adjacent graduations of the engraved scale.

The sensitiveness of a tube should accord with that of the instru-



INSTRUMENTS—CONSTRUCTION AND ADJUSTMENT 37

ment mounting it. If the sensitiveness is greater than is really necessary, time is wasted in levelling the instrument, and the accuracy of the observations is in no way increased.

Measurement of Sensitiveness.—*Laboratory Method.*—The apparatus, called a level trier, consists of a beam, 18 to 21 in. long supported on two points at one end and by a micrometer screw at the other. Rotation of this screw tilts the beam, the inclination being recorded by the micrometer, which reads to one or two seconds. Having placed the instrument on which the bubble is mounted, or the tube alone, on the beam, the movement of the bubble corresponding to an observed change of inclination is quickly determined.

Field Method.—If the tube is attached either directly or indirectly to the telescope of an instrument, the tilt corresponding to an observed movement of the bubble may be measured by the resulting motion of the line of sight on a levelling staff held at a known distance from the instrument. A base of about 300 ft. is measured on level ground by steel tape. With the instrument at one end of the base, and the staff held at the other, staff readings are observed (*a*) with the bubble near one end of its run, (*b*) with the bubble moved towards the other end by means of the levelling screws. In each case the position of both ends of the bubble is recorded, and that of its centre is deduced in terms of the tube graduations, it being insufficient to observe one end only owing to possible change in length with varying temperature. The observations should be repeated several times for different positions of the bubble, and the results are averaged.

Expressions for the radius of curvature and the angular value of one division of the tube may be obtained from a consideration of Fig. 29, in which the line of sight is made coincident with the tube axis, and *O*, the centre of curvature of the tube, is shown stationary. *AB* represents the staff, and *EF* the average run of the centre of the bubble.

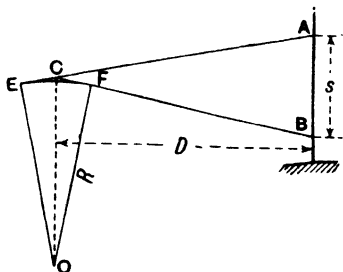


FIG. 29.

Let *D* = the horizontal distance between the instrument and the staff,

s = the average length of staff intercepted between the upper and lower lines of sight,

n = the average number of divisions through which the centre of the bubble is moved,

R = the radius of curvature of the tube,

d = the length of one division on the tube, expressed in the same unit as *D* and *s*.

The angle between the lines of sight is practically $\frac{D}{s}$ radians, since its value is small, and this angle equals the tilt of the tube, or

$$\frac{s}{D} = \frac{EF}{R} = \frac{nd}{R};$$

$$\therefore R = \frac{Dnd}{s}$$

The angular value of one division, expressed in radians, = $\frac{s}{Dn}$, or $\frac{d}{R}$,

and, expressed in seconds, = $\frac{s}{Dn \sin 1''} = \frac{206,265 s}{Dn}$.

The Chambered Level Tube.—Since increase of temperature shortens the bubble, and therefore reduces its sensitiveness, tubes used in precise instruments are commonly provided with means for regulating the length of the bubble. An air chamber is fitted at one end, so that, by tilting the tube, the required quantity of air may be added to or taken from the bubble.

The "Constant" Level Tube.—With a view to eliminating the loss of sensitiveness caused by shortening of the bubble at high temperatures, Messrs. E. R. Watts & Son, of London, have developed an interesting design of level tube, in which the bubble remains of constant length throughout a range of 130° F. or more.

Rise of temperature causes not only expansion of the liquid in a level tube but also decrease of its surface tension. The effect of the former is to decrease the length of the bubble; that of the latter is to increase the length owing to the cylindrical form of the upper surface of the tube. By suitably proportioning the volumes of the liquid and of the air bubble, it is possible to balance the two effects and secure a constant length of bubble. Messrs. Watts found that accurate results were attained with about 71% of spirit, such as petroleum ether, and 29% of air, but that, to obtain constancy of bubble length with a tube of circular cross section, it was necessary to use a longer tube than would be suitable for attachment to a surveying instrument. This difficulty is overcome by departing from the circular form, either by placing within a level tube of circular section a glass rod or sealed glass tube of smaller circular section, or, more neatly, by using a level tube of non-circular section. In the "Constant" bubble tube (Fig. 94) the upper wall is of relatively large radius, the side walls are sharply curved, and the bottom is flat, but may be curved like the top to form an oval section, or it may be convex upwards, so long as the required proportion between the volumes of liquid and air is obtained.

The Circular Level.—This level, Figs. 90 and 101, consists of a circular box, the upper glass surface of which is a portion of a sphere,

concave on the lower side. The bubble is circular, and, when central, shows that the axial *plane* is horizontal. Although it is usually of low sensitiveness, the circular level forms a convenient adjunct to various instruments, as it is equivalent to two tubes at right angles.

THE VERNIER

The vernier is a device for measuring fractional parts of the divisions of a graduated scale. Although invented by Vernier as early as 1631, it still remains the most commonly used means for this purpose.

In reading the position of an index relative to the divisions of a straight or circular scale, it may for certain purposes be sufficient to estimate by eye the amount by which it is past a graduation of the scale. The probable error made in the estimation of the fractional part of a division will depend primarily upon the observer, but also upon the circumstances of the observation: for any particular set of conditions it may be expressed as a constant fraction of a scale division. If, therefore, the scale divisions are comparatively large, the absolute probable error of a measurement made in this way may become serious, and if a high degree of precision were required, very close division of the scale would apparently be necessitated. Close graduation has, however, serious disadvantages. The division and engraving present mechanical difficulties. Further, the eye is confused by close spacing, and is unable to distinguish clearly one space from another. The use of a sufficiently powerful magnifier involves waste of time, and does not altogether eliminate the confusion.

These defects are overcome by the use of the vernier, the principle of which is based on the circumstance that the eye can perceive without strain and with considerable precision when two graduations coincide to form one continuous straight line. It consists of a small scale arranged to slide in contact with the fixed scale, which latter may be distinguished as the main or primary scale. The vernier carries the index point, the position of which is to be recorded, and which forms the zero of the vernier divisions. If the graduations of the main scale are numbered in one direction only the vernier used is of the single vernier type. If the graduations are figured in both directions, a double vernier, consisting of two single verniers, is preferable, one being read when the scale is used from left to right, and the other for the reverse direction.

Whether single or double, verniers may also be classed as direct and retrograde.

Direct Vernier.—Let d = the value, linear or angular, of the *smallest* spaces into which the main scale is divided,

v = the value of the smallest spaces on the vernier scale,

n = the number of spaces on the vernier.

In the direct vernier, v is such that the n vernier spaces occupy a length equal to that of $(n-1)$ main scale spaces,

$$\text{i.e. } nv = (n-1) d,$$

$$\text{or } v = \left(\frac{n-1}{n} \right) d;$$

and the difference between the value of a main scale division and that of a vernier division

$$= (d-v) = d \left(1 - \frac{n-1}{n} \right) = \frac{d}{n}$$

This quantity is termed the least count of the vernier. The position of the index can be read by means of this vernier to $1/n$ of a main scale division.

The method of using the vernier can be followed most easily from a simple example. In Fig. 30, S is the main scale, and V is a single vernier provided with 10 spaces of such size that they altogether occupy the same length as 9 main scale divisions. The vernier is therefore a direct one with a least count of $\frac{1}{10}$ th of a main scale division. In Fig. 30 the reading is exactly 11, and, since

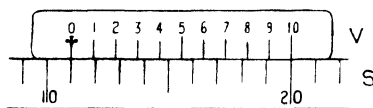


FIG. 30.—DIRECT VERNIER.

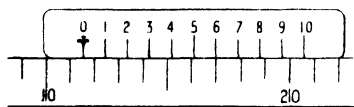


FIG. 31.—DIRECT VERNIER.

the vernier spaces are $\frac{9}{10}$ ths the length of the main scale spaces, graduation 1 of the vernier falls short of a scale graduation by $\frac{1}{10}$ th of d , graduation 2 is $\frac{2}{10}$ ths of d short of another, and so on. If, therefore, the vernier is moved forward a distance equal to $\frac{1}{10}$ th of d , graduation 1 of the vernier falls into coincidence with a main scale mark, and, in general, if the p th graduation on the vernier is in the same straight

line as one of the main scale marks, the index has passed a main scale graduation by $p/10$ ths of d . In taking a reading, therefore, when the index does not happen to record an exact number of scale units, first note the main scale graduation beyond which the index lies, in the direction of increase of the numbers on the scale (the approximate reading), and then observe where the coincidence of a vernier mark with a scale mark takes place. The number of the coinciding graduation on the vernier scale multiplied by the least count is the fractional part to be added to the approximate reading. Thus, in Fig. 31 the reading is 11.6. No account is taken of the main scale graduation in coincidence with that of the vernier.

The converse of reading a vernier is setting it to a given reading. Having set the index approximately in place, a small adjustment is made by means of the screw provided for the purpose until the desired coincidence of the vernier graduation representing the given

fractional part is obtained. It is well to check the approximate reading after the fractional part has been set.

Fig. 32 shows a double vernier, the single verniers composing it being similar to that just described. The part in advance of the index in the direction of measurement is that to be used. The readings illustrated are 15·6, from left to right, and 84·4, from right

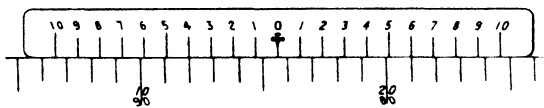


FIG. 32.—DOUBLE VERNIER.

to left. The adoption of two different characters of figures on the scales tends to obviate confusion : in the diagram the main scale numerals slope the same way as those on the half of the vernier to be used.

Retrograde Vernier.—In this type the n vernier divisions occupy a length equal to that of $(n+1)$ main scale divisions, so that, with the previous notation,

$$nv = (n+1)d,$$

$$v = \left(\frac{n+1}{n}\right)d,$$

and, as before, the difference between the value of a main scale division and that of a vernier division

$$= (v-d) = d \left(\frac{n+1}{n} - 1\right) = \frac{d}{n} = \text{the least count.}$$

In this vernier the numbering of the vernier divisions increases in the opposite direction from that of the main scale. Fig. 33 illustrates a retrograde vernier reading to tenths, the reading shown being 20·7. This type of vernier may also be doubled for use with scales figured in both directions.

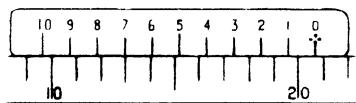


FIG. 33.—RETROGRADE VERNIER.

Examples of Verniers in Surveying Instruments.—Before using a particular vernier for the first time, it must be examined to ascertain :

(1) Whether it is direct or retrograde.

This is at once determined by observing whether the number of main scale spaces subtended by the vernier is one less or one more than the number of vernier spaces. The great majority of verniers used in surveying instruments are direct.

(2) Its least count.

The value of the smallest division on the main scale and the number of spaces on the vernier scale must be noted. By the

latter is meant the number intercepted between the zero and the last numbered mark, as sometimes one or two divisions are engraved beyond the end of the scale proper. The last numeral may not indicate the number of spaces, as it does in Fig. 34.

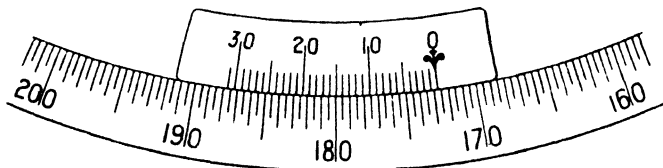


FIG. 34.—VERNIER READING TO 1 MINUTE.

In Fig. 34 a theodolite circle is shown divided to half degrees. The vernier contains 30 spaces, and these occupy a length of 29 circle divisions, or $14^{\circ} 30'$. The vernier is therefore direct, and reads to $\frac{1}{30}$ of half a degree, i.e. to one minute. The index is

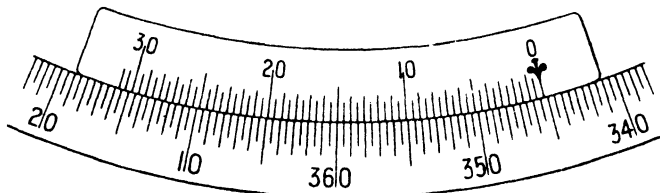


FIG. 35.—VERNIER READING TO 30 SECONDS.

situated between $172^{\circ} 30'$ and 173° , so that the approximate reading is $172^{\circ} 30'$, and, since the 14th graduation of the vernier is that in coincidence, the complete reading is $172^{\circ} 44'$.

The circle of Fig. 35 is similarly divided, but in this case there are 60 vernier spaces with a total length of 59 circle divisions. The

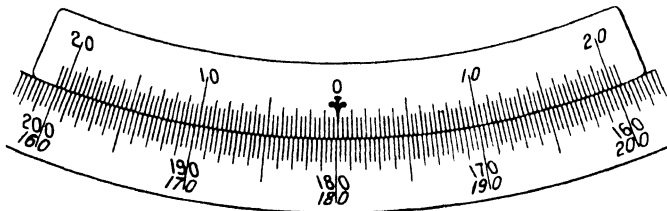


FIG. 36.—DOUBLE VERNIER READING TO 20 SECONDS.

vernier is therefore direct, and reads to $\frac{1}{30}$ of half a degree, or $30''$. The approximate reading is 345° , and the vernier reading $14' 30''$, giving a total of $345^{\circ} 14' 30''$. It will be observed that the longer graduations and the figures on the vernier scale represent whole minutes.

In Fig. 36, the circle is figured in both directions, and the accompanying vernier is of the double type. The 60 spaces on either

half of the vernier are equivalent to 59 circle divisions, so that the vernier is direct, and consequently that half of it in advance of the index, in the direction of increase of the set of circle figures being used, falls to be read. The least count is $20''$. If the measurement is being made clockwise, the figures which slope to the left are read, and the approximate reading is $179^\circ 40'$, which with the vernier reading of $11' 20''$ gives a total of $179^\circ 51' 20''$. In the counter-clockwise direction, the reading is $180^\circ 8' 40''$.

Special Forms of Vernier.—An extended vernier is one in which n vernier spaces occupy the same length as $(2n-1)$ main scale spaces. With the previous notation (page 39),

$$nv = (2n-1)d,$$

$$v = \left(2 - \frac{1}{n}\right) d,$$

and in this case the difference between two main scale spaces and one vernier space

$$= (2d - v) = \frac{d}{n} = \text{the least count.}$$

The extended vernier is therefore equivalent to a simple direct vernier of which only every second graduation is engraved, the object being to afford a small least count without having the vernier graduations too close together for convenience in reading. The

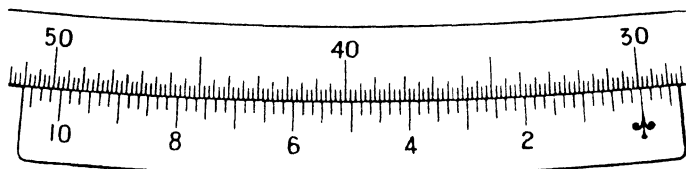


FIG. 37.—EXTENDED VERNIER.

extended vernier is regularly employed in the astronomical sextant. In the commonest form the vernier has 60 spaces equivalent to 119 arc spaces, each of $10'$, and the least count is $10''$. This pattern is illustrated in Fig. 37, in which the approximate reading is $29^\circ 50'$, and the vernier reading $4' 10''$, giving a total of $29^\circ 54' 10''$.

A form of vernier sometimes used in compasses has the zero in the middle of the length. It is known as a folded vernier, and may be either direct or retrograde. In the case of a direct folded vernier dividing half degrees to minutes and reading from left to right, the values of the fifth divisions are marked 15, 20,

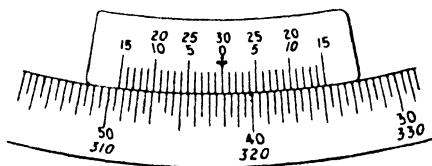


FIG. 38.—DOUBLE FOLDED VERNIER.

25, 0, 5, 10, and 15 minutes. When the main scale reads both ways a double folded vernier has the advantage over an ordinary double vernier in occupying less room. Fig. 38 shows such a vernier, the readings being $42^{\circ} 7'$ clockwise and $317^{\circ} 53'$ anti-clockwise.

Reading the Vernier.—In obtaining the vernier reading to apply to the approximate reading, there may be a little difficulty in judging where the coincidence takes place. It will be found that the time spent in searching for the coincidence may be reduced by making a rough estimate, from the position of the index, of the vernier reading to be expected. If the graduation is somewhat open, none of the vernier marks may appear to coincide with a scale graduation, but two successive marks may seem to be about equally off exact coincidence. The mean value of those two vernier graduations will be more nearly correct than either. On the other hand, with close graduation, there may appear to be more than one coincidence. If two or more vernier marks are seemingly coincident with scale marks, their mean is to be taken. In any case it is always advisable, before deciding upon the reading, to examine a few marks on either side of that which at the first glance is thought to be in coincidence.

A reading glass is a necessity except when the graduation is open and the lines are bold. The accuracy and ease of reading are increased if the silvered surfaces upon which the scales are cut are shaded from brilliant side illumination, which, since it is reflected brightly from one side of the engraved marks, makes accurate reading somewhat troublesome. Reflectors of white celluloid or opal glass provide a soft and satisfactory illumination.

Mistakes in Reading Verniers.—Of the various mistakes made by beginners in using a vernier, two are much more common than others, and, since they refer to the reading of the main scale itself, lead to serious error.

(1) Assuming a wrong value for the approximate reading, particularly by the omission to record one or more subsidiary scale divisions.

Thus, in the case already cited of a circle divided to third parts of a degree, when the index lies between a $20'$ and a $40'$ mark, the approximate reading is $(n^{\circ} + 20')$, and if it lies beyond the $40'$ mark, the reading is $(n^{\circ} + 40')$, because the vernier subdivides the smallest main scale divisions only.

(2) Reading the main scale in the wrong direction.

If the main scale is graduated in one direction only, the reading must be made in that direction irrespective of that in which the vernier has been moved. If the direction of motion of the vernier is opposite to that of the scale graduation, the difference of the readings before and after movement represents the required measurement. Thus, if the vernier of Fig. 34 has been brought to

the position illustrated by a counter-clockwise rotation, say, from the 180° graduation, the mistake might be made of recording the angle as $7^\circ 14'$, instead of $(180^\circ - 172^\circ 44')$. Special care must be exercised in reading double verniers.

THE MAGNETIC NEEDLE

The magnetic needle is an essential feature of the various forms of compass used by the surveyor, and is a convenient adjunct of several instruments, notably the theodolite and the plane table. It consists of a piece of magnetised cast steel supported at its centre on a sharp, hard steel pivot. In obedience to the directive influence of the earth's magnetism, such a magnet floats in a definite position, its longitudinal axis lying in the plane of the magnetic meridian and exhibiting the direction of magnetic north and south. This direction is not usually coincident with that of the true or geographical meridian, or true north and south, the difference being explained on page 204.

A second effect of the earth's magnetism is that a symmetrical floating magnet will not usually rest in a horizontal position, but will be inclined towards the nearer of the earth's magnetic poles. This deviation from the horizontal is termed the dip of the needle. Its amount varies from 0° at the magnetic equator to 90° at the magnetic poles, but does not remain constant at any place. At present, 1944, at Abinger, the north end of the needle dips downwards at $66^\circ 45\frac{1}{4}'$ to the horizontal, and this dip is increasing at the rate of about $1'$ per annum. To balance the needle, it is weighted on one side by the maker. This is best done by means of a sliding weight, or rider, which, for use in different parts of the world, can be adjusted to counteract the dip obtaining in the locality.

Construction.—The commonest types are (a) the broad needle (Fig. 88), (b) the edge-bar needle (Fig. 84); but tubular forms are also used. In each case the centre is fitted with an aluminium or brass cell containing a hollow conical bearing of agate or other hard mineral to receive the fine point of the support pivot, which should be in the same horizontal plane as the ends of the needle. Extreme fineness of the supporting point is necessary to prevent the possibility of the needle coming to rest other than in the magnetic meridian, and hardness of both bearing surfaces is therefore essential to minimise wear with consequent friction and sluggishness of the needle. To prevent unnecessary wear, the needle should never be allowed to bear on the pivot except when in use. Needles of surveying instruments are usually provided with means for lifting them from the pivot: the device may take the form of a lever or a cam.

To observe directions with reference to magnetic meridian, the

needle is either read against the graduations of a compass box or it may itself carry a graduated card or ring to be read against an index. The former arrangement is applied in the surveying compass and the theodolite, an edge-bar needle being used. The box may be circular with a completely graduated circle, so arranged that by reading the position of the north end of the needle, the magnetic direction of any line can be obtained. In the trough compass the needle is mounted in a narrow rectangular box (Fig. 59) or is enclosed in a tube. The trough compass is a very useful form for the establishment of magnetic meridian, directions with reference to magnetic meridian being measured by the instrument to which the compass is attached. The broad needle, usually of uniform breadth, is employed when a graduated card or ring has to be carried, as in the prismatic compass (Fig. 88) and the mariner's compass. Aluminium or cardboard is used to minimise the weight on the pivot.

Requirements of the Magnetic Needle.—The principal requirements of the needle are :

1. It should be sensitive.
2. The magnetic axis should coincide with the geometrical axis.
3. The ends should lie in the same horizontal and vertical planes with the pivot point (page 105).
4. The centre of gravity should for stability be as far below the pivot point as possible.
5. The compass box should be entirely non-magnetic.

Sensitiveness.—If a needle, on being lowered on its pivot, comes to rest quickly, the sluggishness may arise from (a) loss of polarity, (b) wear of pivot. The latter is the more common defect.

To re-magnetise a needle, it is best to place it in the magnetic field of a dynamo, taking care not to reverse the poles. If this method is impracticable, procure a bar magnet, and draw the N. end of the magnet several times from the centre of the needle towards its S. end, using an axial stroke. Repeat with the S. end of the magnet on the N. side of the needle.

To sharpen the pivot, use a piece of very fine oil-stone, and rub the point all round until it will adhere to the finger nail at the slightest touch. This test for sharpness should be frequently repeated during grinding, as over-grinding reduces the height of the pivot appreciably, and lowers the needle relatively to the graduations of the compass box.

Magnetic Axis.—The axis of magnetism may not coincide with the geometrical axis of the needle, and, as it is the former which lies in the magnetic meridian, a needle defective in this respect will not record magnetic bearings correctly. The error, however, is of constant amount, and therefore does not influence the values of the included angles of a compass survey.

Non-magnetism of Compass Box.—Cases are not infrequent of the faulty performance of a compass being due to the presence of iron in the alloys used in the manufacture of the instrument. The defect may be tested for in instruments having a horizontal circle and vernier by setting off successive angles of, say, 5° right round the circle and reading the position of the needle on the compass circle each time. Irregularity of the intervals between the needle readings indicates that a source of attraction is present.

THE TRIPOD STAND

The most important requirement of a tripod is rigidity or steadiness, but lightness and portability are also to be desired. Although varying in detail, tripods may be classed as solid leg or split leg. The former is the more convenient to carry, as the legs, when brought together, form a cylinder, and mainly for this reason it is the more common type, particularly for small instruments. The split leg or framed tripod possesses the advantages of superior rigidity and lightness, and is to be preferred for precise work.

The wood of which the legs are made is commonly mahogany, but ash, yellow pine, and cedar are also utilised. The lower ends of the legs are fitted with pointed steel shoes so that they may be thrust into the ground. Some makers provide projections near the bottom by which the user can press down the legs with his foot. At the top, the legs are screwed to a casting which forms the immediate support for the instrument. The connection of the legs to this piece is the weak point in many tripods, the rapid development of looseness in the screws necessitating their frequent tightening. If the line of sight of the telescope is directed towards a fixed mark, the quality of the tripod as regards rigidity may be tested by grasping one of the legs and giving it a twist. The intersection of the cross-hairs will appear to leave the mark, but, on removal of the twist, should return to the point if the tripod is stable. Accurate work is impossible with a tripod which has developed shakiness at the head or in the shoes.

CHAIN SURVEYING INSTRUMENTS

Chain.—The chain (Fig. 39) is formed of 100 pieces of straight wire, the ends of which are bent to connect to each other through small oval rings, which for flexibility should number three at each joint. The wire is of iron or steel of from 8 to 12 gauge ($\cdot 16$ to $\cdot 10$ in.), the use of steel enabling weight to be reduced without sacrificing strength. The ends of the chain consist of brass handles, each with a swivel joint to eliminate twist.

Chains used in English-speaking countries are of two lengths, the 100-ft. chain divided into feet, and the 66-ft. or Gunter's chain,

which measures one *chain* and is divided into 100 *links*. The *length of a chain* is the total length from outside to outside of the handles. At every tenth foot or link is attached a distinctive tag or tally of brass of the patterns shown in Fig. 39. As each tag represents its distance from both ends of the chain, either handle can be regarded as the zero, so that a little care is necessary to avoid misreading. In taking readings between tags, one must count the

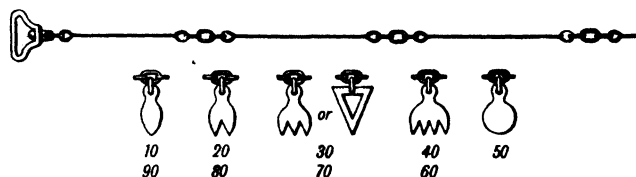


FIG. 39.—CHAIN.

number of feet or links from the previous tag, estimating fractions of a unit if necessary. Time is saved if the intermediate fifth divisions are distinguished by a small oval or circular tally, as inserted by some makers.

In countries in which the metre is the standard unit of length, chains of 20 and 25 metres are employed, the former being the more generally used. It is divided into 100 parts of 2 decimetres with a tally at every two metres.

Steel Band.—The long steel band or band chain consists of a ribbon of steel with brass handles attached by swivel joints to the ends (Fig. 40). It is obtainable in lengths of from 50 to 1000 ft., with widths

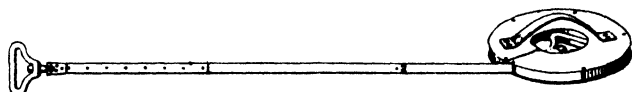


FIG. 40.—STEEL BAND.

ranging from $\frac{1}{8}$ in. to $\frac{3}{4}$ in., and thicknesses from .01 to .03 in. The effective length may include the handles, but greater accuracy is possible if the ends are marked on the tape itself. The steel band is wound either upon an open cruciform frame or on a reel in a case as shown.

The marking of the graduations is performed in various ways—by etching with acid to leave marks and figures in relief, by brass rivets, by brass sleeves soldered on, or by stamping attached pieces of solder. Etching and riveting have a weakening effect, but the former is necessary when each foot or link is figured and subdivided. In the other systems only the foot or link at each end is subdivided, decimally in the case of the link, and decimally or duodecimally for the foot.

Comparative Utility of the Foot and the Link as a Unit.—The foot is the unit employed in all structural work, so that surveys made in connection with construction are usually executed with the 100-ft. chain or band, which in consequence is sometimes referred to as the engineer's chain. For purely land surveys, on the other hand, Gunter's chain is often preferred owing to the facility with which areas may be computed from the measurements, since the unit of land measurement, the imperial acre, = 10 sq. chains = 100,000 sq. links. Gunter's chain has the further advantage of subdividing the mile exactly. On this account it is sometimes used by engineers in this country in the pegging out of railway centre lines. By virtue of its shorter length, it can be more rapidly manipulated than the 100-ft. chain, but, as it has to be put down so much oftener, the measurement proceeds more slowly.

Relative Merits of the Chain and the Steel Band.—The principal comparative features of the chain and band may be summarised as follows.

Advantages of the Chain.—(1) It can withstand rather rough treatment.

(2) If broken, it is easily repaired temporarily with wire or even with string.

(3) It is easily read.

Disadvantages.—(1) It is very subject to alteration in length. As it has some 800 wearing surfaces, continued use increases the length. In iron chains especially, the application of excessive pull may cause stretching of the small rings by flattening and, unless brazed, by opening out. Causes tending to shorten it are bending of the links and the adhesion of mud between bearing surfaces.

(2) It is comparatively heavy, and therefore takes some time to lay straight on the ground, while it sags considerably when suspended.

(3) It is liable to become entangled in shrubbery and to collect mud, etc.

Advantages of the Steel Band.—(1) It maintains its length very much better than the chain, the stretching with prolonged use being negligible for ordinary work.

(2) For the same strength it is lighter than the chain.

(3) Both foot and link graduations may be furnished on opposite sides.

Disadvantages.—(1) If carelessly used, it is liable to be broken, especially by pulling it when twisted into loops or by stepping upon it.

(2) It can be repaired only by riveting or soldering, and this cannot be done in the field unless a repairing outfit is carried.

(3) It is less easily read than the chain.

(4) It must be kept free from rust. If the graduations are etched, special care must be exercised to preserve their legibility.

The steel band yields so very much better results than the chain that its employment is a necessity in such operations as city surveying and setting out works. The chain is, however, still used to a large extent for ordinary small surveys, principally on account of its durability.

Accessories for Use with the Steel Band.—Various accessories are necessary for use with the steel band or tape when the more refined type of work is being done. These include a spring balance, a special fork for attaching the balance to the tape, a thermometer and special clips for holding the tape at points other than at the ends. These articles are described and their use explained in Chap. II, which deals with linear measurements.

Testing and Adjustment of the Chain.—The length of a chain must be checked at frequent intervals. It may be compared with : (a) an official standard ; (b) a good steel band or tape ; (c) a chain of known length, reserved for the purpose ; (d) a levelling staff laid down successively. Official standards are established in several large towns as the distance between marks on metal plugs, but the surveyor may with a steel tape set out a private standard by marks on a corridor or pavement. In extensive surveys a field standard should be established as the distance between tacks driven into two stout pegs fixed in the ground, a similar point being fixed midway.

Before making the comparison, the chain should have all bent links straightened and the bearing surfaces freed from mud. On stretching it with a moderate pull, it will probably be found too long. To save the trouble of computing corrections from the known error and applying them to measured lengths, the end of the nominal length may be marked on one handle, but it is better to adjust the length by removing one or more connecting rings. In doing so, an endeavour should be made to keep the 50 tally midway and the others as nearly right as possible. Some chains are provided with means for adjustment at the handles.

Steel bands or tapes, especially if used for the better classes of work, also require to be carefully standardised at reasonably frequent intervals. Official standards, established by municipal or local authorities, are sufficient for testing chains, but should not be relied on for really accurate work with the steel tape, and, for such work, it is far better for the surveyor to compare his tape himself with one that has been properly standardised at the National Physical Laboratory and is kept solely for standardising the tapes that are to be used in the field. Methods of doing this are described in Chap. II.

Arrows.—For marking the ends of chain lengths and recording the number of times the chain is laid down in measuring a line,

set of marking pins or arrows, generally ten in number, is used. These are formed of iron or steel wire, preferably a little heavier than that of the chain, and are pointed at one end for thrusting into the ground, while the other end is bent to form a ring for facility in carrying. The usual lengths are 12 and 15 in., but 18-in. arrows may advantageously be used in long grass. Arrows should have a small strip of red bunting tied to the ring to make them conspicuous on the ground.

Linen Tape.—In ordinary surveying, short measurements are made by means of a linen tape, which consists of a painted and varnished strip of woven linen usually $\frac{1}{2}$ in. wide. It is attached to a spindle in a leather case, into which it is wound when not in use (Fig. 41). The usual lengths are 33, 50, 66, and 100 ft., the 66-ft. tape being the most serviceable on the whole. The graduation is in links on one side and feet and inches on the other. In place of figuring each inch, the marking shown in Fig. 41 is less liable to lead to mistakes in reading, and is preferred for surveying tapes.

Linen tapes should not be used where precise results are required, as they are subject to serious variations in length. They stretch when pulled, and may easily be permanently elongated, since a considerable pull is required to straighten them in wind. Exposure to wet causes them to shrink. Linen tapes are not very durable, and in time become frayed and illegible, particularly towards the zero end. Short end lengths, complete with ring, are obtainable, and a worn end portion may be replaced by a new length sewed



FIG. 41.—LINEN TAPE.

on in the proper position. A complete refill tape may be mounted by unscrewing the spindle of the box and attaching the end of the tape to it, either by sewing or by insertion in a slit for the purpose. If treated with care, a tape box will serve for several refills.

Metallic Tape.—The so-called metallic tape is a linen tape into which brass or copper wires have been woven for the purpose of promoting constancy of length. The device is only partially successful.

Steel Tape.—Far superior to the others is the short steel tape, which is of lighter section than the steel band and is mounted similarly to the linen tape. The graduations, links on one side, and feet, inches, and eighths on the other, are marked by etching, and the

tape must be kept well oiled and clean to preserve their legibility. The precautions against breaking apply as in the band.

Ranging Poles.—For the conspicuous marking of points and the ranging of lines, wooden poles or pickets are used. They are generally made of red pine, but ash, lancewood, and bamboo are also used. They are of circular or octagonal cross section, tapering towards the top, and are fitted with a strong iron or steel shoe. The more commonly used lengths are 6 ft. and 10 links. To render them as conspicuous as possible they are painted black and white, or red and white, in alternate bands, or red, white, and black successively. By having the painted divisions 1 ft. or 1 link long, a ranging pole can be used as a measuring rod. Poles which have to be observed from a long distance should have a small flag attached.

Ranging poles made of steel, and of light, but strong, section, are now available and last much longer than the ordinary wooden ranging pole. Some are provided with a sighting device for setting out lines at right angles to one another. This consists of two pairs of slots cut through the tube at eye height, a line projected through one pair being at right angles to that projected through the other pair. One advantage of these poles, which are obtainable from Messrs. Cooke, Troughton & Simms, is that they may be driven into hard ground with a mallet. They can be had in sections which screw together for convenience in transport and a coupling attachment can be obtained when it is desired to use lengths of from 12 to 16 ft.

Offset Rod.—An offset rod is simply a ranging pole, similarly shod and painted, which is primarily intended for the measurement of short distances. The usual length is 10 links. It may be jointed in the middle, and it is fitted with a recessed hook for pulling or pushing the chain through a hedge or other obstruction.

Line Ranger.—The line ranger is a small instrument whereby intermediate points can be established in line with two distant signals without the necessity of sighting from one of them. It

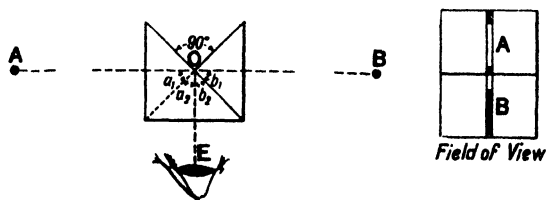


FIG. 42.—LINE RANGER.

consists essentially of two reflecting surfaces, either small plane mirrors or square prisms, as in Fig. 42, one above the other, and with their reflecting surfaces normal to each other.

In locating an intermediate point in line with the poles A and B, the observer stands approximately in line and places the instrument on a rod, or holds it at the level of the eye, turning it until the image of one of the poles, A, is seen in the field of view. He then moves backwards or forwards at right angles to the line until the image of B appears. A point in the line AB is reached when the images of A and B lie in the same vertical line. The reflected rays from both A and B are then situated in the same vertical plane OE. The direction of OE depends upon the position of the eye, and is not necessarily normal to AB, but, by the laws of reflection, $a_1 = a_2$ and $b_1 = b_2$, and, since $(a_2 + b_2) = 90^\circ$, we have $(a_1 + a_2 + b_1 + b_2) = 180^\circ$. AOB is therefore one straight line, and O can be transferred to the ground by pole.

One of the mirrors or prisms is commonly made adjustable for securing the necessary perpendicularity between the reflecting surfaces. To perform the adjustment, three poles are ranged by theodolite, and the line ranger is held over the middle one. If the images of the others do not lie in the same vertical line, they are made to do so by turning the adjusting screw.

Cross Staff.—The cross staff is the simplest instrument used for setting out right angles. It is mounted on a pole shod for fixing in the ground, and is arranged in the form of a frame or box furnished with two pairs of vertical slits yielding two lines of sight

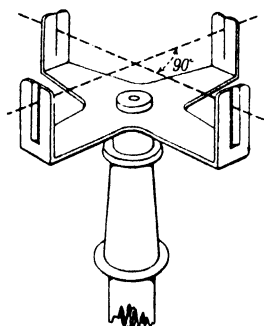


FIG. 43.—CROSS STAFF.

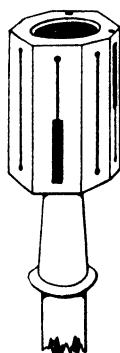


FIG. 44.—CROSS STAFF.

mutually at right angles. Figs. 43 and 44 illustrate two common types. The open form has usually a longer base between the slits than the octagonal box pattern. The latter has additional openings, so that angle of 45° may also be set out. In another form of the instrument, the head consists of two cylinders of equal diameter placed one on top of the other. Both are provided with sighting slits. The upper cylinder carries an index or vernier, and can be rotated relatively to the lower, to which is attached a graduated

circle. This arrangement enables any angle to be set out or measured roughly.

To set out a perpendicular to a line from a point on it, it is only necessary to fix the staff in the ground at the given point and bring one pair of slits in range with the poles marking the line. The slits at right angles then define a perpendicular line of sight, in which poles or marks may be set. To drop a perpendicular to a line from a point outside it, the surveyor, holding the staff on the line, must proceed by trial and error until a point on the line is obtained at which the poles in the line and the point outside it are simultaneously in the two lines of sight of the instrument.

The cross staff is non-adjustable, and is not susceptible of high accuracy, but is useful for setting out long offsets.

The Optical Square.—This is a compact hand instrument for setting out right angles, and is capable of greater accuracy than the cross staff. It is based on the principle that a ray of light reflected successively from two surfaces undergoes a deviation of twice the angle between the reflecting surfaces. In the instrument these surfaces are placed exactly at 45° to each other, and belong

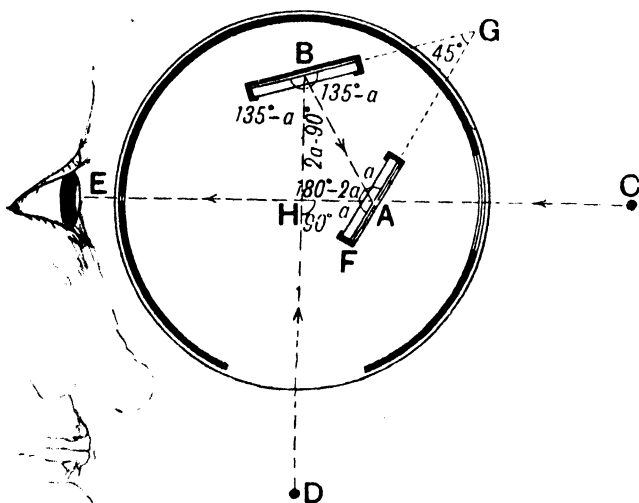


FIG. 45.—OPTICAL SQUARE.

to small mirrors mounted in a circular box or on an open frame, or, alternatively, they form two sides of a prism, the instrument in this case being sometimes distinguished as a prism square. The latter is the more modern and better form of the instrument.

Fig. 45 shows a plan of the essential features of the box form, the top cover being removed. The periphery is formed of two cylinders, of about 2 in. diameter and about $\frac{1}{2}$ in. deep, the one

capable of sliding on the other, so that, when not in use, the eye and object openings can be closed to protect the mirrors from dust. Mirror A is silvered over half the depth only, and the other half is of plain glass. Mirror B is completely silvered. To an eye placed at E objects such as C are visible through the transparent half of A, and at the same time objects in the direction of D are seen in the silvered part after reflection at B. If the object D is so placed that its image appears directly above (or below) C, so that the eye received the rays from D in the same vertical plane AE as those from C, then C and D subtend a right angle at the instrument. For the angle G formed by the reflecting surfaces is made 45° , and if EAF be denoted by a , then $BAG = a$ by the laws of reflection of light, and the values of the remaining marked angles are readily deduced, giving CHD a constant value of 90° .

Use of Optical Square.—To set out a perpendicular to a line EC from a point on it, the observer holds the instrument over the point, preferably resting it against a ranging pole, and turns it until the pole C, marking the line, is seen through the clear glass. A chainman, having been sent out with a pole in the direction of D, is then directed to right or left until the reflected image of his pole appears coincident with the line pole.

To find where a perpendicular from a given point D would meet the line EC, the surveyor, holding the instrument to his eye, walks along the line until he obtains the coincidence as before, when the instrument will be over the required point.

Notes.—(1) It is always advisable to have two forward poles marking the survey line. By keeping them in range, the surveyor can maintain the instrument in the line without trouble.

(2) When the perpendicular lies to the other side of the survey line, the instrument is held upside down.

Testing and Adjusting of Optical Square.—The instrument is often made with both mirrors permanently fixed, but some makers mount mirror B (Fig. 45) so as to permit adjustment of the angle between the mirrors. An adjusting key is fitted in the box, and is readily available when required. The test and adjustment are as follows:

Object.—To place the mirrors at 45° to each other, so that the angles set out shall be right angles.

Test.—1. Range three poles A, B, and C in line, preferably at least 300 ft. apart.

2. With the instrument at B, sight A, and set out a right angle ABD.

3. From the same point sight C, and if the pole at D appears in coincidence with C, the instrument is correct.

Adjustment.—1. If not, mark opposite D the point D' which appears coincident with C, and erect a pole E midway between D and D' to mark the true perpendicular BE.

2. By means of the key, turn the screw controlling the adjustable mirror until the image of E is made coincident with C.

3. On again sighting A, E should appear in coincidence: if not, repeat the test and adjustment until the error is eliminated.

Note.—The above test is a straightforward case of reversal, and corresponds exactly to the testing of a set-square (page 10).

The Prism Square.—The same principle applies to the prismatic

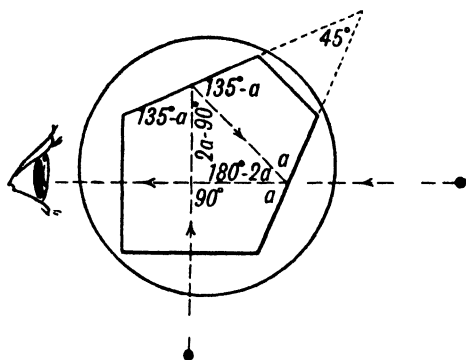


FIG. 46.—PRISM SQUARE.

form of the instrument, shown diagrammatically in Fig. 46, and it is used in the same manner. The prism square has the merit that no adjustment is required, since the angle between the reflecting surfaces cannot vary. The deviation of this angle from 45° , due to errors of workmanship, may be regarded as negligible for the purposes of the instrument.

The Box Sextant.—The sextant, of which the box sextant is the most compact form, is a reflecting instrument capable of measuring angles up to about 120° . Although the box sextant is not strictly a chain surveying instrument, it is commonly included in the equipment in place of an optical square, and may appropriately be dealt with here. The principle that the deviation of a ray of light reflected successively from two mirrors is twice the angle between them is applied as in the optical square, except that in this case one of the mirrors, called the index glass, is mounted on an axis about which it can be rotated. The variable angle between the mirrors is set off on the instrument, which therefore measures that subtended by the objects.

The box has a diameter of about 3 in. and a depth of about $1\frac{1}{2}$ in., and is provided with a cover, which is removed and screwed on underneath to form a handle when the instrument is in use. Fig. 47 represents an interior plan of the instrument. The fixed mirror, or horizon glass, is silvered on the top half only, and the index glass is silvered all over. Attached to the latter is a toothed segment gearing with a small pinion which is actuated by the milled head 3 on the top of the box, shown in the exterior plan (Fig. 48). The axis of the index glass carries an index arm with a vernier, which on rotation of the mirror is carried over the graduated arc.

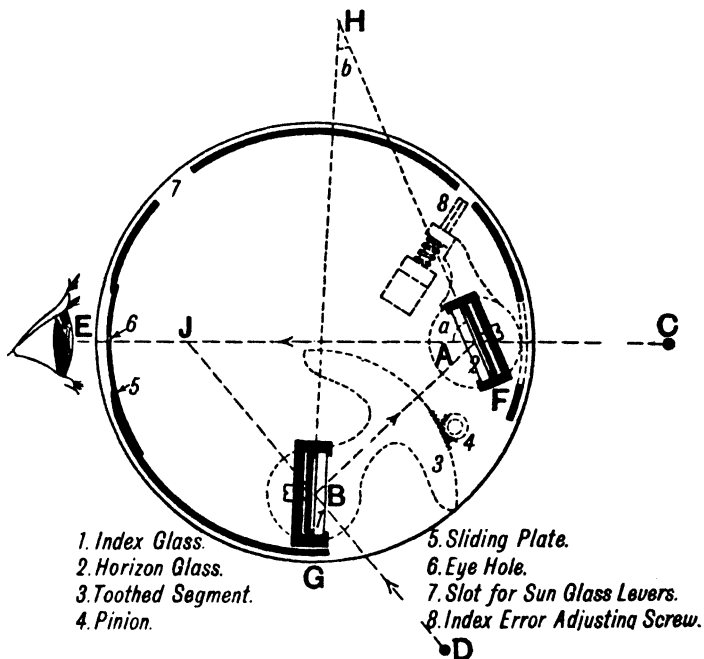


FIG. 47.—INTERIOR PLAN OF BOX SEXTANT.

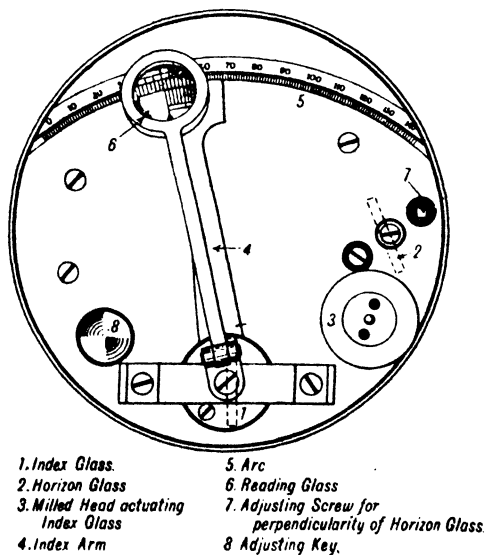


FIG. 48.—EXTERIOR PLAN OF BOX SEXTANT.

The graduation is carried to half degrees, subdivided to single minutes by the vernier, which is viewed through the hinged reading glass.

The observations are usually made by sighting through an eye-hole. A small telescope is provided for long-distance sighting, and this is attached either by inserting it in the circular opening exposed on pushing back the sliding plate or by means of an external bracket screwed on when required. The telescope has a dark glass for use in the solar observations required in adjusting the instrument, and, to intercept the sun's rays reflected from the index mirror, dark glasses are mounted in the box on hinges so that they may be lowered out of the way for terrestrial observations.

The theory of the sextant is as follows (Fig. 47).

Let a = the angle EAH between the horizon glass and the line of sight EC,

b = the angle AHB between the mirrors when the image of a pole D appears in line with pole C.

By the laws of reflection, $BAF = a$, and $\therefore EAB = (180^\circ - 2a)$.

But BAF is an exterior angle of triangle ABH,

$\therefore ABH = (a - b) = GBD$, so that $ABD = 180^\circ - 2(a - b)$.

But ABD is an exterior angle of triangle ABJ, whence

AJB, the angle between the signals, $= 180^\circ - 2(a - b) - (180^\circ - 2a)$
 $= 2b$ = twice that between the mirrors.

The point J is not fixed in position, and, in general, the angle observed between the objects is not the angle subtended by them at the centre of the box. The difference, however, is negligibly small, unless the objects sighted are very near the instrument.

To avoid the necessity of doubling the angle between the mirrors at each observation, the graduated arc has the angles figured twice their real values, so that the readings give the required measurements.

The sextant differs from other angular instruments in that it measures the actual angle between objects of different elevations instead of its horizontal projection. As the latter is required in surveying, it may be necessary, when the levels of the two signals differ considerably from each other, to reduce the observed angle to its horizontal equivalent. Thus, in Fig. 49, let A be the point of observation and AZ the zenith line at A. In AZ take any convenient point Z and, with A as centre and AZ as radius, describe an imaginary sphere about A. From A draw lines AB and AC to the distant objects

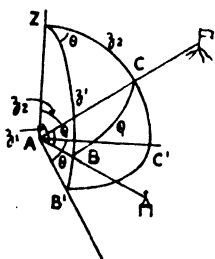


FIG. 49.

to cut the sphere in B and C. These lines then represent the lines of sight. From Z draw the great circles ZBB' and ZCC' through the points B and C to cut the horizon plane in B' and C'. Then the

angle BAC is the measured angle ϕ and the horizontal projection of this angle, the angle B'AC' = θ , is the angle we require.

Let the lines of sight AB and AC be inclined at angles z_1 and z_2 to the zenith. Then angle BAZ = z_1 , and angle CAZ = z_2 . Hence, in the spherical triangle ZBC, angle BZC = θ and the sides are BC = ϕ , ZB = z_1 , ZC = z_2 . Solving the triangle,

$$\cos \theta = \frac{\cos \phi - \cos z_1 \cos z_2}{\sin z_1 \sin z_2}$$

or, in the ordinary form for solution by logarithms,

$$\tan \frac{\theta}{2} = \left[\frac{\sin (s-z_1) \cdot \sin (s-z_2)}{\sin s \cdot \sin (\phi-s)} \right]^{\frac{1}{2}}$$

where $s = \frac{1}{2} (\phi + z_1 + z_2)$.

Observing with the Box Sextant.—The instrument is rested against the station pole, and one hand is left free to turn the milled head 3 (Fig. 48). To measure the angle subtended by two distant signals, the instrument is turned in the hand until one of them is visible through the transparent part of the horizon glass, and screw 3 is then slowly rotated until the image of the second signal is brought in line with the first. Unless the signals are at the same level as the instrument, the latter must be tilted to lie in their plane. The angle is given by the vernier index.

In setting out an angle, the index must first be set to read its given value. Keeping the line of sight, through the clear glass, on the given signal, the surveyor directs a chainman into the position at which a pole held by him appears coincident with the signal.

Notes.—(1) In measuring an angle, an inexperienced observer may find difficulty in picking up the point to be sighted by reflection. This frequently arises through inattention to the holding of the instrument in the correct plane, and it is well to keep tilting it up and down as the index glass is being turned. Time is saved if the value of the angle is first estimated and the index set roughly to this reading.

(2) When the signals are not very clear, the more distinct of the two should be that sighted by reflection. When the left-hand object is thus sighted, it is necessary to hold the instrument upside down.

(3) The reflected image loses in distinctness when the angle between the objects exceeds 110° , and the measurement of angles greater than about 120° should be made in two parts by erecting a temporary pole in an intermediate position or by using a well-defined permanent point in the same way.

Testing and Adjustment of the Box Sextant.—The requirements of the astronomical sextant (Vol. II, Chap. II) are applicable to the box sextant, but in the latter instrument both the index glass and the telescope are mounted without provision for their adjustment. The horizon glass has the two movements required for setting it perpendicular to the plane of the arc and for the elimination of index error.

First Adjustment.—*Object.*—To set the horizon glass perpendicular to the plane of the instrument, so that both mirrors may be perpendicular to the same plane.

Test.—1. Set the vernier to zero.

2. Sight the sun or any terrestrial object, and observe whether there is any vertical gap or overlapping of the reflected image relatively to the object (*a*, Fig. 50).

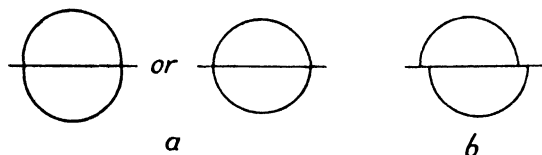


FIG. 50.

Adjustment.—If so, by means of the key, kept at 8 (Fig. 48) when not in use, turn screw 7 until the vertical displacement is eliminated.

Second Adjustment.—*Object.*—To eliminate index error, so that, when the mirrors are parallel, the vernier will read zero.

Test.—1. Set the vernier accurately to zero.

2. Sight the sun or any distant object furnishing a definite line, and observe whether the reflected image appears displaced laterally with respect to the object (*b*, Fig. 50).

Adjustment.—If so, by means of the adjusting key turn the screw 8 (Fig. 47) until the lateral displacement is eliminated.

THE THEODOLITE

The theodolite is an instrument designed for the measurement of horizontal and vertical angles. It is the most precise instrument available for such observations, and is of wide applicability in surveying.

The general constructive arrangement will first be considered, beginning with the parts relating to horizontal measurements.

Parts for Horizontal Measurement.—The line of sight is telescopic, and, in observing the horizontal angle subtended at the instrument by two objects, the telescope is directed first to one and then to the other. In order that the angle may be recorded, the instrument must evidently be furnished with:

- (1) A horizontal graduated circle.
- (2) A vertical axis passing through the centre of the circle and about which the telescope may be turned from the direction of one object to that of the other.
- (3) A horizontal axis about which the telescope may be moved in a vertical plane, since the points sighted may be at different elevations.

(4) An index, connected with the telescope, and placed against the graduations of the circle, so that the horizontal angular movements of the telescope may be exhibited by the change of reading of the index on the fixed circle. (The possible alternative of attaching the circle to the vertical axis of the telescope and observing readings against a fixed index is not adopted.)

Perhaps less obvious at the outset is the necessity for adding the following features :

(5) A clamping device for holding the index at a given reading, and some form of slow motion, or tangent, screw to facilitate adjusting it exactly to that reading.

(6) Mounting of the graduated circle so that it may, when required, be rotated about the vertical axis of the instrument, as it is commonly necessary that the graduations should lie in particular directions from the centre. As the circle must be stationary during the measurement of an angle, it must be fitted with a clamping arrangement and also with a slow motion screw for its exact orientation.

The manner in which these features are embodied in the theodolite is illustrated diagrammatically in Fig. 51, which indicates an instrument provided with four levelling screws: the slow motion screws are not shown. The index forms the zero of a vernier, of which there are usually two. The provision of an inner and an outer axis, coned to eliminate looseness due to wear, is necessary to meet the requirements of paragraphs 5 and 6 above. Keeping in view the fact that *A* is prevented from rotating by its connection to the tripod and by the grip of the levelling screws, the effects of tightening one or both clamps can be followed from the diagram. When the lower clamp is tightened, *B* is fixed, but, if the upper clamp is slack,

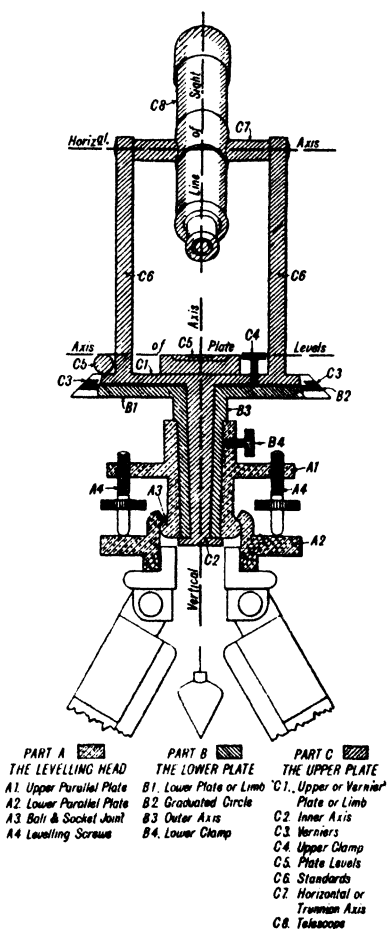


FIG. 51.—DIAGRAMMATIC SECTION OF THEODOLITE.

When the lower clamp is tightened, *B* is fixed, but, if the upper clamp is slack,

C can be rotated because the inner axis is free. If now the upper clamp is also tightened, no part can be rotated horizontally, but, by releasing the lower clamp, while the upper is tight, *B* and *C* can be rotated as one upon the outer axis. By actuating the appropriate tangent screw, however, a piece which is clamped may be moved through a small angle. Thus, if both clamps are tight, turning the lower tangent screw will have the effect of moving *B* and *C* together relatively to the levelling head. In consequence, the line of sight may be adjusted exactly on an object without altering the vernier reading. The upper tangent screw controls part *C* only, and using it to direct the line of sight on to a signal necessarily changes the position of the index on the fixed circle.

Parts for Vertical Measurement.—The observation of vertical angles differs from that of horizontal angles in that the former are always measured from a fixed reference plane, *viz.* the horizontal plane through the horizontal axis of the telescope. The further essentials of the instrument are therefore :

- (1) A vertical graduated circle, or, since very large vertical angles have rarely to be measured, an arc or arcs in place of a complete circle.
- (2) An index.
- (3) A clamp for fixing the circle and index together, and a slow motion screw for fine adjustment.
- (4) A sensitive spirit level for the definition of the horizontal plane.

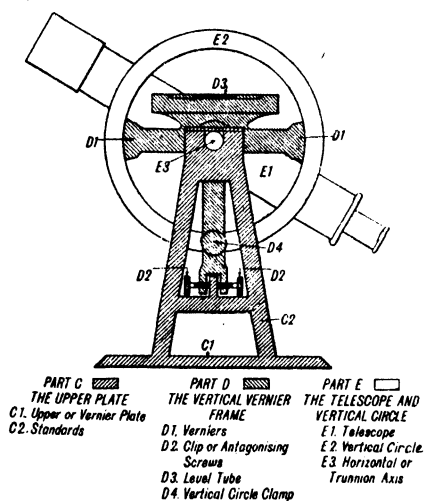


FIG. 52.

One arrangement of these parts is sketched in Fig. 52. The relation between the vernier index and the circle is the reverse of that obtaining with the horizontal circle. The vertical circle is rigidly connected with the telescope, and the verniers remain stationary during an observation. The two verniers are mounted upon a T-shaped frame attached to one standard by clip screws, by means of which adjustment may be effected when required. The level tube is either attached to the telescope or mounted on the vernier frame as shown : some instruments

have two tubes, one in each position. It will be seen that the effect of clamping is to fix *E* relatively to *D* and *C*, but a small motion of *E* can be obtained by actuating the tangent screw. When the clamp

is fixed, slackening one clip screw and tightening the other causes a movement of *D* and *E* together relatively to *C*.

Types of Theodolite.—The arrangement shown in Figs. 51 and 52 indicates that the telescope can transit, *i.e.* make a complete rotation about its horizontal axis. The possibility of reversing the line of sight by rotation of the telescope constitutes the necessary and distinguishing feature of the *Transit Theodolite* or *Transit*. It is not essential that the rotation should be complete, and frequently the proportions of the telescope and standards are such that reversal can be effected only with the eyepiece end downward. The standard British transit is furnished with a complete vertical circle, but this feature does not constitute the distinctive characteristic of the type. A transit without means for measuring vertical angles is known as a *Railway Transit*.

Non-transiting patterns comprise the obsolete *Wye* or *Plain* and *Everest* theodolites. In these, the manner of mounting the telescope is such that the line of sight cannot be reversed by rotation of the telescope, but the effect of transiting is obtained by removing the telescope from its supports and turning it end for end. Although these instruments possess the merit of compactness, they are inferior in utility to the transit, which has entirely superseded them.

Transit Details.—Fig. 53 shows the construction of an ordinary pattern of transit theodolite, of which important fittings are further described.

Levelling Screws should be of fine pitch, $\frac{1}{32}$ in. being suitable. Delicacy of action is also dependent upon the distance of the screws from the vertical axis; the greater the distance the smaller will be the tilt caused by one turn. Either three or four screws are used, and the construction of the levelling head differs in the two types.

A three-screw head is illustrated in Fig. 53, the foot plate 4 having three arms which receive the lower ends of the levelling screws. The foot plate forms a separate part, which, in the assembling of the instrument, has to be screwed to the tripod head. The locking plate is provided with slots of keyhole shape, as in Fig. 54; by sliding it round sufficiently to bring the wider part of the slots opposite the recesses in the foot plate, the levelling screws may be entered. On sliding back the plate, and clamping it, the screws are clipped, but the connection is not very firm if the locking plate or the balls of the levelling screws have become worn. Greater rigidity is obtained by having the ball ends of the screws permanently fitted into the foot plate, as illustrated in Figs. 71 and 92, which also show the encasement of the threads of the screws as a protection against dust. The screws operate in bushes fitted into, and extending above and below, the levelling head, and dust caps are screwed on to the upper ends of the bushes.

Four levelling screws necessitate the arrangement of parallel

plates and ball and socket joint outlined in Fig. 51. As constructed, the four-screw instrument has the more compact levelling head, but

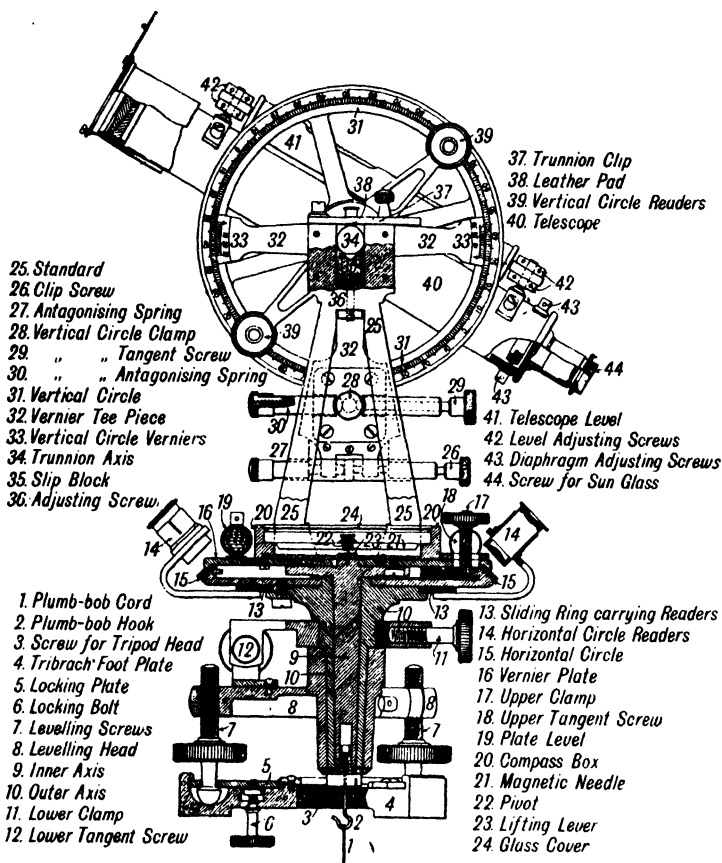


FIG. 53.—TRANSIT THEODOLITE.

this is its only merit. The use of four points of support, when three are sufficient for stability, leads to uneven distribution of pressure on, and excessive wear of, the screws, and is likely to strain both the axes and the screws. The three-screw instrument is free from these objections, and has the important advantage of being more rapidly levelled (page 86).

A *Plumb Bob* is suspended from the vertical axis to enable the instrument to be centered over the ground point from which the measurements are required. The cord must have a slip knot or other means to permit the plummet to be adjusted to the proper level. The operation of centering the instrument (page 85) is

facilitated by the provision of a *Movable Head* or centering arrangement, whereby the instrument and attached plumb line may be moved independently of the tripod and clamped thereto when the plumb bob is over the mark.

Several patterns of movable head are in use. Fig. 54 shows a simple form. The available movement consists of a straight line motion controlled by the pin 7 in the slot 8 formed in the bottom plate, which is screwed to the tripod, and a rotation about the pin

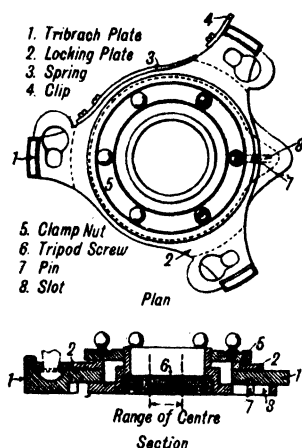


FIG. 54.—MOVABLE HEAD.

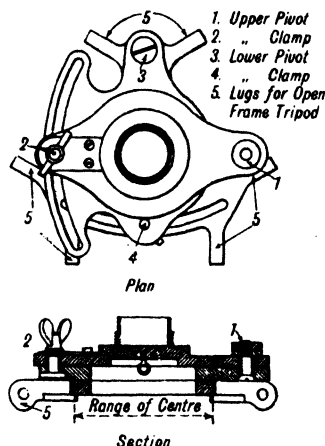


FIG. 55.—TROUTON AND SIMMS' CENTERING ARRANGEMENT.

limited by the projections on the underside of the arms of the tribrach plate.

Fig. 55 illustrates Messrs. Troughton and Simms' centering arrangement, which permits a much greater range. It consists of two plates movable relatively to the tripod head and to each other. The upper plate, carrying the instrument, can be rotated about pivot 1 and clamped to the lower plate, which is provided with a similar movement at right angles to the first.

An improvement is made in some instruments by introducing the centering arrangement just above the levelling screws instead of on the tripod head.

In circumstances requiring the instrument to be placed at some height above the station point, it is a difficult matter to perform the centering by means of a long plumb line. An optical plumbing device is fitted in some modern theodolites, and consists of a small telescope which can be attached horizontally to the levelling head, an axial, vertical line of sight being obtained by means of a prism. Optical plummets may be adapted for sighting upwards, as well as downwards, so that in tunnel work the instrument may be centered below a mark in the roof.

Level Tubes.—Two plate levels are provided for the levelling up of the instrument, and sometimes a small circular level is fitted to the foot plate to facilitate the setting up of the instrument. Of the plate levels, one is placed at right angles, and the other parallel, to the line of sight, the second level being sometimes mounted on one of the standards. The other level should be the more sensitive, since it controls the horizontal axis and therefore the verticality of the plane of rotation of the line of sight. The sensitiveness of both should depend upon the resolution of the telescope and the delicacy of the instrument generally. It ranges from 20 to 60 sec. per 2 mm. division in instruments for ordinary work. The bubble tube attached to the telescope (Fig. 53) or to the vertical vernier frame (Fig. 52), being used in vertical angulation and occasionally for ordinary levelling, should be more sensitive, a value of 10 to 20 sec. per 2 mm. division being required. This altitude level, as it is called, is better placed on the vernier frame, since in that position it exhibits the stability of the instrument while the telescope is tilted for the observation of vertical angles.

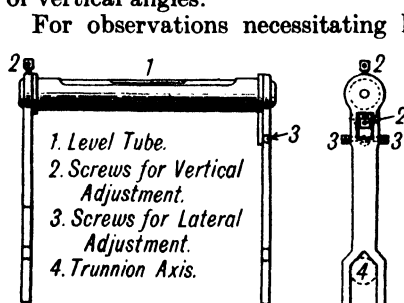


FIG. 56.—STRIDING LEVEL.

For observations necessitating levelling of the horizontal axis with greater delicacy than is possible by means of the plate level normal to the telescope, a *Striding Level* (Fig. 56) is required. This level is mounted on two legs with V-shaped ends, which can be placed upon the trunnion axis. The level tube is adjustable so that its axis and the feet of the legs may be parallel. By reversing the

striding level on the trunnion axis, the latter may be accurately levelled. The sensitiveness of the striding level is usually equal to, or greater than, that of the altitude level.

Circles.—The diameter of the horizontal circle between the reading edges of the graduations serves to designate the size of an instrument: this ranges from 3 in. to 12 in. The $4\frac{1}{2}$ -in., $4\frac{3}{4}$ -in., and 5-in. sizes are those most commonly used in ordinary surveying and engineering work. The 6-in. instrument is employed when a higher degree of accuracy than usual is required, but 6-in. vernier theodolites have been largely superseded by smaller instruments in which the circles are read by micrometer microscopes (page 72) instead of verniers. Instruments of sizes smaller than 5 in. have increased greatly in popularity of recent years, largely owing to modern improvements in graduating machines. The use of instruments larger than 6-in. is confined principally to large triangulation surveys. The vertical circle is commonly of the same diameter as the horizontal circle, but is sometimes made rather smaller.

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The graduations usually are cut on a ring of silver, but in some modern instruments, such as the "Tavistock" and Zeiss theodolites, the circles are of glass, with the graduations marked directly on them. These glass circles have the advantage that they can be divided with extreme accuracy, and at the same time the marks themselves made finer and sharper than is possible on silver, so that, when micrometers are used to read them, a higher magnification is possible. In some parts of the tropics, however, they are liable to fungus (page 34) unless special precautions are taken to avoid it.

For instruments read by verniers the division is generally as follows :

Diameter	Circle Division	Verniers read to
3"	30'	1'
4" to 5"	30' or 20'	1', 30", or 20"
6"	10' or 20'	10" or 20"

The figures increase clockwise from 0° to 360° on the horizontal circle. In some instruments two sets of figures are engraved, increasing in the clockwise and counter-clockwise directions respectively, and double verniers (page 41) are used. This graduation enables angles measured counter-clockwise to be read as directly

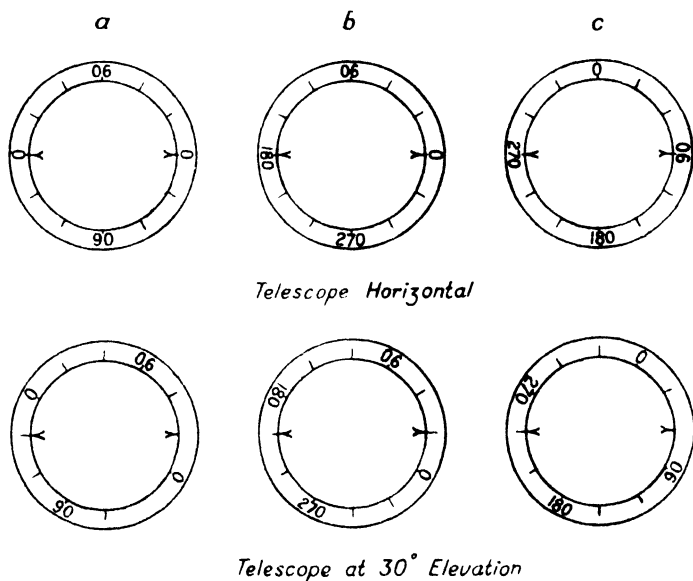


FIG. 57.

as those measured clockwise, but the risk of mistakes in reading is greater than with the single figuring, and the latter is to be preferred.

The vertical circle is divided and figured either in quadrants or

continuously from 0° to 360° . The former system is generally preferred in small instruments, the readings at the horizontally opposite verniers both being 0° when the telescope is horizontal (*a*, Fig. 57). With this arrangement, the same reading is obtained, on both verniers, for an angle of elevation as for an angle of depression of the same magnitude. It is therefore necessary to distinguish between elevations and depressions in booking, and in the observation of small vertical angles care has to be taken to notice whether they are above or below the horizontal, especially if the circle is not marked + and - on either side of the zeros. To obviate mistakes from this source, and to afford different readings at the opposite verniers, a continuous graduation round the circle is sometimes preferred. The figuring may be that of *b*, Fig. 57, giving readings of 0° and 180° for a horizontal line of sight, or as shown at *c*, in which the corresponding readings are 90° and 270° .

When an angle of elevation is being measured with the telescope pointing from right to left, the position of the figures is as shown on the lower half of Fig. 57, since the circle rotates with the telescope. For an elevation of n° , the readings given by both verniers of circles *a*, *b*, and *c* respectively, taking the vernier next the eyepiece first, are : n°, n° ; $n^\circ, (180^\circ + n^\circ)$; $(90^\circ - n^\circ), (270^\circ - n^\circ)$. For the same angle below the horizontal, the readings would be : n°, n° ; $(360^\circ - n^\circ), (180^\circ - n^\circ)$; $(90^\circ + n^\circ), (270^\circ + n^\circ)$. There is evidently little possibility of confusion between elevations and depressions with the whole circle figurings, of which *b* is preferable to *c*. The risk of mistakes in the extraction of trigonometrical ratios is, however, greater than in the quadrantal system, which is the more convenient for ordinary work.

To facilitate reading, British instruments with metal circles have the graduated face of the horizontal circle made in the form of the frustum of a cone : the vertical circle is plane. It is desirable to protect both circles by a solid guard in which glass covered openings are provided at the verniers. The horizontal circle of Fig. 53 is shown covered in : the vertical circle is most commonly left exposed.

The two *Horizontal Verniers* are placed diametrically opposite each other, and do not appear in Fig. 53 as the line joining their zeros is perpendicular to the line of collimation. This is the usual arrangement, but it necessitates the observer's moving round from his position at the eyepiece in order to read even one of the verniers, which is sometimes all that is required. A position with the zeros at 30° to 40° with the line of sight is most convenient for work in which only one vernier is consulted, and this arrangement is adopted in some patterns. The plates carrying the vernier scales have the same bevel as the circle, and they may be fitted with a weak flat spring to maintain contact with it. In modern designs, however, the verniers are not actually in contact with the circle, so that wear and frictional resistance to rotation are minimised.

The magnifiers used for reading the verniers are mounted either

on small brackets hinged to an arm which can turn about the centre of the upper plate or as shown in Fig. 53. The optical arrangement of the ordinary pattern is that of the Ramsden eyepiece (page 27), and, in taking a reading, the eye must be brought close to the eye lens. An alternative pattern of reader, consisting of an achromatic doublet, enables the graduations to be focussed with the eye 6 or 8 inches away, and is more convenient to use.

The *Vertical Verniers* are given a slight bevel only, and are rigidly connected to their frame. If the vertical circle is divided in quadrants, the readings have to be taken clockwise and counter-clockwise in alternate quadrants. Instead of fitting double verniers, however, a common arrangement is to have single verniers with two sets of figures, reference being made to those which increase in the same direction as the figures on the quadrant being read.

Clamp and Tangent Screw fittings may take several forms. In Fig. 53, the lower clamp 11 is screwed into a circular casting which embraces the outer axis, and which is connected to the fixed levelling head through the tangent or slow motion screw 12, the latter being threaded through a lug projecting from the levelling head. On applying the clamp, a pad is pressed against the outer axis, which is then secured through the tangent screw to the levelling head. On turning the tangent screw, the outer axis, circle, and clamp fitting are rotated together relatively to the levelling head. In an alternative, but less satisfactory, pattern, the clamp is placed tangentially, and tightens a split ring round the axis as shown in Fig. 58.

In the upper clamp and tangent screw and those of the vertical circle in Fig. 53 the tangent screw operates directly on the clamp. The tangent screws are protected from dust, and are fitted with opposing springs to take up wear and give a uniform motion.

The *Telescope*, to permit of transiting, is limited in length by the height of the standards, which must be reasonably low for stability and lightness. If the telescope is of the internal focussing type, it will be capable of being transited both ways, *i.e.* with the eyepiece or the object glass downwards. With an external focussing telescope the design may admit of this, but a common arrangement is to make the telescope to transit only with the eyepiece downwards, and therefore the focussing movement takes place at the object glass end. Two Ramsden eyepieces are generally provided, giving a high and a low power. The magnifications range from about 15 to 30 diameters, but should accord with the capabilities of the instrument in other respects (page 91). Stadia hairs should always

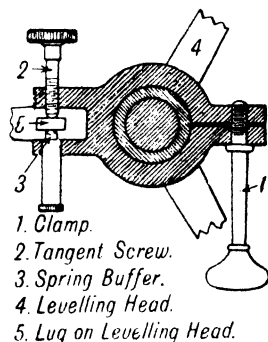


FIG. 58.—CLAMP AND TANGENT SCREW.

be fitted, and, if the instrument is being used for astronomical observations, a diagonal eyepiece and a sun cap are necessary. Small open sights are sometimes fixed on the outside of the telescope to enable the surveyor to direct the line of sight rapidly towards a mark, and they prove a convenience for work in wooded country. They should be fitted on both the upper and lower sides, so that they may be equally available in the direct and reversed positions of the telescope.

If the instrument is to be used at night or in underground work, provision must be made for illuminating the cross-hairs, which would not otherwise be visible. The simplest method is to attach a reflector in front of the object glass in order to project a small amount of light from a lamp or candle into the telescope. The reflector may take the form of a very small mirror or prism, a silver bead, or simply a piece of white paper with a sufficiently large opening through which to sight. Otherwise, the telescope trunnion is made hollow on the side remote from the vertical circle, and a lamp, supported from the standard, projects light along the hollow axis into the telescope. The rays are focussed on to a metal reflector of about $\frac{1}{8}$ -in. diameter placed in the longitudinal axis of the telescope, and are reflected on to hairs. Some theodolites are fitted with a complete lighting set with lamps for the illumination of the circles at the verniers or microscopes, as well as of the reticule. A cell is carried on the tripod, and the current is transmitted through the vertical axis, the intensity of the light being controlled by a rheostat.

The *Compass* needle of Fig. 53 floats in a circular box, the circumference of which is divided to degrees. This second graduated circle is somewhat unnecessary, since magnetic bearings may be read upon the larger horizontal circle when oriented by the needle, and a better arrangement is to contain the needle in a narrow rectangular box provided with a scale of only a few degrees on either side of zero (Fig. 59). Such a compass is called a trough compass,



FIG. 59.—TROUGH COMPASS.

and has the advantage that a longer needle can be employed, while weight is reduced. It is attached when required to either the upper limb or the lower, according to the design of the theodolite. In the former case, the compass is screwed on to one of the standards so that its axis is parallel to the line of sight of the telescope. In the latter, it is fitted to the underside of the circle plate, the projecting flanges shown in Fig. 59 being slid into grooves on the plate. These grooves must be so situated that, when the needle floats centrally

in the box, the circle is oriented for the reading of magnetic bearings, *i.e.* the line of collimation lies in the magnetic meridian when the circle verniers read 0° and 180° . Screws are provided for adjusting the position of the scales.

Although they are sufficiently accurate for most purposes, neither the circular compass nor the ordinary trough pattern lends itself to very precise setting, owing to parallax arising from the difficulty of ensuring that the eye is in the vertical plane of the needle. A greater degree of accuracy is attained by the use of a so-called tubular compass. In this case, the needle is contained in a tube at one end of which there is fitted an eyepiece and a diaphragm carrying a glass plate with vertical rulings, which is nearly in the same plane as one end of the needle. The reticule being suitably illuminated by a reflector, the observer, on looking through the eyepiece, sees the end of the needle, which may be formed with a sharp edge on top or may be fitted with a light frame carrying a reference line.

Testing the Trough Compass.—In most cases the magnetic axis of the needle of a trough compass does not coincide exactly with the geometrical axis, so that the line joining the ends of the needle does not indicate the true magnetic meridian. In some compasses, the needle is accessible and it is possible to unscrew the agate bearing and replace it on the under side of the needle. When this is so, the compass can be tested as follows :

Set the instrument, with the needle swinging freely, so that the end of the latter is approximately opposite the zero mark of the scale. Tighten all clamps of the theodolite and note the readings on the scales at each end of the needle. Now, without disturbing the theodolite in any way, lift the needle very carefully off its pivot, unscrew the agate bearing and replace it so that the needle will swing with what was its under side on top. Read the scales at both ends. If the readings are not exactly the same as before, the geometrical and magnetic axes do not coincide and the true position of the magnetic axis is the division half-way between the first and second readings at either end. Repeat the test several times and take the mean of all results. The test may also be repeated by turning the theodolite through 180° about its vertical axis and replacing the needle so that the scale or mark, originally opposite the north end, is now opposite the south end of the needle.

The reason for this method of testing can easily be seen by drawing two lines inclined at a small angle to one another, the one to represent the geometrical axis and the other the magnetic axis, and then imagining the paper to be turned upside down.

A *Quick Levelling Head* may be fitted to facilitate levelling up, but these are not so common in theodolites as in levels (page 120), principally on account of the increase of weight—an important consideration in the theodolite.

Wall Tripod.—It may be necessary to set the instrument over a

point where the use of the ordinary tripod is impossible. In the case of the three-screw levelling head, the tribrach plate is usually provided with three points which can be used as supports, but the four-screw head may require the attachment of a separate piece, called a wall tripod (Fig. 60). The instrument is centered when the three supporting points are equidistant from the station point.

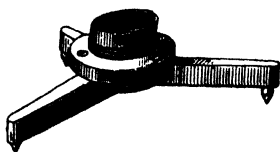
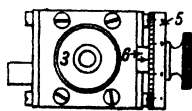
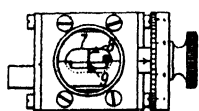


FIG. 60.—WALL TRIPOD.

The Micrometer Microscope.—As the required fineness of reading increases, the vernier decreases in utility as a means of measuring fractional parts of the circle because of the difficulty of estimating the position of coincidence. Recourse may be had to the repetition method of angle measurement (Vol. II, Chap. III), but in geodetic and astronomical instruments the vernier has long been superseded by the micrometer or measuring microscope, which is superior not only in accuracy but also in the rapidity with which readings may be taken. In recent years, the micrometer has been increasingly used in ordinary work as a fitting for small theodolites.

Principle.—The principle of the micrometer screw is widely used in physical and workshop instruments for the accurate determination of length, the required measurement being primarily obtained as a certain number of times the pitch of a fine screw working in a fixed nut, plus a fraction of a pitch or turn. By dividing the circumference of the head of the screw into a number of equal spaces, a convenient and accurate means is afforded for estimating the fractional part by reference to a fixed index mark. As applied in the theodolite, the micrometer screw is used in conjunction with a microscope, and is arranged to measure parts of the real image of the circle divisions which is formed by the objective of the microscope.

Details of Micrometer.—Figs. 61 to 64 illustrate the micrometer as used in theodolites for ordinary work. The microscope consists of an achromatic objective 1 and a Ramsden eyepiece 3. In front of the latter the microscope tube is enlarged to form a rectangular box, through one side of which the micrometer screw passes. The screw is provided with a milled head and a graduated drum 5, the rotation of which can be read relatively to the fixed index arrow 6. By rotating the head a lateral motion is imparted to the slide 7, across the opening in which are stretched either two closely spaced parallel webs 8, cross-hairs, or a single hair. The slide is controlled by two light spiral springs (not shown), with the object of eliminating backlash. By suitably arranging the position of the entire

Elevation
FIG. 61.Elevation - Eyepiece Removed
FIG. 62.

microscope relatively to the circle and the distance of the objective from the hairs, (a) the real image of the circle graduations may be formed in the plane of hairs, and (b) the size of the image of a circle division may be made such that an exact number of turns of the screw is necessary to carry the hairs from the image of one graduation to that of the next. When these two requirements are met, fractional parts of the scale divisions can be measured on the image, the circumstance that they are measured on a chord instead of an arc leading to no appreciable error.

As in the vernier, there must be provided a point or line of reference from which to obtain both the approximate and fractional readings. This zero is defined, for the reticule illustrated, by the imaginary line midway between the parallel hairs when these occupy the middle of the field of view, the drum reading zero. For convenience, this point is approximately marked by a notch 9, cut in a small plate fixed immediately in front of the hairs, it being impracticable to place it in their plane. When the eyepiece is focussed for the hairs and the image, the notch is not therefore in precise focus, but *the whole function of the notch is to facilitate the determination of the approximate reading, and it is not referred to for the measurement of the fractional part.*

Least Count of Micrometer.—To determine the least count, the nature of the circle division must first be ascertained by examination through the microscope. In the case of ordinary patterns of instruments up to 5-in. diameter, the circle spaces represent 10 min. The 6-in. theodolite is usually divided to 10 min., but the division may be carried to 5 min. The latter graduation is adopted in instruments larger than 6 in. Since the Ramsden eyepiece is non-erecting, the figures are engraved in the inverted position, so that they appear right side up when viewed through the microscope. The inversion of the image makes the circle appear to be graduated counter-clockwise instead of clockwise.

By rotating the milled head, the number of turns of the screw necessary to carry the hairs from the image of one graduation to that of the next should then be found. If t turns are required, and the circumference of the drum is divided into n parts, the value of a drum division is $\frac{1}{nt}$ of a circle division. In theodolites with circle

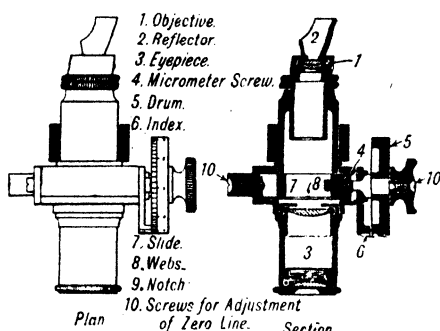


FIG. 63. FIG. 64.
MICROMETER MICROSCOPE.

spaces of 10 min., t is made 1, and the drum has altogether 60 divisions, so that the value of each is 10 sec. In more precise instruments t is 5, and the drum divisions represent 1 sec. or 2 sec.

To Read the Micrometer.—As an example, let the circle be divided to 10 min., and let one turn of the screw carry the hairs across a circle division, the drum having 60 parts. The eyepiece should, if necessary, first be adjusted until the hairs are very sharply focussed. The appearance presented through the microscope may then be as in Fig. 65. The hairs may be in any position, but if they were brought to the middle of the field of view, so that the drum reads zero, they would appear to lie in the notch. The approximate reading,

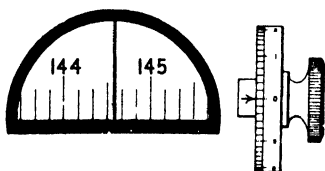


FIG. 65.

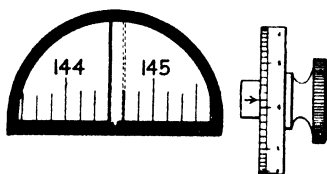


FIG. 66.

given by the position of the notch, is $144^{\circ} 30'$, and, to obtain the additional part, all that is necessary is to move the hairs until they are centered over the graduation next the notch, as shown in Fig. 66. The drum now records $4' 10''$, so that the complete reading is $144^{\circ} 34' 10''$.

Notes.—(1) The object of using two parallel hairs, instead of cross-hairs or a single hair, is to facilitate centering over a graduation. The distance between the hairs is very little more than the width of the image of an engraved line, and it is thus possible to estimate the bisection by the latter of the space between the hairs with considerable precision.

(2) Since one turn of the screw corresponds to a motion of the hairs across a division, it is immaterial on which graduation they are centered, as the drum will register the same reading, if the micrometer is in adjustment. Thus, in Fig. 66, in taking the reading, the hairs might have been moved to the position shown dotted, with the same result as before.

(3) For the same reason, it is altogether unnecessary to put the hairs over the notch before taking a reading. They have simply to be moved from the position in which they were left from the previous observation. For the avoidance of lost motion, the hairs should, however, always be brought on to a graduation by screwing against the spring. It may therefore be necessary to run the hairs past a graduation in order to bring them on to it in the constant direction.

(4) Care should be exercised not to press upon the head of the screw. It should be fingered as lightly as possible.

(5) The caution, mentioned on page 44, against side illumination of the circle, applies with special force to micrometer reading. The illumination, whether daylight or artificial, must be thrown upon the circle in a radial direction by turning the reflector (Fig. 64) into the proper position. The reflector is usually formed of plaster of Paris, and its matt surface affords a soft uniform illumination.

(6) Apparently the reading could be carried to the nearest second by estimation of parts of the drum divisions. With small instruments, however,

the readings cannot in general be relied upon to less than 10 sec., as the accuracy of an observation depends primarily upon the theodolite itself. The magnifying power of the micrometer microscopes should accord with the sensitiveness of the instrument to which they are fitted, and for instruments of standard pattern smaller than 8 in. this is insufficient to permit of reading to a single second with accuracy.

To Set the Circle to a Given Reading, the angle must be considered as consisting of a number of circle divisions (the approximate reading) and a fractional part (the micrometer reading). The hairs have first to be placed to the left of the centre point by a distance corresponding to the latter reading, and then the graduation representing the approximate reading must be brought to bisect the space between the hairs, the final adjustment being made by the upper tangent screw.

Errors and Adjustments of the Micrometer. See Vol. II, Chap. III.

The Estimating Microscope.—A simple method of reading the circles by microscope without using a micrometer screw and drum has developed widely on the Continent. The microscope is fitted with a diaphragm carrying a glass plate on which is ruled a scale of equal parts, the total length of the scale being equal to that of the real image of a circle space produced by the microscope objective. The scale, being placed at the common focal plane of the objective and eyepiece, enables estimates to be made of the fractional parts, the zero of the scale forming the reference point.

Thus in Fig. 67, which represents the view presented by the microscope in the case of a circle divided to 20 min., the scale is divided into 10 parts, each representing 2 min. The approximate reading given by the position of the zero is $144^{\circ} 20'$, and, if the scale reading is estimated to be 7.1, the complete reading is $144^{\circ} 20' + 14' \cdot 2 = 144^{\circ} 34' 10''$, to the nearest 10". If the readings of opposite microscopes are taken for averaging, the reading is simplified by regarding the scale divisions as single minutes and adding the sum of the two scale readings to the approximate reading. Thus, if the scale readings are 7.1 and 7.2 respectively, giving a sum of $14' \cdot 3$, the mean reading is $144^{\circ} 34' \cdot 3$.

In the case where the circle is divided to 10 min., a scale of ten parts enables estimates to be made at each microscope to 0.1 min. Apparently by increasing the power of the microscope, so that a more finely divided scale can be used, any degree of refinement of reading is possible, provided that the microscope is adjusted so that the length of the image of a circle space equals that of the scale. Messrs. Cooke, Troughton and Simms, of London, have patented a design of estimating microscope, described in Appendix I, page 598, in which the scale can be moved parallel to the circle graduations and is so designed that it affords intersections at 20 sec.

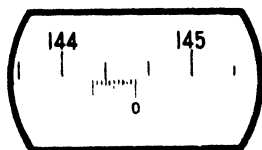


FIG. 67.

intervals from which to estimate. Actually, however, high magnification makes the engraved lines of the circle appear too coarse for refined estimations, unless the lines are very accurately tapered at the end or are ruled on a glass circle instead of a silver ring, as is done in some modern theodolites. The estimating microscope is therefore inferior to the micrometer microscope for precise readings, but it forms a very simple substitute for the vernier.

Double Reading Theodolites.—Important angle measurements, especially such as warrant the use of a micrometer theodolite, always require that readings at diametrically opposite points on the circle should be taken for averaging (page 90). This necessitates the observer's moving round the instrument, and introduces the possibility of his disturbing it, particularly when working in the dark. To minimise this risk, and to save time in reading the circles, various designs of theodolite have been introduced in which it is possible to observe the readings of the opposite points of both circles from one position.

Casella's Double Reading Theodolite.—In the instrument designed by Messrs. C. F. Casella & Co., of London, the images of the opposite parts of the horizontal circle are brought into the field of a single microscope by means of prisms placed in the vertical axis. These images are presented side by side to the observer along with an image of the micrometer drum, as shown in Fig. 68. The two readings are obtained by adjusting the micrometer hairs on the graduations of both images successively in the usual manner. In the case illustrated, the micrometer hairs are centered on a graduation of the left-hand image, and the reading is $144^{\circ} 34' 10''$. The vertical circle microscope, which is of the same pattern, is placed with its axis in that of the telescope trunnion, and is just above the horizontal circle microscope. With

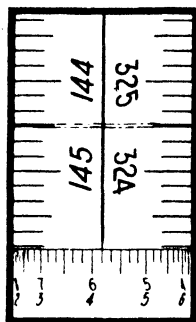


FIG. 68.

this arrangement the observer can obtain all the angle readings from one position.

The Wild Universal Theodolite.—This instrument, designed by Dr. H. Wild, the inventor of the type, and made by his firm at Heerburg, Switzerland, embodies a number of novel features tending to precision of reading and saving of time in observing. The horizontal circle is only $3\frac{1}{2}$ in., and the vertical circle 2 in., in diameter. Both circles are read through a single microscope placed alongside the telescope, the movement of a prism determining which is viewed. A noteworthy feature is that the same micrometer system serves for both circles, their different sizes being compensated for by magnifying the graduations different amounts.

The optical arrangement may be followed from Fig. 69. External

light is received by the adjustable prism *2a*, and is transmitted through the system *2b, c, d* to opposite points of the horizontal circle *1*, being reflected to right and left by the prisms *2d*. The glass ring forming the horizontal circle is graduated on the upper side, and it is silvered on that face. The rays from the illuminated graduations at diametrically opposite points are reflected downwards into

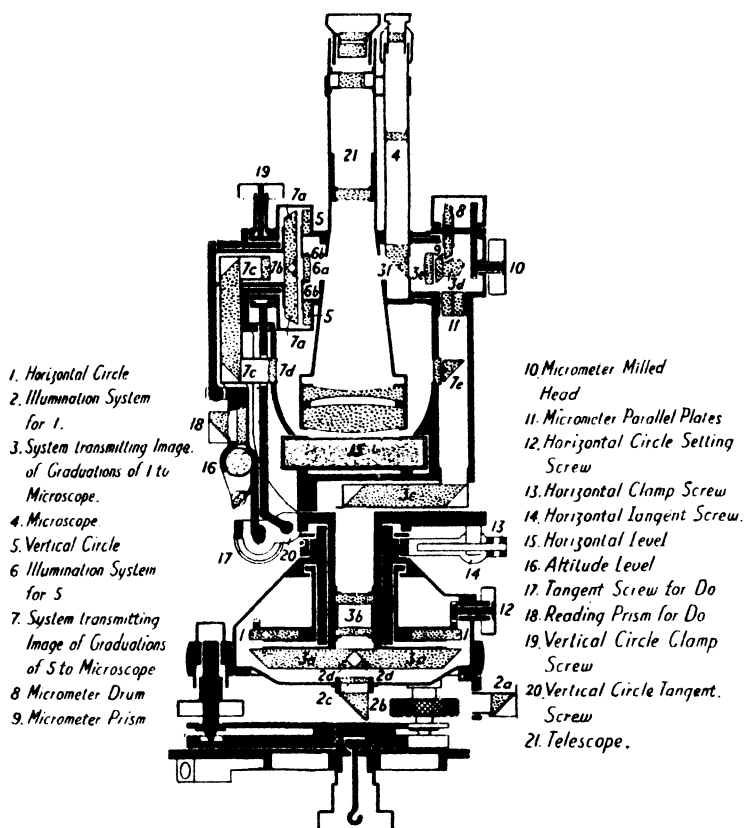


FIG. 69.—WILD UNIVERSAL THEODOLITE.

the prisms *3a*, which transmit them upwards side by side through the lenses *3b* and the prism *3c*. Prism *7e* forms part of the vertical circle system, and is withdrawn while the horizontal circle is being read. The rays traverse plates of parallel glass *11*, which form part of the micrometer device, and are reflected in prism *3d*. They are projected through lens *3e* on to prism *3f*, which transmits them along the reading microscope *4* to the eye.

A similar scheme enables the images of opposite points of the

vertical circle 5 to be viewed in the microscope when the prism 7e is placed, by the movement of a button, in the position illustrated. Light is introduced by a prism, not shown, and travels by way of lens 6a and prisms 6b to the circle. The rays reflected from the illuminated graduations proceed through 7a, b, c, d to the prism 7e, and thence through the parallel plates to the microscope as before.

The micrometer drum 8, consisting of an unsilvered glass ring, is suitably illuminated. Light passing through it is reflected by prism 9, which carries the index mark, proceeds through the object lens 3e, and is transmitted along the microscope by prism 3f together with the circle images. The circles are divided to 20 min., and the approximate reading is estimated to 10 min. The micrometer drum is divided into 10 single minute spaces, each of which is subdivided to single seconds. The turning of the micrometer screw 10 rotates the micrometer drum, and simultaneously tilts the plates 11, moving the images of the two parts of the circle in opposite directions.

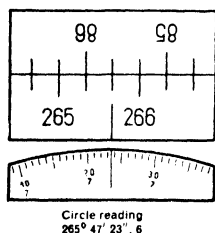


FIG. 70.

tion corresponding to the figure diametrically opposite to it on the graduated circle. This number multiplied by 10 gives the tens of minutes. The single minutes and the seconds to be added are then read on the scale in the lower window, the lower figures being minutes and the upper figures seconds. In the example, the reading is 265° plus $4 \times 10'$ plus $7' 23'' \cdot 6 = 265^\circ 47' 23'' \cdot 6$.

The Tavistock Theonolite.—This instrument (Fig. 71), made by Messrs, Cooke, Troughton and Simms, Ltd., incorporates the method of reading circles patented in 1927 by Instructor Captain T. Y. Baker and Mr. R. W. Cheshire whereby opposite parts of the circle are viewed side by side with the reference mark between them. A feature of this system is that the fine reading device simultaneously moves the circle images in the same direction. Readings are obtained by displacing the images until the graduations on either side of the index mark are equidistant from it. The fractional part of the mean reading is obtained on the image of the micrometer scale, which appears in the field.

The horizontal and vertical circles are divided on glass to 20 min., and are of $3\frac{1}{2}$ in. and $2\frac{1}{2}$ in. diameter respectively. The former is carried on an independent axis, as in the ordinary pattern, and the optical system for the illumination and reading of the circle rotates with the telescope as the latter is wheeled horizontally. The circles

are read by means of independent micrometers direct to single seconds. The reading microscopes are pivoted from the standards, and can be adjusted to suit the observer whether he is using the telescope in the direct or reversed position.

Fig. 72 illustrates the optical arrangement as far as it can be shown on one view. In the case of the horizontal circle, light entering by the reflector *2a* is transmitted to diametrically opposite points of the horizontal circle by a series of prisms of which *2b* is the first. The rays reflected from the silvered surface of the graduated circle are, in the course of their path, reflected by prisms *3a*, and transmitted through movable wedges, or travelling prisms, *3b*, to the prisms *3c*. The latter lie just below the glass disc *4* on which the fine reading scale is engraved, and the rays from the circle are brought to a focus in the plane of the micrometer scale. By means of the system *5a, b, c*, rays from the fine reading scale and from the images of the opposite points of the circle are transmitted side by side into the horizontal circle microscope *6*. The vertical circle reading system is designed on similar lines.

The manner in which the readings are shown in the microscopes is illustrated in Fig. 73, which shows the field of the horizontal circle microscope. Of the three apertures, the bottom one shows the approximate reading, that at the top presents an image of part of the micrometer scale, and in the middle one are brought side by side images of the diametrically opposite points of the circle separated by a relatively broad line forming the index. The three index marks and the mask defining the apertures are placed on a thin glass plate between *3c* and *4* (Fig. 72). To minimise the possibility of the wrong microscope being read, the relative position of the apertures is made different in the case of the vertical circle, and the letter *V* is shown.

To read, say, the horizontal circle, the milled head *7* is first adjusted until graduations from opposite points of the circle are placed symmetrically about the reference line. This is effected by equalising the narrow bands of light between the index and the graduations on either side. The rotation of *7* required to do so

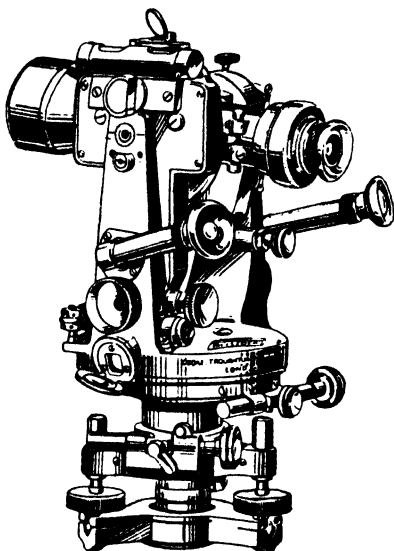
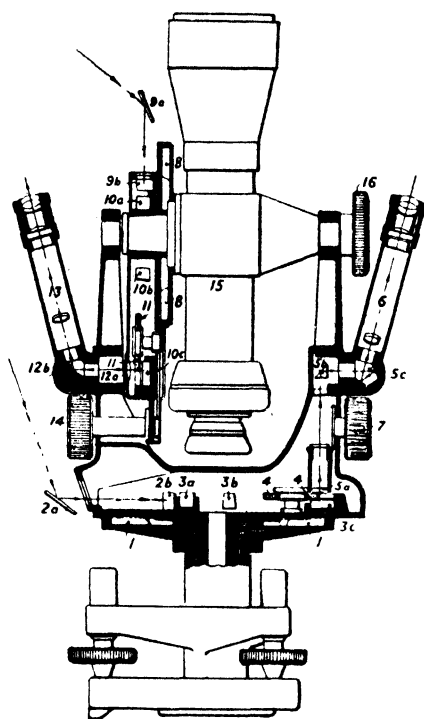


FIG. 71.—THE TAVISTOCK THEODOLITE.

displaces the images by moving the travelling prisms *3b*, turns the fine reading scale to the mean of the opposite readings, and at the



1. Horizontal Circle.
2. Illumination System for 1
3. System transmitting Image of Graduations of 1 to Plane of 4.
4. Horizontal Fine Reading Circle
5. System transmitting Images into 6
6. Horizontal Circle Microscope
7. Horizontal Circle Micrometer Milled Head.
8. Vertical Circle
9. Illumination System for 8
10. System transmitting Image of Graduations of 8 to Plane of 11
11. Vertical Fine Reading Circle.
12. System transmitting Images into 13
13. Vertical Circle Microscope.
14. Vertical Circle Micrometer Milled Head
15. Telescope
16. Telescope Focussing Screw

FIG. 72.—TAVISTOCK THEODOLITE—
PART SECTION.

same time places the graduation representing the coarse reading exactly opposite the index point in the bottom aperture. In the example illustrated the reading is $144^{\circ} 20' + 14' 12'' = 144^{\circ} 34' 12''$. To set the circle to a given reading, the micrometer scale is set to the fractional reading by means of the micrometer milled head, and the circle is swung until the approximate reading is brought roughly to its index mark. The final setting is effected by turning the upper tangent screw until the opposite graduations appear equidistant from the reference line in the small aperture.

The makers claim the following advantages for the Tavistock theodolite.

(1) For convenience of reading, both horizontal and vertical circles are read from the eyepiece end of the telescope.

(2) The two diameters of a circle, 180° apart, can be brought together or slightly separated and viewed simultaneously in a single eyepiece.

(3) Micrometers can be built up inside the instrument to give a mean reading of one second or arc or less.

(4) Much greater magnification can be obtained of the circles and fine reading scales than is possible with graduations cut in silver.

(5) The illumination is brilliant and evenly spread over the field of view;

illumination for night observation is easily arranged and ample.

(6) The instrument can be rendered dust-tight and waterproof, and also smaller and more compact, without loss of accuracy.

The above is a description of the original Tavistock theodolite which at the time of writing is in extensive use all over the world. Messrs. Cooke, Troughton & Simms, however, are now putting on the market an improved model which is described in Appendix I, pages 598 to 601. Some smaller theodolites by the same firm, read by optical micrometer or optical scale, including ones suitable for the three-tripod system of observing (page 84), are described in Appendix I, pages 601 to 603.

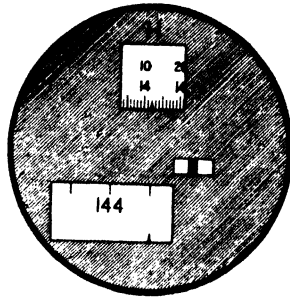


FIG. 73.

Watts Microptic Theodolites.—Besides the Watts Microptic

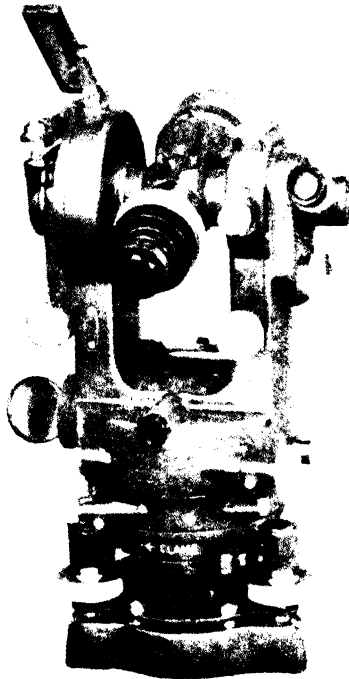
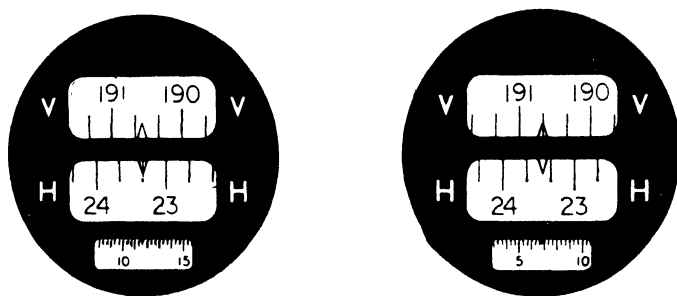


FIG. 74.—WATTS MICROPTIC THEODOLITE No. 1.
(By courtesy of Messrs. Hilger & Watts, Ltd.)

Theodolite No. 2 which is described in Chapter III of Vol. 2, and which reads direct to 1" of arc and by estimation to 0".2, Messrs. Hilger & Watts, Ltd., manufacture a smaller instrument provided with glass circles and optical micrometers which is suitable for ordinary work where geodetic accuracy is not needed. This is Microptic Theodolite No. 1 which reads direct to 20" and by estimation to 5". In this instrument the readings are taken from only one end of a diameter of the horizontal circle but special precautions are taken in the design and manufacture of the instrument to keep centering errors very small. The micrometers are read through an eyepiece which can be rotated to lie parallel to the vertical plane containing the axis of the telescope, so that it is not necessary to move to the side of the instrument to take a reading, both horizontal and vertical circles being read in the same eyepiece. Each instrument is fitted with a built-in optical plummet for centering over a ground mark and for overhead plumbing a centering thorn is fitted on top of the telescope.

Microptic Theodolite No. 1 is shown in Fig. 74. This is the ordinary model, No. ST156, with centering above the footscrews: if the instrument is to be used with the three-tripod system of observing, two other models fitted with a different base are available in one of which centering is above the footscrews and in the other it is below them.



Horizontal Circle

23°	20'	00"
	11'	40"
<hr/>		
23°	31'	40"

Vertical Circle

190°	40'	00"
	6'	50"
<hr/>		
190°	46'	50"

FIG. 75.—SYSTEMS OF CIRCLE READINGS ON THE WATTS MICROPTIC THEODOLITE No. 1.

(By courtesy of Messrs. Hilger & Watts, Ltd.)

INSTRUMENTS—CONSTRUCTION AND ADJUSTMENT 83

The appearance in the field of view of the micrometer is shown in Fig. 75. Here there are three windows, the upper of which gives the degrees and twenty-minute divisions of the vertical circle, the middle one the degrees and twenty-minute divisions of the horizontal circle, and the lower one the minutes and twenty-second readings for each circle. The hollow triangular index mark seen in the upper two windows is patented and it is claimed that it gives a very accurate bisection. Readings are taken by bringing a division of the circle into the hollow index mark so that it exactly bisects the mark. This is effected by rotating the micrometer milled head below the circle reading eyepiece.

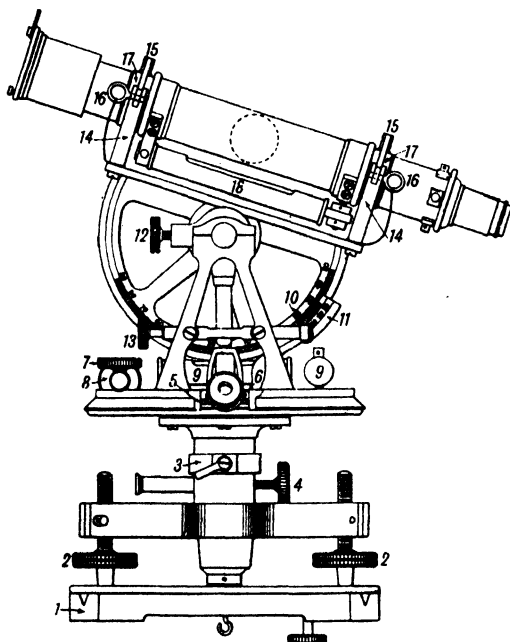
Two important improvements in this instrument are:—a new telescope with a 25-diameter magnification and a gun metal instrument body instead of aluminium, which, by a new casting technique, has resulted in scarcely any increase in weight.

The American Transit.—The ordinary American transit is commonly lighter than, and differs in several respects from, the British model. The trunnion axis is not usually removable from the standard supports, and there is no fitting corresponding to 26, 27 (Fig. 53). The four-screw levelling head is employed to a far greater extent than in this country, but three screws are favoured for precise instruments. The graduated faces of the horizontal circle and vernier are placed in a horizontal plane, this arrangement having the merit of minimising wear between the circle and verniers. The circle is commonly figured in both directions, the verniers being of the double type; and a somewhat open graduation, usually with verniers reading to 1 min., is preferred, so that reading microscopes are commonly dispensed with. A much used pattern is fitted with a vertical semicircular arc, read by a single vernier attached to the standard. The American type of transit is manufactured in England, principally for export to Canada.

Non-transiting Theodolites.—In these the general constructive arrangement of the parts relating to horizontal measurements is the same as in the transit. The manner of mounting the telescope creates differences in the method of adjustment. The Wye pattern is still in use to a limited extent, and its adjustments are given on page 100.

The Wye Theodolite.—The essential difference between this instrument (Fig. 76) and the transit is that the telescope is not directly mounted on the trunnion axis, but is supported on two U- or Y-shaped forks, called wyes. Although held by clips, the telescope is capable of rotation about its longitudinal axis, and, on opening the clips, it can be removed and placed end for end to reverse the line of sight. The collars which rest in the wyes must be truly cylindrical and equal, so that these movements of

the telescope may not alter the direction of the axis. The stage carrying the wyes is attached to the trunnion axis, and carries a graduated semicircle for the measurement of vertical angles. Only one vertical vernier is fitted, and that is attached directly to the horizontal vernier plate.



- | | | |
|----------------------|--------------------|----------------------------|
| 1. Tribrach Plate | 7. Upper Clamp | 13. Vertical Tangent Screw |
| 2. Levelling Screws | 8. " Tangent Screw | 14. Wyes |
| 3. Lower Clamp | 9. Plate Levels | 15. Wye Clips |
| 4. " Tangent Screw | 10. Vertical Arc | 16. " Pins |
| 5. Horizontal Circle | 11. " Vernier | 17. Telescope Collars |
| 6. " Vernier | 12. " Clamp | 18. " Level Tube |

FIG. 76.—WYE THEODOLITE.

The Three-Tripod System of Observing.—The three-tripod system of observing is peculiarly suitable for the measurement of traverse angles when legs are very short—a case that often occurs in underground mining and tunnelling surveys—and in recent years it has also been used to save time and maintain the accuracy of the angular work on ordinary traverses. The equipment consists of special “targets” and extra tripods, the theodolite and targets

being interchangeable on the tripods. A full description of the method is given in Chap. IV, page 211.

TEMPORARY ADJUSTMENTS OF THEODOLITE

The temporary adjustments are :

- (1) Setting over the station.
- (2) Levelling up.
- (3) Elimination of parallax.

Setting Up.—This includes both centering of the instrument over the station by the plumb bob and its approximate levelling by manipulation of the tripod legs only. The accuracy of centering should depend upon the required precision of the observations. In the majority of ordinary small surveys refined centering is not required unless the sights are short. Centering is facilitated if the instrument has a shifting head, but, even without one, rapid setting up should present no difficulty if performed systematically.

Notes.—(1) The legs of the tripod can be moved not only radially but also circumferentially or sideways. The first movement shifts the plumb bob, and tilts the instrument. The second causes a considerable change of inclination without disturbing the plumb bob, and this movement is therefore effective in the approximate levelling of the instrument.

(2) First hold the instrument off the ground with the legs spread out and the plumb bob hanging approximately over the station. On being lowered to the ground, it will not be seriously out of centre, and the exact centering and approximate levelling may be completed by small radial and tangential motions of two legs and the pushing of all three into the ground.

(3) On a hillside, place one leg uphill.

(4) A heavy bob is preferable to a light one, especially when working in wind. In windy weather spread the legs well out, and push them firmly into the ground.

(5) If ground chaining is to be conducted from the station, avoid setting a leg in the chain line.

Levelling Up.—The manner of levelling the instrument by the plate levels depends upon whether there are three or four levelling screws.

Four-Screw Head.—(1) Turn the upper plate until one level tube is parallel to a pair of diagonally opposite screws. The other tube will be parallel to the remaining pair.

(2) By manipulating one pair of screws, as in Fig. 77, bring the bubble to the centre of its run in the tube to which they are parallel. In turning the screws, the thumbs move towards or away from each other, and the left thumb must be moved in the direction in which the bubble is required to travel.

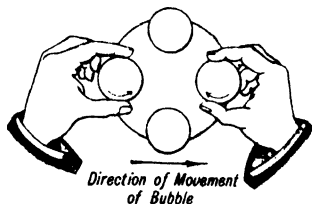


FIG. 77.—MANIPULATION OF LEVELLING SCREWS.

(3) Level the other bubble tube with the remaining two screws, and then releve the first tube if necessary.

Notes.—(1) If there is only one plate level, it will be necessary to rotate the plate through 90° after step (2) to bring the level parallel to the second pair of screws.

(2) If the instrument has been set up much out of level, jamming of a pair of screws may occur, and this may necessitate resetting the instrument. If one screw jams, turn the other only, or slacken the other pair. Do not force the screws.

(3) Screws should be left bearing firmly upon the lower parallel plate so that there is no tendency to rock. To ensure that all screws are gripping, it is well to finish the working of a pair by turning both clockwise for a small fraction of a turn.

Three-Screw Head.—(1) Turn the plate until one level tube is parallel to any pair of screws. The other tube will be parallel to the line joining the third screw and the point midway between the first pair.

(2) Manipulate the pair of screws as before until the tube parallel to them is levelled.

(3) Level the other tube with the remaining screw by using one hand only.

Note.—With the plate in the above position, both bubbles may be centered coincidentally by turning one screw of the pair simultaneously with the third screw, the plate tilting about the second screw of the pair.

Elimination of Parallax.—See page 28.

ANGLE MEASUREMENT BY THEODOLITE

To Measure a Horizontal Angle.—The temporary adjustments having been made, the usual procedure in measuring the horizontal angle subtended at the instrument by two signals is as follows :

(1) With the upper and lower clamps slack, bring the index of one of the verniers approximately to the zero (commonly marked 360°) of the circle. Fix the vernier plate to the circle by the upper clamp, and, by turning the upper tangent screw, set the zero of the vernier exactly to 360° .

(2) The circle, verniers, and telescope can now revolve as one upon the outer axis. Turn them by hand until the left-hand signal appears in the field of the telescope. Fix the outer axis by the lower clamp, and direct the line of sight exactly upon the signal by turning the lower tangent screw.

(3) Release the upper clamp, and swing the telescope horizontally until the second object appears in the field. Fix the upper clamp, and centre the intersection of the hairs on the signal by the upper tangent screw.

(4) Read the angle on the vernier which was originally set at 360°

Notes.—(1) Instead of setting the vernier initially at the zero of the circle, it may be clamped in any position, and the reading noted, this procedure usually being adopted in all precise work. The required angle is obtained by subtracting the initial from the final reading. If the 360° graduation lies between the two readings, subtract their difference from 360° .

(2) By sighting the left object first, the line of sight has to be moved clockwise, *i.e.* in the direction of graduation of the circle, and, if the vernier was first at zero, the final reading gives the required angle. When the right-hand object is taken first, the final reading must be subtracted from 360° . If the circle is figured in both directions, it is equally convenient to start from the right or the left point. Measuring an angle from left to right does not necessarily involve turning the telescope in that direction. The same final reading is obtained by swinging the telescope in either direction.

(3) To avoid improper use of the clamps and tangent screws, their functions should be clearly understood. The lower clamp and tangent screw are used only for the orientation of the circle, *i.e.* the setting and fixing of it in position for the measurement. Turning the wrong tangent screw necessarily introduces error. Thus, if the exact pointing to the first signal is effected by the upper tangent screw instead of the lower, the vernier is moved, and the measurement is made from the new position of its index instead of from the zero of the circle, as supposed. Erroneously using the lower tangent screw in the second pointing disturbs the necessary fixity of the circle during the measurement.

(4) Sight as far down the poles or signals as practicable, to reduce errors due to their possible non-verticality. The vertical circle clamp and tangent screw may be used to increase the delicacy of vertical movement of the line of sight.

(5) The accuracy of the measurement may be greatly increased by a system of repeated observations designed to minimise the effects of instrumental and other errors (see page 224).

To Set Out a Horizontal Angle.—This operation is the converse of the above. Being given one point, it is required to locate the direction in which a second should lie from the instrument so that a given angle is subtended between them. Let the given angle be a measured clockwise.

(1) Set the index of one of the verniers to 360° by the upper clamp and tangent screw.

(2) Direct the line of sight to the given signal, fix the lower clamp, and bisect the signal exactly by using the lower tangent screw.

(3) Release the upper clamp, and turn the vernier to read a approximately. Fix the upper clamp, and obtain the exact reading by turning the upper tangent screw.

(4) The telescope now points in the required direction. Mark a point in the line of sight.

Notes.—(1) If the angle has to be set out counter-clockwise, the second pointing must be obtained by setting the vernier to $(360^\circ - a)$.

(2) In guiding the staffman into the line of sight in step (4), it will be observed that, with the usual inverting eyepiece, it is necessary to order him to move apparently away from the line of sight.

To Measure a Vertical Angle.—Vertical angles of elevation or depression are measured from the horizontal plane through the trunnion axis. This plane must be established by careful levelling up of the instrument by means of the level tube mounted either on the telescope or the vertical vernier arm.

If the level tube is attached to the telescope, the procedure is as follows :

(1) Having levelled the instrument by the plate levels, bring the zero of the vertical circle approximately to the index of one of the verniers by moving the telescope by hand. Apply the vertical circle clamp, and by means of the vertical circle tangent screw adjust the zero to exact coincidence with the vernier index.

(2) The bubble of the telescope level should now be nearly, if not quite, central. Complete the levelling in two positions at right angles by means of the levelling screws.

(3) Swing the instrument through 180° . If the bubble deviates from the central position, bring it half-way back by means of the clip screw 26 (Fig. 53) and the remainder by the levelling screws. Repeat until the bubble remains central while the telescope is rotated horizontally.

(4) Release the vertical circle clamp, and tilt the telescope to bring the image of the distant point approximately on to the horizontal hair. Clamp, and by means of the tangent screw adjust the horizontal hair exactly upon the point, and read the required angle on the circle.

If the altitude level is mounted on the index arm, the procedure is the same, except that it is unnecessary to set the vertical circle to zero.

Notes.—(1) It is assumed in the foregoing that the altitude level is in adjustment, and in the case where this level is mounted on the vernier arm it is further supposed that index error has been eliminated. These adjustments (page 97) have to be attended to before proceeding to measure vertical angles.

(2) Some instruments do not possess the clip screw fitting whereby the telescope bubble may be levelled without altering the reading of the vertical circle. In this case the levelling is performed by the vertical circle tangent screw, and note is made of the reading when the telescope is horizontal. This index error must be applied positively or negatively to each reading obtained to give the required angles.

TESTING AND ADJUSTMENT OF THE THEODOLITE*

Requirements of the Theodolite.—Through imperfections in workmanship and the development of defects by continued use, the ideal requirements of a theodolite are only partially fulfilled, and its successful operation is very largely dependent upon a knowledge of the nature and relative importance of the effects produced by instrumental errors. Provision is made for the elimination of the more serious errors by adjustment, while the

* For additional notes on some of these adjustments see Appendix II.

influence of certain defects in the non-adjustable parts can be reduced or completely eliminated by adopting a particular routine in observing.

The more important requirements in the non-adjustable parts are :

1. The whole instrument should be stable, *i.e.* devoid of slackness.
2. The inner and outer axes should have the same geometrical axis of rotation, *viz.* the vertical axis of the instrument.
3. All movements should be truly circular.
4. The centre of graduation of the horizontal circle should lie in the vertical axis, and that of the vertical circle in the horizontal axis.
5. The zeros of the verniers should be diametrically opposite each other.
6. The division of the circles should be accurate.
7. The plane of the reading edge of the horizontal circle should be at right angles to the vertical axis, and that of the vertical circle at right angles to the horizontal axis.
8. The resolving power of the telescope should be so related to the least count of the verniers or microscopes that a just discernible movement of a vernier or microscope produces a perceptible motion of the cross-hairs on a signal, and *vice versa*.
9. The resolving power should also be such that a perceptible change of position of any of the bubbles produces a visible movement of the line of sight.

The following further conditions may be established by adjustment :

1. When the plate level bubbles are in the centres of their runs, the vertical axis should be truly vertical.
2. The line of collimation should coincide with the optical axis of the telescope, and should be perpendicular to the horizontal axis.
3. The horizontal axis should be perpendicular to the vertical axis.
4. The line of collimation should be parallel to the axis of the telescope level.
5. When the line of collimation is horizontal, the vertical circle reading should be zero.

Testing of Non-adjustable Parts.—1. **Stability** increases in importance with increase of sensitiveness in the various parts. Slackness may be present in the tripod (see page 47) and in various parts of the instrument, but particularly at the trunnion axis supports.

Test.—Mount the instrument on a rigid tripod, and sight a point with all clamps fixed. Apply a gentle, but firm, lateral pressure on the eyepiece with a finger. The intersection of the hairs will probably leave the point, but should return exactly to it on removal of the pressure.

Elimination of Error.—The error can be eliminated only by repair, but its effect may be minimised by avoiding pressing unduly on any part. Clamps should be applied gently, and the tripod should not be touched unnecessarily.

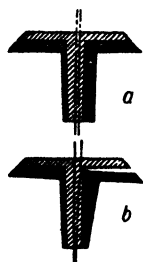


FIG. 78.

2. **Incoincidence of the Inner and Outer Axes of Rotation** may be of two kinds: (a) the axes may be parallel (Fig. 78a), this being one of the causes of eccentricity between the circle and vernier plates (4, *infra*); (b) the axes may be non-parallel (Fig. 78b), and not only is there eccentricity, but, if one of the axes is made truly vertical by the plate bubbles, rotation about the other is oblique. For test and elimination of error, see Note 2, page 93.

3. **Circularity of the Movements** is dependent upon the cross sections of the axes being circular. A simple test is not available.

Elimination of Error.—The effect of unequal wear of the axes is reduced, and in certain cases completely eliminated, by reading both verniers. For horizontal angles, repetition of the measurement on different parts of the circle is an additional precaution.

4. With **Eccentricity of the Horizontal Circle**, the line of sight and the verniers have an eccentric motion relatively to the graduations, and an angle read on either vernier is incorrect. In eccentricity of the vertical circle, the circle and line of sight have an eccentric motion with respect to the verniers, and the effect is the same.

Test.—Read both verniers in several positions on the circle, and note the differences between the readings. If the difference remains constant, there is no eccentricity.

Thus, in Fig. 79, let A be the centre of graduation of the circle, and B that of the verniers, the distance AB being greatly exaggerated. When the line of sight occupies the position CBD, the vernier readings at E and F differ by 180° . On turning the line of sight through a into the position GBH, the verniers are actually moved through a to K and L, but they do not record a . The reading at K is EAK, say $(a - e)$, and that at L is $(180^\circ + \text{FAL}) = (180^\circ + a + e)$, since $\text{AKB} = \text{ALB} = e$. The readings therefore now differ by $(180^\circ + 2e)$.

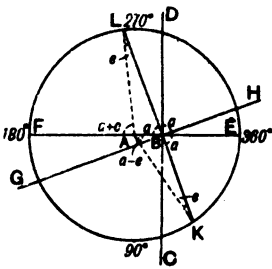


FIG. 79.

Elimination of Error may evidently be completely effected by averaging the values of the angle given by each vernier.

5. In **Eccentricity of the Verniers**, their zeros are not situated at the ends of the same diameter, but as K and L (Fig. 79). The

readings in any position therefore differ by a constant angle (KAL), other than 180° , provided B coincides with A .

Test.—Read both verniers in several positions on the circle, and note the differences between the readings. A constant difference, other than 180° , indicates eccentricity of the vernier zeros.

Elimination of Error.—Error from this source can arise only in orienting by one vernier and reading the angle on the other. Measurements are given correctly by either vernier. By drawing a diagram similar to Fig. 79, it will be seen that reading both verniers eliminates errors due to eccentricity of the circle although the verniers are eccentric.

6. **Errors of Graduation** of a circle can be analysed as (1) periodic errors, which recur at regular intervals according to some law, (2) accidental errors, which are quite irregular. Although a vital matter, accuracy of division cannot readily be tested by the surveyor, but the refinement of modern dividing engines is such that the errors are unlikely to influence his results, particularly in vernier, as distinct from micrometer, instruments.

Elimination of Error.—The effect of graduation errors may be reduced to any required extent by repeating the measurement on various parts of the horizontal circle and averaging. For vertical angles, measurement by both verniers is the only available precaution in ordinary instruments.

7. **Perpendicularity of the Planes of the Circles to their Respective Axes** is a condition easily fulfilled by the maker within limits of error which will have no appreciable influence on observations. If, for instance, the condition does not obtain in the case of the horizontal circle, then, when the vertical axis is placed truly vertical, the circle is oblique, and an observed horizontal angle is not measured on the horizontal plane; but the reading will differ inappreciably from the true value unless the obliquity is very serious.

8. **The Relationship between Resolution and Vernier Least Count** should obtain for convenience in observing. A magnification capable of showing movements of the line of sight too small to be recorded by the verniers entails loss of time in sighting, and has the further disadvantage of reducing the brilliancy of the image. On the other hand, if the resolving power is too low, advantage cannot be taken of fine subdivision of the circles, and time is wasted in reading the vernier.

Test.—Centre the hairs exactly on a well-defined distant point with all clamps fixed, and read the vernier. Make as small a deviation of the line of sight as possible, and observe if the vernier reading has changed. Alternatively, move the vernier through its least count or a fraction of it, if discernible, and note whether the line of sight has moved perceptibly.

9. Relationship between Resolution and Sensitiveness of Levels.—The level attached to the telescope or to the vertical vernier frame, being used in vertical measurements, should be capable of indicating the horizontal direction with as much accuracy as setting of the line of sight can be performed.

Test is similar to the foregoing, a just perceptible movement of the bubble being made.

The plate levels are less sensitive, but they should be able to pass the following :

Test.—Sight a well-defined point with both bubbles central and all clamps fixed. By turning the levelling screws, throw the bubbles out of centre. On relevening, the line of sight should be brought back to the point.

Adjustment of the Transit Theodolite*

In the examination and elimination of errors in the adjustable parts of the theodolite, considerable use is made of the principle of reversal (page 10). An important reversal is that produced by transiting the telescope and then turning it horizontally (termed "wheeling" or "swinging") through 180° , so that, if the vertical circle is initially on the right-hand side of the telescope, it will lie to the left after reversal. These positions are distinguished as "face right" and "face left," the operation being termed "changing face."

1. Adjustment of the Plate Levels.—*Object.*—To set the axes of the plate level tubes perpendicular to the vertical axis.

Necessity.—If this requirement is met, levelling the plate bubble tubes levels the horizontal circle and also the horizontal axis of the telescope, provided these are respectively perpendicular to the vertical axis. Horizontality of the trunnion axis is an important requirement in all work involving vertical movement of the telescope, as in the measurement of the horizontal angle between points at considerably different elevations (see page 225).

Test.—1. Set up the instrument on firm ground. Fix the lower clamp, and level the plate bubbles carefully.

2. Swing the upper plate through 180° . If the bubbles remain central, the adjustment is correct.

Adjustment.—1. If not, bring the altitude level, attached to the telescope or the index arm, parallel to a pair of levelling screws. If this tube is mounted on the telescope, set it approximately level by hand, clamp the vertical circle, and complete the levelling by the vertical tangent screw or the levelling screws : if it is mounted on the vernier frame, use the levelling screws. Turn through 90° , and centre the bubble by the levelling screws. Repeat until the bubble is central in these two positions.

2. Swing through 180° : the bubble will leave the centre of its run. Bring it half-way back by the levelling screws and the

* For additional notes on the theory of Adjustments 1, 2 and 3 see Appendix II.

remainder by the vertical circle tangent screw or the clip screws. Repeat until the bubble remains central in any position.

3. The vertical axis is now truly vertical. By means of the adjusting screws of the plate levels bring the bubble of each to the centre of its run.

Notes.—(1) The long bubble is used merely to secure greater delicacy in setting the vertical axis truly vertical. The adjustment is commonly performed by levelling the plate bubbles and correcting the error shown by reversal, half by their adjusting screws and half by the levelling screws.

(2) On completion of the adjustment, the test may be repeated with the lower clamp slack and the upper fixed. If the bubbles do not remain central on reversal, the outer axis of rotation is not vertical, and is not therefore parallel to the inner axis (Fig. 78). If the error is serious, the instrument should be sent for repair.

(3) The adjustment is sometimes referred to as the setting of the plates truly horizontal when the plate bubbles are central, but in reality this condition is secured only if the plates are perpendicular to the vertical axis. If either plate is oblique, its diameter generates a cone about the vertical axis on being swung, and the bubbles remain throughout in the centres of their runs. The only method of testing the horizontality of a plate is by examining whether a level tube can be reversed on it without deviation of the bubble, and this cannot be satisfactorily performed for the plane of the reading edge of a bevelled limb. Practically it may safely be assumed that the plates are horizontal when the axis has been set vertical, and that the plate levels are parallel to the plates.

2. Adjustment of the Line of Sight, or the Collimation Adjustment.

—*Object.*—To make the line of sight coincide with the optical axis of the telescope, i.e. to place the intersection of the hairs in that axis. This involves the adjustment of both the horizontal and the vertical hair. If two inclined hairs take the place of a single vertical hair, their intersection is adjusted as for a vertical hair.

Necessity.—Horizontal Hair: If, in an external focussing telescope, the horizontal hair does not intersect the optical axis, the direction of the line of sight will vary slightly with varying position of the objective due to focussing, the movement of the objective being assumed along the optical axis. In the case of an internal focussing telescope in which the horizontal hair is not in the axis, the tilting of the line of sight caused by movement of the internal lens is much less, and the necessity for the horizontal hair passing through the optical axis is correspondingly less urgent.

The requirement with respect to the horizontal hair is quite immaterial in horizontal measurements, but affects vertical measurements and levelling when focussing is necessary.

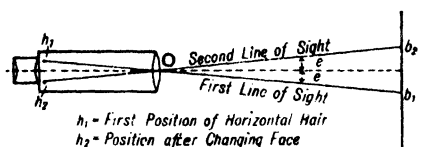
Vertical Hair: If the maker has set the optical axis of the telescope perpendicular to the trunnion axis, adjustment of the vertical hair amounts to the placing of the line of collimation perpendicular to the horizontal axis, and *this constitutes the primary object of the test and adjustment.* If the focussing movement is not in the adjusted line of collimation, this defect may be discovered by test, and can be eliminated in certain instruments. It is likely to be inappreciable if focussing is effected by an internal lens.

When the line of collimation is perpendicular to the horizontal axis, rotation of the telescope about the latter must cause this line to sweep out a plane, but, if not, on transiting the telescope, the line will generate a cone, the axis of which is the horizontal axis of the telescope. This defect introduces error in the measurement of the horizontal angle between points at different elevations, and is very important in work involving transiting of the telescope, as in the prolongation of a straight line through the instrument. It also occasions errors in setting out a straight line from one end owing to the change of inclination of the telescope necessary to sight the several pegs.

Horizontality and Verticality of Hairs.—Since the adjustment is to be made by means of the capstan screws controlling the diaphragm, it may be desirable before proceeding to the test proper to examine whether the hairs are respectively horizontal and vertical when the instrument is levelled up, as it is convenient to have them so. To test the horizontal hair, level the plate bubbles carefully, clamp the vertical motion, and sight a levelling staff or any definite point on one side of the field of view. Turn the telescope about the vertical axis, and observe whether the same point appears intersected by the other end of the horizontal hair. If not, slacken the four capstan screws, and rotate the diaphragm until the condition is secured. The hairs being fixed relatively to each other, the vertical hair should now appear truly vertical on comparing it with the image of a plumb line suspended in front of the instrument.

In making the adjustment proper, the horizontal hair is examined first, as it is of less importance for ordinary work than the vertical hair. This part of the adjustment is usually omitted if the instrument has an internal focussing telescope.

Horizontal Hair.—*Test.*—1. Set up and level the instrument carefully. With all the clamps fixed, take a reading on a levelling staff



held upon a firm point a few hundred feet away (Fig. 80). Note the reading and the vertical angle.

2. Unclamp, transit the telescope, and swing through 180° . Set the vertical circle to the same angle as before.

3. Again read the staff. If the previous reading b_1 is obtained, the adjustment is correct.

Adjustment.—1. If not, move the diaphragm by the top and bottom capstans until the staff reading is the mean of those previously obtained.

2. Repeat until no error is discernible on changing face.

Notes.—(1) An alternative method of reversing the line of sight consists in placing the telescope in improvised wyes and rotating it through 180° about its longitudinal axis as in adjustment 2 of the wye theodolite (page 100).

(2) Having completed the adjustment, a test may be performed to discover whether there is droop of the slide in focussing. Proceed as before, but use short sights, so that the object glass must be well racked out. If the instrument does not now pass the test, the error must be due to droop, which in certain instruments can be corrected by adjusting screws bearing upon the slide. Generally, however, the defect can be remedied only by a maker.

(3) To adjust the horizontal hair in an internal focussing telescope after a replacement, it is adequate for ordinary work to sight a staff sufficiently near that it fills the field of view. The readings at the extreme top and bottom of the field are noted, and the horizontal hair is adjusted to intersect the mean reading.

(4) If, through the horizontal hair being out of position, the line of collimation makes an angle e with the optical axis, as shown in an exaggerated manner in Fig. 80, it must not be thought that vertical angle measurements will be that amount in error. The altitude level will be adjusted so that its axis is parallel to the line of collimation, and it is only the variation of e , due to the focussing movement, which constitutes the error.

Vertical Hair.—Test.—1. Set the instrument on a fairly level stretch of ground and in such a position that a sight of about 300 ft. may be obtained on either side. Level up.

2. Establish a point A at about 300 ft. from the instrument by thrusting a chaining arrow into the ground, or otherwise. Sight A, and clamp the horizontal movement.

3. Transit, and mark B at about the same level as A and so that $OB = OA$ approximately (Fig. 81a).

4. Unclamp, swing through 180° , and again sight A and clamp.

5. If, on again transiting, the line of sight intersects B, it is perpendicular to the horizontal axis.

Adjustment.—1. If not, mark C in the line of sight (Fig. 81b). Measure out from C towards B a length $CD = \frac{1}{4}CB$, and mark D.

2. By means of the diaphragm side capstans bring the vertical hair to D.

3. Repeat until no error is perceptible on changing face (Fig. 81c, in which B and C coincide at A').

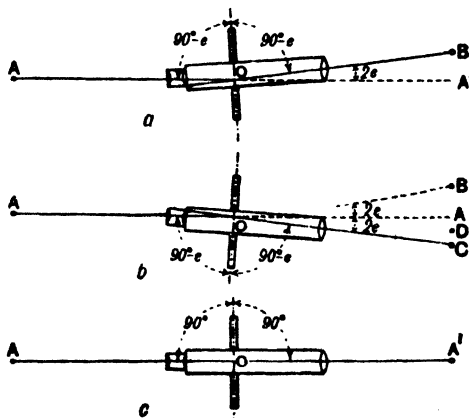


FIG. 81.

Notes.—(1) Transiting of the telescope again constitutes the reversal, and, as this is performed twice, the apparent error BC is four times the real error.

(2) If the adjustment is out, the line of sight will not intersect the vertical axis, as assumed in Fig. 81, but the small discrepancy is gradually eliminated in making the adjustment.

(3) Selecting A and B at the same level is a precaution against the possibility of adjustment 3 being out.

(4) By making the two sights of equal length the necessity for focussing is eliminated, and the adjustment is made correct for an average length of sight. If, however, there is slackness or lateral deviation in the focussing movement, the instrument will exhibit collimation error for sights of different length from those of the test. Examination should therefore be made for this defect by repeating the test as above, but with one of the sights very short. If the instrument no longer passes the test, the error lies in the focussing movement. In some instruments an adjusting screw is threaded through the side of the telescope and bears on the focussing slide, so that the defect can be remedied. In instruments having interior focussing this error may be regarded as negligible.

3. Adjustment of the Horizontal Axis.—*Object.*—To make the horizontal axis perpendicular to the vertical axis, so that it is truly horizontal when the instrument is levelled up.

Necessity.—Adjustment 2 having secured that rotation of the line of sight shall be in a plane, this adjustment ensures that such rotation shall lie in a *vertical* plane when the instrument is levelled. This condition is essential in all work necessitating motion of the telescope in altitude.

Test.—1. Set the instrument in such a position that a highly inclined sight may be obtained to a well-defined point A, *e.g.* the finial of a steeple. Level up very carefully, and sight A.

2. With both clamps fixed, depress the telescope, and mark a point B on the ground in the line of sight (Fig. 82a).

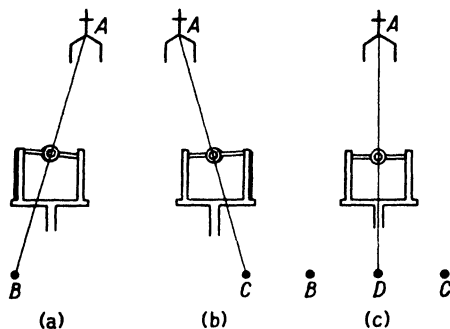


FIG. 82.

3. Unclamp, transit the telescope, and swing through 180° . Sight on A, and, with the horizontal movements clamped, depress the telescope. If the line of sight again cuts B, the horizontal axis is truly horizontal, and is therefore perpendicular to the vertical axis.

Adjustment.—1. If not, make a mark C on the ground where the line of sight intersects a horizontal line through B perpendicular to the vertical plane containing the line joining the theodolite to the distant point (Fig. 82b). Bisect the line BC and make a mark D at the point of bisection. Using the upper tangent screw, set the instrument to sight on D and then, by means of the adjusting screw at the trunnion support on one standard, raise or lower the

adjustable end of the axis until the line of sight when raised intersects A.

2. Again perform the test, marking a new point B' on the ground, and repeat test and adjustment until no error is detected on changing face (Fig. 82c). The theory of this adjustment will be found in Appendix II, pages 624-627.

Notes.—(1) The larger the vertical angle between A and B the better; the instrument should therefore be placed as near the high object as possible.

(2) Owing to the short length of the sights to B, that point should be marked carefully, a pencil mark or a pin being used.

(3) Since in sighting A, after changing face, the horizontal axis will not lie exactly in the same plane as it did for the first sight if the adjustment is out, the point D, in the same vertical plane as A, is not precisely midway between B and C. The conditions are greatly exaggerated in the diagram, and the divergence is of no account since the adjustment must be effected gradually by repeated trial.

(4) If a striding level is provided with the instrument, the horizontal axis may be adjusted by the method described in Vol. II, Chap. III.

(5) Some makers omit means for adjusting the axis, on the ground that it will thereby better maintain its adjustment. Should such an instrument require adjustment, the axis should be removed from the standards and the higher pivot wiped with fine emery until, on replacing the axis, the test is passed.

4. Adjustment of the Telescope Level.—*Object.*—To place the axis of the level tube attached to the telescope parallel to the line of sight.

Necessity.—This adjustment secures that the line of sight shall be horizontal when the bubble of the telescope level is central, and is of prime importance in the measurement of vertical angles and when the instrument is used as a level.

Test and Adjustment.—The procedure is exactly the same as in the "two-peg" adjustment of the dumpy level (page 124), the adjustment being made by means of the screws attaching the level tube to the telescope.

Note.—When the long level tube is mounted on the vertical index arm, its adjustment is performed along with that of index error as given below.

5. Adjustment of the Vertical Index Frame.—*Object.*—To make the vertical circle read zero when the line of collimation is horizontal.

Necessity.—If, when the line of collimation is horizontal, the vertical circle reading is not zero, such reading is called an index error. On eliminating index error from an instrument otherwise in adjustment, readings of the vertical circle will represent without correction the values of observed vertical angles. No error of measurement need, however, be caused by the existence of index error if its amount is determined and applied as a positive or negative correction to the observed values of vertical angles.

In certain instruments having no means for the elimination of index error, all that can be done in testing the instrument is to determine the value of the error. To do so, set the vertical axis

truly vertical by the method given in adjustment 1 (page 92), using the vertical circle tangent screw for correcting half the movement of the bubble on reversal. The reading of the vertical circle is the required index error.

If, as in the British transit, adjustment can be made between the vertical index frame and the standards, index error can be eliminated. The procedure depends upon whether the altitude level tube is mounted on the telescope or on the index frame, and to a greater extent upon the slow motion design adopted for the vertical vernier frame and the circle. In Fig. 53 the vertical circle tangent screw 29 and the clip screw 26 are on the same side of the telescope, and, if the vertical circle clamp is applied, screw 26 imparts a slow motion to the verniers, circle, and telescope without altering the reading. In more recent designs, with the object of securing a better balance, the vertical circle clamp and tangent screw are usually placed on one side of the telescope, and the clip screw fitting on the other, the tangent screw and clip screw operating on different vertical arms. With this arrangement, movement of the clip screw moves the vernier along the circle and tilts the bubble tube attached to the vernier arm, but does not move the circle and telescope.

For an instrument having the clip and tangent screws on one arm, as in Fig. 53, the elimination of index error is performed as follows, considering first the case in which the altitude level is mounted on the telescope :

1. By means of the vertical circle clamp and tangent screw set the vertical circle to read zero.
2. Place the bubble of the telescope level tube central in two positions at right angles by means of the levelling screws.
3. Swing the telescope through 180° . If the bubble does not remain central, bring it half-way back by the levelling screws and half-way by the clip screw attaching the index frame to the standard, repeating until the bubble remains central in all positions.

The instrument is now accurately levelled for vertical measurements without index error.

If the altitude level tube is mounted on the index arm, the usual procedure is to combine the adjustment of the level with that of index error, so that the line of collimation may be horizontal when the bubble is central and the vertical circle reading is zero.

Test.—1. Level the instrument by the plate levels, and by means of the vertical circle clamp and tangent screw set the vertical circle to read zero.

2. Bring the bubble on the index arm exactly central by means of the clip screw at the standard. Observe a levelling staff held 200 or 300 ft. away, and note the reading.

3. Release the vertical circle clamp, transit the telescope, and again set the vertical circle to read zero. Swing through 180° ,

relevel if necessary, and again read the staff held on the same point. If the reading is unchanged, the adjustment is correct.

Adjustment.—1. If the readings differ, bring the line of collimation on to the mean reading by turning the clip screw.

2. Return the bubble of the altitude level to the central position by means of the adjusting screws attaching it to the index arm.

3. Repeat until no error is discovered in the test.

Note.—In the case of instruments in which the upper portion, consisting of the telescope, vertical circle, and index frame, is disengaged from the standards in packing, the elimination of index error partakes of the nature of a temporary adjustment since it requires attention on each occasion of assembling the instrument for vertical angle measurement. In any case, it is advisable to examine the instrument with reference to index error at least once a day.

Considering now the case of an instrument in which the clip and tangent screws operate on separate arms, the elimination of index error and the adjustment of the altitude level are performed as follows, whether the level is mounted on the vernier arm or the telescope.

Test.—As in the last case if the level is on the vernier arm, but, if a telescope level only is fitted, set the bubble central by the levelling screws.

Adjustment.—1. If the readings differ, bring the line of collimation on to the mean reading by turning the vertical circle tangent screw.

2. Return the vernier index to zero by means of the clip screw.

3. Bring the bubble of the level central by means of the adjusting screws attaching it either to the vernier arm or the telescope.

4. Repeat until no error is discovered in the test.

Notes.—(1) If the instrument is fitted with levels on both the vernier arm and the telescope, only one of them need be referred to in making the adjustment. When one level is adjusted, and the bubble is placed central, it is only necessary to centre the bubble of the other by means of its adjusting screws.

(2) Instruments having the tangent screw and clip screw on opposite sides are usually packed as one piece, so that the clip screw fitting is used only for the purpose of adjustment. It is provided with a lock nut, which must be released before the screw can be operated. In performing the adjustment, the lock nut should be applied before the bubble is finally centred.

Elimination of Residual Errors of Adjustment.—To obtain the best possible results in angle measurement, it must be recognised that the relationships which are aimed at in the various adjustments may not have been precisely established, and that errors may have developed since the adjustments were made.

In horizontal angle work, only adjustments 1 and 3 and that part of 2 which is concerned with the vertical hair can have any bearing on the result. If a horizontal angle is measured as the mean of two observations, one face right and one face left, the effects of errors of adjustments 2 and 3 are eliminated. Changing face does not, however, get rid of the error caused by the vertical axis not being truly vertical, and it is therefore advisable to test the adjustment of the plate levels frequently.

In the measurement of vertical angles, adjustments 4 and 5 are of special importance, and residual errors of the others produce small effects. Change of face eliminates the principal errors in this case, but not the others, which, however, are of little consequence in vertical angulation.

If, as in highly refined horizontal or vertical measurements, a number of observations is taken for averaging, it is always essential that as many measurements should be made face right as face left.

Adjustment of the Wye Theodolite

The special features of service in the testing and adjustment of this instrument are: (a) the telescope can be removed from the wyes and replaced end for end; (b) it can be rotated in the wyes about its longitudinal axis.

1. Adjustment of the Plate Levels.—As for the transit theodolite.

2. Adjustment of the Line of Collimation.—*Object.*—To make the line of collimation coincide with the axis of the telescope collars. The construction of the telescope should be such that the axis of the collars which rest in the wyes coincides with the optical axis, and the wyes, which are rigidly connected to the horizontal axis, should be so placed with respect to it that the optical axis of the telescope is perpendicular to the horizontal axis.

Necessity.—As in adjustment 2 of the transit theodolite. Further, rotation of the telescope in its supports causes an unadjusted line of sight to generate a cone, and, unless the telescope can be clipped in one position, small rotations are likely to occur in observing.

Test.—1. Plant the instrument firmly, and carefully sight a definite small mark. Fix all clamps.

2. Rotate the telescope in the wyes through 180° . If the intersection of the hairs remains on the mark, the adjustment is correct.

Adjustment.—1. If not, by means of the four screws controlling the diaphragm move the intersection of the hairs half-way towards the point.

2. Set the line of sight again on the mark by the tangent screws, and repeat the test and adjustment until correct.

Notes.—(1) It is unnecessary to level the instrument.

(2) Unlike the same adjustment of the transit, both hairs are adjusted in the one operation. The effect of the reversal is that illustrated in Fig. 81.

(3) The adjustment has secured perpendicularity between the line of collimation and the horizontal axis only if the optical axis and that of the collars are at right angles to the horizontal axis. If difficulty is experienced in effecting adjustment 3, there will be reason to suspect that those relations are not fulfilled, in which case the correction cannot be undertaken by the surveyor.

(4) Having performed the adjustment, a test for deviation of the object glass slide in focussing should be made by repeating the procedure with a new length of sight necessitating a considerable change of focus.

3. Adjustment of the Horizontal Axis.—As in the transit theodolite, but in place of transiting the telescope in the test, it is removed from the wyes and replaced end for end.

Notes.—(1) An alternative method is to sight a plumb line suspended in front of the instrument and observe whether the intersection of the hairs remains on the line while moving the telescope through as large an angle as possible about the horizontal axis.

(2) In wye theodolites the horizontal axis is not readily removable from the standards, and few are provided with capstans for making this adjustment. The caps at the top of the standards must be unscrewed, and the high end of the axis reduced with emery.

4. Adjustment of the Telescope Level.—The axis of the telescope level is placed parallel to the line of collimation by the method of adjustment 2 (*A* and *B*) of the wye level (page 127). Alternatively, but less conveniently, *B* may be performed by the peg method used for the transit and the dumpy level.

5. Determination of Index Error.—The usual construction of the instrument does not enable the surveyor to eliminate index error, and its value must be determined by the method on page 98.

THE COMPASS

The compass is an instrument designed for the measurement of directions with reference to the magnetic meridian. The essential parts are a magnetic needle, a graduated circle, and a line of sight. The various types may be classified as :

1. The surveying compass, circumferentor, or miner's dial.
2. The prismatic compass.

Instruments in the first category are commonly used in mine surveying, but for general surveying are now employed to a much smaller extent than formerly. Some of these instruments are adapted for the measurement of horizontal and vertical angles, but they are inferior to the small theodolite.

The prismatic compass is a lighter and simpler instrument, and is of the greatest service to the engineer or surveyor for rapid traversing.

Plain Compass.—Fig. 83 illustrates a simple form of surveyor's compass with plain sights. The compass box, of about 5 in. diameter, has the circle graduated to degrees, and mounts an edge bar needle. The sight vanes are screwed to a plate attached to the underside of the compass box, and are hinged to fold down upon a metal cover which protects the compass box when not in use. The sight vanes are sometimes adapted for measuring slopes by having the object vane engraved with a scale of tangents and the eye vane provided with a pin-hole sight.

The compass is graduated either on the whole circle or the quadrantal system (page 206). The graduation increases counter-

clockwise, so that the 90° or E. mark is to the left, and 270° or W.

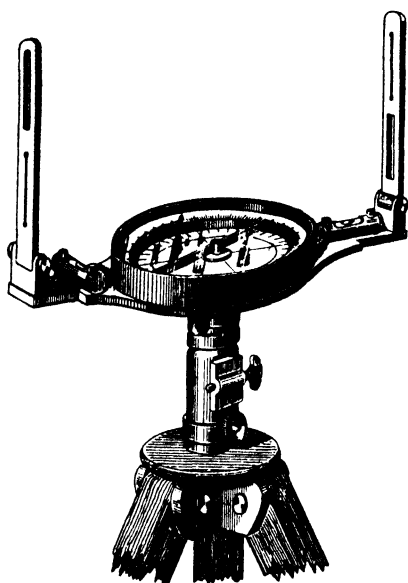


FIG. 83.—PLAIN COMPASS.

to the right, of zero or N., this reversal being required because the graduated circle moves with the line of sight, while the needle remains stationary. On swinging the line of sight from north towards east, for example, the increasing graduations on the left of zero are successively brought opposite the north end of the needle. Some patterns are furnished with means whereby the graduated circle may be turned, independently of the line of sight, through an angle corresponding to the magnetic declination, so that the needle may record true bearings.

The level tubes are sometimes omitted, and the instrument is levelled by adjusting it until the ends of the floating needle lie in the plane of the

graduations. Levelling is performed by means of a levelling screw head or, alternatively, by a ball and socket joint, as in Fig. 83.

The instrument is mounted on a single rod, called a Jacob staff, or on a light tripod. A form of tripod common in mining instruments has the legs jointed in the middle of their length. For observations where headroom is limited, the lower sections are unscrewed, and points are screwed to the upper lengths to form a low tripod. The low tripod sometimes proves very convenient in surface work for sighting through woods by enabling the surveyor to keep his line of sight below the foliage.

Other Forms of Compass.—The more elaborate forms of compass are better adapted for taking inclined sights. Usually both a telescope and vanes are provided, one or the other being used according to circumstances. These instruments can be employed in the manner of a theodolite for the measurement of horizontal and vertical angles. Although varying in detail, many of the patterns available are modifications of the two type instruments, Lean's dial and Hedley's dial.

In Lean's dial (Fig. 84), the compass base plate carries either folding vanes or a telescope and vertical arc, the same screws 7 being used for the attachment of either. By means of a pinion 1, gearing with the toothed vertical arc, the telescope can be moved tangentially round the arc about a short horizontal axis or hinge 3.

The proportions are such that a vertical sight can be taken, as required in transferring a line down a shaft in mine surveying.

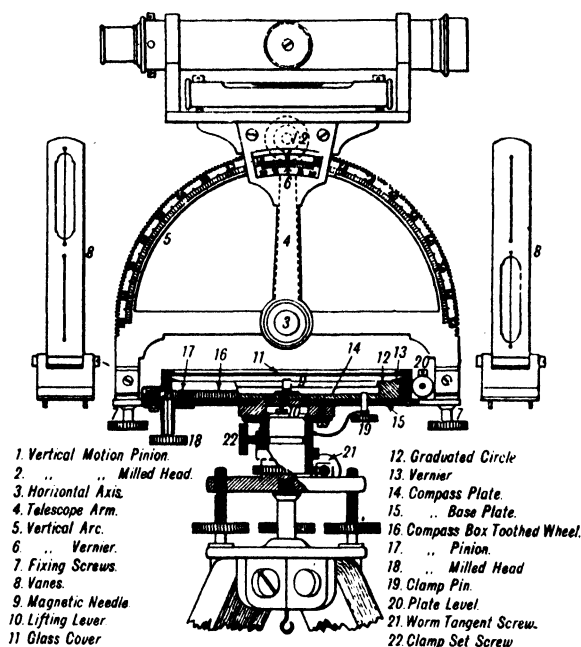


FIG. 84.—LEAN'S DIAL.

The needle is of edge-bar form, and the bottom plate of the compass box is graduated counter-clockwise to degrees, usually on the quadrantal system. On the step, at the same level as the upper edge of the needle, there is a circle 12, graduated clockwise on the whole circle system. The compass plate 14 is provided with a toothed wheel in gear with a pinion 17 actuated by the milled head 18. With this arrangement, when the clamp pin 19 is withdrawn, the base plate, with the outer portion of the compass box and the telescope, can be

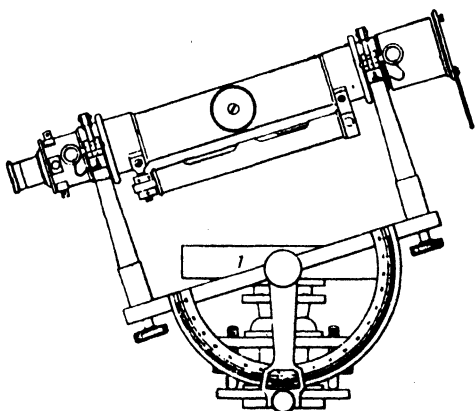


FIG. 85.—HEDLEY'S DIAL.

rotated about the fixed compass plate and graduated circle. The compass box carries a vernier 13, so that bearings and horizontal angles may be measured as with the theodolite. Otherwise, with the pin 19 locking the compass plate to the base plate, the instrument is used as a plain compass, and bearings are read directly from the needle.

In Hedley's dial (Fig. 85), the characteristic feature is the method of supporting the frame carrying the telescope or sight vanes. It is mounted on two diametrically opposite pivots which project from the underside of the compass box 1, and about the horizontal axis so formed the line of sight may be moved in the vertical plane. At one side there is fitted a vertical arc, which moves with the line of sight and is read against a vernier, the arm of which is rigidly fixed to the pivot. In other respects this instrument does not differ essentially from Lean's dial.

Testing and Adjustment of the Compass.—In addition to the properties which should be fulfilled by the needle (page 46), the following requirements of the compass can be established by adjustment :

1. When the plate bubbles are in the centres of their runs, the vertical axis should be truly vertical.

2. Sight vanes should be vertical when the instrument is levelled.

3. The telescopic line of collimation should be perpendicular to the horizontal axis, which should itself be perpendicular to the vertical axis.

4. The extremities of the needle should lie in the same vertical plane with the pivot.

5. The pivot point should be vertically over the centre of the graduated circle.

1. Adjustment of the Plate Levels.—*Object.*—To set the axes of the plate level tubes perpendicular to the vertical axis.

Necessity.—Non-adjustment in this respect introduces error whenever the line of sight is moved in altitude. Further, since the compass box and plate are placed by the maker perpendicular to the vertical axis, the adjustment practically ensures their horizontality when the bubbles of the plate levels are centered.

Test.—1. Set up the instrument on firm ground, and level carefully.

2. Swing through 180° . If the bubbles remain central, the adjustment is correct.

Adjustment.—1. If not, correct half the errors by the level tube capstans and half by the levelling screws or ball and socket joint.

2. Swing through 180° , and, if necessary, repeat the adjustment until the bubbles remain central throughout a rotation.

2. Adjustment of the Sight Vanes.—*Object.*—To make the sight vanes vertical when the instrument is levelled.

Necessity.—The error caused by non-verticality of the vanes will generally be negligible so long as the slit and sighting line are in the same place. Otherwise, the direction of the line of sight from a particular point on the eye vane will vary according to which point on the object vane is referred to in sighting.

Test.—1. Level the instrument carefully, and look through the sights at a plumb line or the corner of a building. Note whether the line on the object vane coincides with the vertical line observed.

2. Swing through 180° , and test the other vane in the same way.

Adjustment.—If necessary, either file one side of the bottom of the vane where it rests on the plate or insert a paper packing. Repeat the test and adjustment until the error is eliminated.

3. Adjustment of the Telescope.—In compasses having a telescope, the line of collimation must be perpendicular to the horizontal axis, which should be perpendicular to the vertical axis. The necessity for these is the same as in the theodolite (pages 93 and 95), and the method of testing and adjustment follows that of the transit or wye theodolite according to the manner in which the telescope is mounted. Usually no provision is made for adjusting the horizontal axis.

4. Adjustment of the Needle.—*Object.*—To straighten the needle.

Necessity.—A needle bent horizontally will not have the magnetic axis coincident with the geometrical axis, and in consequence the readings are in error by an amount which remains constant if the N. end only is read (*cf.* 5, page 90). If the reading ends are not in the same horizontal plane with the pivot point through vertical bending of the needle or faulty construction, they will be found to quiver when the needle swings, and this proves inconvenient in reading. This defect should be eliminated before proceeding to remedy horizontal bending.

Test for Vertical Bending.—Lower the needle on the pivot, and observe whether trembling of the needle is accompanied by rocking of its upper surface at the ends.

Adjustment.—If so, remove the needle, and bend it in a vertical direction until no rocking is noticeable on replacing it.

Test for Horizontal Bending.—1. Read both ends of the needle in any position.

2. Rotate the instrument until the graduation originally opposite the N. end of the needle comes opposite the S. end.

3. If the reading at the N. end is now that originally at the S. end, the needle is straight. If not, half the discrepancy represents the deviation from straightness.

Adjustment.—1. Remove the needle, and bend the N. end horizontally through this angle in the direction which would carry

it from the N. end reading after reversal towards the new position of the original S. end reading.

2. Repeat test and adjustment as often as necessary.

Note.—The above method of testing for horizontal bending is designed to be independent of whether the pivot is in the centre of the graduated circle or not, and should therefore be applied in place of the more obvious method of noting whether the end readings differ by 180° .

5. Adjustment of the Pivot.—*Object.*—To set the pivot point in the centre of the graduated circle.

Necessity.—If the pivot is not central, the true reading will be obtained only when the needle happens to lie in the same vertical plane as the line joining the centre and the pivot (*cf.* 4, page 90).

Test.—Read both ends of the needle for various positions of the circle, and note whether the readings differ by a constant amount,

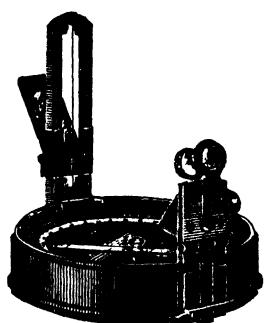


FIG. 86.—PRISMATIC COMPASS.

which should be 180° when the needle is straight. If so, the pivot is central.

Adjustment. — 1. If the difference between the readings is variable, ascertain as nearly as possible the position of the needle at which the discrepancy is greatest.

2. Remove the needle, and with a small pair of pliers gently bend the pivot at right angles to this position of the needle and towards the greater segment of the circle formed by the needle.

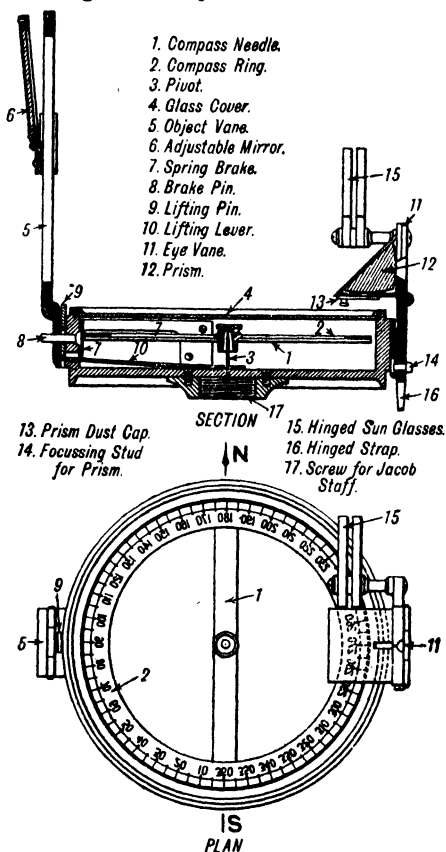


FIG. 87.—PRISMATIC COMPASS.

3. Repeat test and adjustment until the end readings agree in all positions.

The Prismatic Compass.—This extremely useful instrument (Fig. 86) is the most portable form of surveyor's compass, and is specially designed for use as a hand instrument. The construction is illustrated in Fig. 87. The compass needle is of broad form, and carries an aluminium ring of from $2\frac{1}{2}$ in. to 6 in. diameter, graduated to half degrees. The special feature of the instrument lies in the construction of the eye vane, which carries a reflecting prism whereby a view of the compass ring is presented to an eye placed opposite the sighting slit. The observer, while sighting through the slit past the object vane wire or bar, sees the latter cutting the image at the required bearing, which is therefore read simultaneously with sighting (Fig. 88). The compass ring is graduated from the S. end of the needle because the readings are taken at the end of the diameter remote from the object. The prism has both the horizontal and vertical faces convex, so that a magnified image of the graduation is formed, and focussing to suit different observers is effected by moving the prism vertically by means of the stud 14. To reduce excessive oscillation of the compass ring caused by unsteadiness of the hand, a light spring 7, carrying a pin 8, is fitted inside the compass box. On gently pressing the pin inwards, the spring is made to touch the ring and act as a brake.

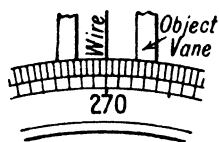


FIG. 88.—APPEARANCE ON SIGHTING.

When the instrument is not in use, the object vane is folded down on the face of the glass cover, and presses against the pin 9, which lifts the needle off the pivot. The eye vane folds outwards, and is held by the hinged strap 16, and a metal lid is placed over the glass cover and object vane. In an alternative arrangement, the brass cover is a fixture, and is fitted with a window under the prism. The standard military pattern has the cover lid designed to serve as the object vane. It is hinged, and is fitted with a glass disc on which is ruled a sighting line. In what is called a liquid compass, the card dial carrying the graduations is floated in liquid with the object of reducing pivot friction.

Some accessory parts are usually added. The dark glasses 15, are so placed that they can be swung into the line of sight when sun observations are required. The object vane is commonly provided with a hinged mirror 6, by means of which points considerably above or below the instrument can be sighted by reflection. The mirror is hinged to a slide which can be placed upwards or downwards on the vane, along which it may be moved as required. No provision is made for adjusting the vanes.

Although commonly used as a hand instrument, the compass may be fitted with a screwed socket for attachment to a Jacob staff or light tripod with ball and socket head. The facility thus afforded

for levelling the instrument makes for increased accuracy, particularly when the line of sight is considerably inclined to the horizontal.

Observing with Prismatic Compass.—The instrument is held with the forefinger over the brake pin, and is turned until the distant signal appears in the line of sight. Care must be exercised to hold it as nearly level as can be judged: if the ring does not oscillate, it is probably in contact with the glass cover through the instrument being out of level. A light touch is applied once or twice to the brake pin until the needle is nearly steady. The readings of the two limits of the swing are estimated to the nearest $\frac{1}{4}^{\circ}$, and their mean is taken as the result.

Connolly Standard Compass.—This instrument, designed by Mr. T. F. Connolly and manufactured by Messrs. E. R. Watts & Son, is not suitable for everyday survey work, but is intended for testing ordinary compasses or for determining the magnetic meridian to an accuracy of about 1 min. of arc, a standard of precision greater than that obtainable with any ordinary compass.

The essential part of the instrument consists of a magnetic and an associated sighting system suspended on a point-and-jewel support. The system consists of a small and very light telescope which carries a number of small magnetic needles attached to the barrel. The telescope, which can be focussed down to a distance of 10 ft. to 50 ft., is mounted on a cradle with two open "V" supports and can be rotated in these supports about its longitudinal axis. The observation consists in putting in a mark in such a position that its image is seen in the telescope, when this is swinging freely, close to, or coincident with, the vertical cross-hair in the diaphragm. This diaphragm, which is of glass, has a scale engraved on it so that the interval between successive divisions corresponds to approximately 10 min. of arc and readings can be estimated to single minutes. Consequently, the angle between the mark and the cross-hair having been observed by means of the scale, the telescope is rotated through 180° about its longitudinal axis, and another reading taken. By taking the mean of the two readings, the error due to the line of collimation not coinciding with the magnetic axis is eliminated and the result is the deviation of the mark from the true magnetic meridian.

The telescope and suspension system are enclosed in a draught-tight box, observations being made through windows at each end. The instrument is also provided with a fixed telescope and with a horizontal circle reading to 1 min. of arc. Consequently, if a line, whose true bearing is known or has been established by astronomical observations, is available, the angle between this line and the magnetic meridian can be measured and the magnetic declination thus determined. In this way, the isogonic lines in any locality can be mapped with considerable accuracy.

Although the Connolly Compass is hardly an instrument which the private surveyor will ordinarily have in his equipment, it is of particular use to firms or bodies employing a large survey staff who wish to have all their instruments properly standardised to yield results that are consistent with each other.

The instrument is described by Mr. Connolly in *The Journal of Scientific Instruments*, Vol. VII, No. 7, July, 1930.

THE LEVEL

The surveyor's level is an instrument designed to furnish a horizontal line of sight. Neglecting for the present various forms of rough levelling instruments, the level consists essentially of a bubble tube attached to a telescope, the axis of the bubble tube and the line of collimation being parallel to each other. The instrument is provided with levelling screws by which the bubble is centered and the line of collimation brought into a horizontal plane.

Types of Levels.—According to their general arrangement, levels may be classified as: (1) the dumpy or Gravatt level; (2) the wye level; (3) levels possessing features of both the dumpy and wye types; (4) the tilting level. The essential feature of the first is the rigid connection of the telescope to the vertical spindle. In the wye level the telescope is mounted as in the wye theodolite. The instruments in the third category are designed to combine the advantages of the solid construction of the dumpy with the ease of adjustment of the wye. The distinguishing feature of the tilting level is that the telescope and attached bubble tube may be levelled without the necessity for setting the rotation axis truly vertical.

General Features.—The several patterns do not differ materially as regards the levelling head, which should be of the three-screw type in preference to one of four screws for the reasons stated on page 64. A quick levelling head (page 120) may be fitted.

The special requirements of the telescope are good definition and illumination, and to promote the latter a positive eyepiece only should be used, and the effective aperture of the objective should be not less than $1\frac{1}{2}$ in. The telescope is not limited in length as in the transit. Focussing is effected either by an internal lens or by movement of the tube mounting the eyepiece and diaphragm. In the former case the length of the telescope, and in the latter the focal length of the objective, serves to designate the size of the level, which varies from 7 in. to 16 in. in instruments for ordinary levelling. The reticule in the dumpy level is usually of the form shown at A (Fig. 18), but instruments with removable telescopes must have the line of sight defined by the intersection point of two hairs. Stadia hairs are commonly inserted.

The sensitiveness of the bubble tube should accord with the delicacy of the levelling screws and the resolving power of

the telescope (page 92). In instruments for ordinary levelling the sensitiveness ranges from 15" to 25" per $\frac{1}{10}$ in. division.

The Dumpy Level, Original Pattern.—The dumpy level, designed by Gravatt, is of the general construction illustrated in Fig. 89, which shows an obsolescent form of four-screw instrument. The telescope tube is firmly secured in two collars fixed by adjusting screws to the stage carried by the vertical spindle. The upper ends of the collars support the bubble tube by a hinge at one end and a screw and locking nuts at the other to permit of the adjustment of the tube relatively to the line of collimation. A circular spirit level, or, in its place, a short bubble tube at right angles to the long tube, is provided as an aid to planting the instrument approximately level.

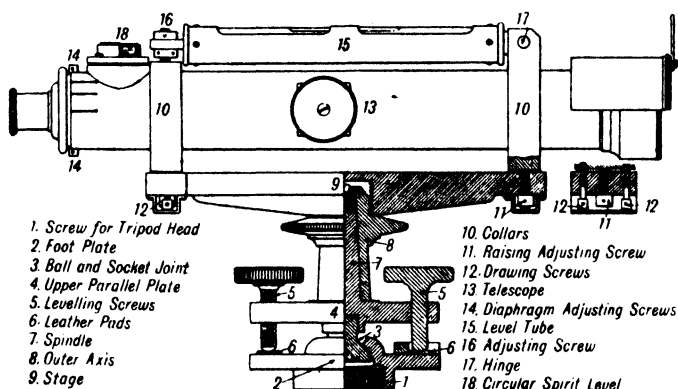


FIG. 89.—DUMPY LEVEL.

Other Types of Dumpy Level.—The modern form of dumpy level, usually termed the solid dumpy (Fig. 90), has the vertical spindle and the telescope barrel cast in one piece. This design affords a

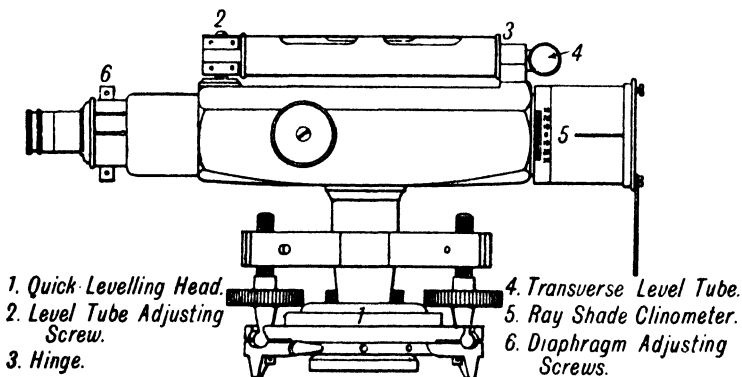


FIG. 90.—SOLID TYPE DUMPY LEVEL.

maximum of rigidity, while the reduction in height adds to the stability of the instrument. Fig. 91 shows a small dumpy level by Messrs. Wild & Co. This has the level tube mounted on the left hand side with a folding reflector to reflect an image of the bubble to the observer at the eyepiece end. A small circular level for rough

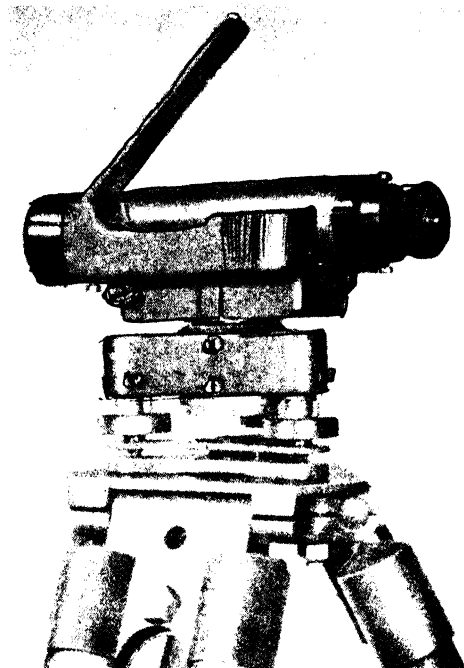


FIG. 91.—SMALL WILD DUMPY LEVEL, NO. NK.0, WITH
HORIZONTAL GRADUATED CIRCLE.

(By courtesy of Messrs. Henry Wild & Co.)

preliminary levelling is fitted beside the main level tube. The instrument in the illustration is provided with a horizontal circle graduated at intervals of 5° and numbered every 30° . A simpler model, without the graduated circle, is also obtainable.

Watts' "Autoset" Level.—This interesting pattern of dumpy level (Fig. 92) was designed by Messrs. Watts to afford a maximum of rapidity in operation. The novel feature is that the bubble need not be exactly central while an observation is being made. The instrument is levelled by a quick levelling head and handle with reference to a small circular level, and errors of levelling, of as much

as 4 min., are neutralised by displacement of the image of the staff relatively to the horizontal hair by the appropriate amount. This is accomplished by means of the device shown in Fig. 93. The prism carries a reference line on the horizontal face and marks for the avoidance of parallax error on its vertical face, and is mounted on a

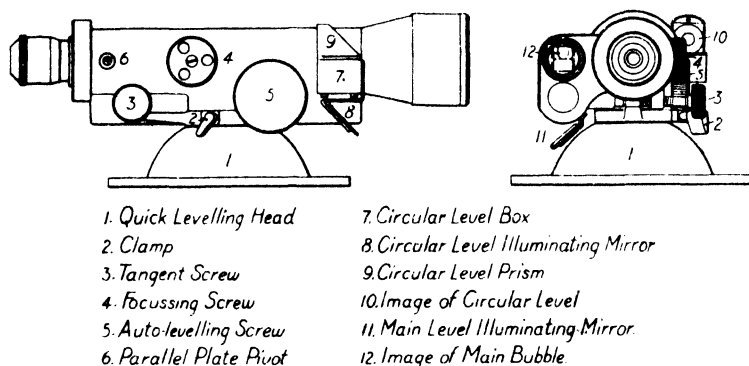
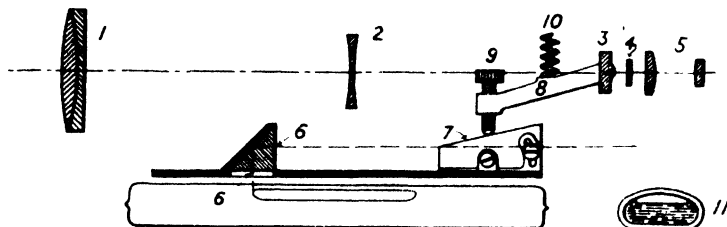


FIG. 92.—WATTS' "AUTOSSET" LEVEL.

slide. By turning screw 5 (Fig. 92) the prism can be placed over one end of the bubble, which is of constant length (page 38). A view of the end part of the bubble is presented to the observer, and the setting is effected when the prism line appears to touch the end of the bubble. The slide mounting the prism also carries a wedge-shaped



- | | |
|-----------------------------|---------------------------------------|
| 1. Object Glass. | 7. Cam. |
| 2. Internal Focussing Lens | 8. Tilting Lever |
| 3. Parallel Glass Refractor | 9. Adjusting Screw |
| 4. Diaphragm Plate. | 10. Spring maintaining Contact of 9. |
| 5. Eyepiece. | 11. Section of "Constant" Level Tube. |
| 6. Prism Index Marks. | |

FIG. 93.

cam 7, which actuates lever 8. On the lever is mounted a parallel glass plate, through which pass the rays forming the image in the plane of the ruled surface of the diaphragm plate. Tilting of the lever, caused by adjusting the reference line on to the end of the bubble, tilts the parallel refracting plate, and produces a displacement

of the image such that the staff graduation actually read against the horizontal hair is that which would be observed with the bubble accurately central. As shown in Fig. 93, the cam is mounted so that it can be set to correspond with the sensitiveness of the particular level tube fitted.

The Wye Level.—In this instrument (Fig. 94), the stage carries two wye supports in which the telescope is clipped. The body of the telescope is fitted with two cylindrical flanged collars of equal

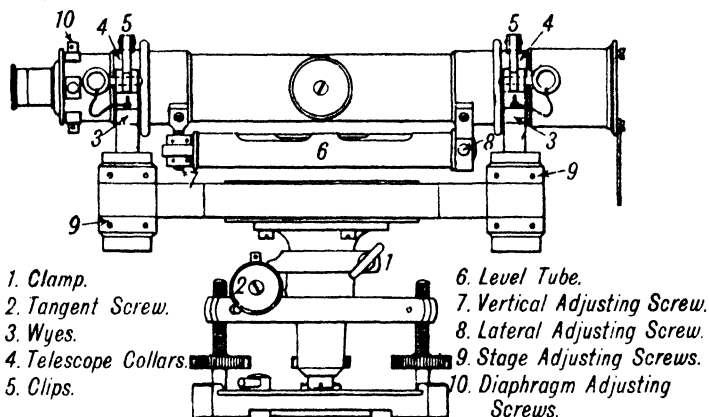


FIG. 94.—WYE LEVEL.

diameter, which rest in the wyes, and the telescope is capable of rotation about the axis of the collars. For purposes of adjustment, the telescope can be lifted from the wyes and replaced end for

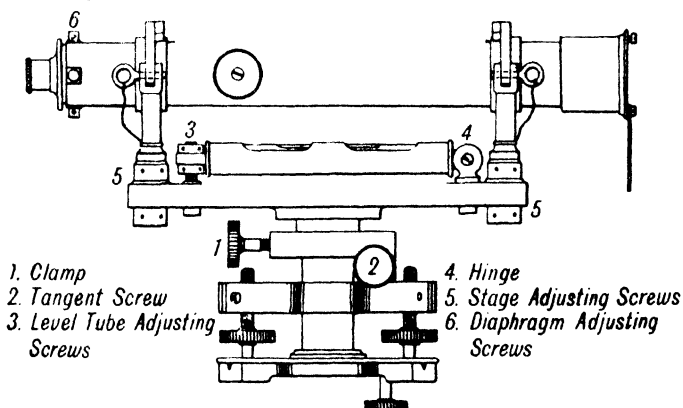


FIG. 95.—TROUGHTON AND SIMMS' WYE LEVEL.

end. A clamp and tangent screw are provided to facilitate pointing of the telescope. In the example illustrated, the bubble tube is suspended from the telescope, but it may be fixed on top of the latter.

In a pattern of wye level formerly made by Messrs. Troughton and Simms (Fig. 95), the bubble tube is mounted on the stage instead of on the telescope.

Comparative Merits of the Dumpy and Wye Levels.—The dumpy level was introduced because of the objections offered by surveyors to the defective performance of the wye level arising from its non-rigid construction; and, by virtue of its superior solidity and compactness, the dumpy has established itself as the favourite level in this country. The wye instrument has an advantage in the rapidity and ease with which the adjustments can be tested, but the dumpy level retains its adjustments better.

Cooke's Reversible Level.—This instrument (Fig. 96) belongs to the class of levels combining features of both the dumpy and wye

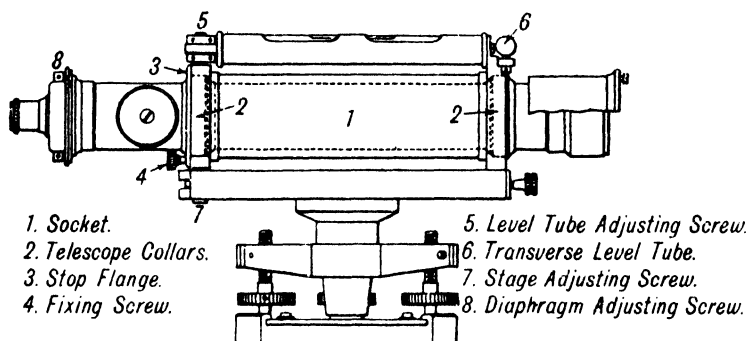


FIG. 96. —COOKE'S REVERSIBLE LEVEL.

instruments. The telescope is similar to that of the wye level, having two equal collars, one of which is formed with a stop flange 3. The telescope supports are connected by a rigid socket into which the telescope can be introduced from either end and pushed home until the flange bears. The telescope can then be fixed in position by screw 4, and the instrument used as a dumpy level. To reverse the line of collimation in the course of testing and adjustment, screw 4 is slackened, and the telescope is withdrawn from the socket and replaced end for end as with the wye level. The telescope can also be rotated within the socket about the axis of the collars.

A later model of this instrument is provided with a reversible tilting level (see below).

The Tilting Level.—In both the dumpy and the wye level the line of collimation is at right angles to the vertical axis if the instrument is in adjustment, so that levelling up of the instrument not only makes the line of sight a horizontal one, but also places the vertical axis truly vertical. In the tilting level the telescope with its attached level tube can be levelled by a finely pitched tilting screw indepen-

dently of the vertical axis, and, in consequence, the line of collimation is not in general at right angles to the vertical axis. This type of level was first designed for precise work, but it has grown greatly in popularity for ordinary levelling, and numerous patterns are available under different names. In many cases the tilting screw carries a drum graduated to give gradients.

In using these instruments, the axis is set only approximately vertical by the levelling screws, or simply by a quick levelling ball and socket head, with reference to a circular level. When a staff reading is about to be taken, the bubble in the main level tube is centered exactly by means of the fine levelling screw, which also tilts the telescope. It is necessary that the observer should have a view of the bubble while sighting the staff, and a bubble reflector is fitted. The circumstances that the level tube axis is not generally at right angles to the vertical axis leads to the necessity for centering the bubble by the fine tilting screw before every observation.

Tilting levels possess the merit that the use of a fine tilting screw facilitates the exact centering of the bubble at the moment of observation. They also save time in the levelling of the instrument, since it is only roughly levelled by the foot screws, and the final levelling can be performed rapidly after a little practice. This advantage is not so apparent in work involving numerous observations from each instrument station. With instruments having the line of sight and the bubble tube axis at right angles to the vertical axis, a series of intermediate sights may be obtained without the necessity for adjusting the levelling screws.

The Reversible Tilting Level.—Although it does not constitute an essential feature of the tilting type of level, some of these instruments have the telescope mounted so that it can be rotated about its longitudinal axis. The bubble tube is connected to one side of the telescope, and a rotation of the latter through 180° moves the bubble tube from, say, the left-hand to the right-hand side of the instrument, and turns it upside down. The names, reversible level and self-adjusting level, are sometimes used to indicate the inclusion of this feature. A bubble tube capable of being used in the reversed position must be barrel-shaped, but, even although a longitudinal section shows the upper and lower walls to have the same curvature, it does not follow that the axes (page 35) of the tube in the direct and reversed positions will be parallel to each other. Nevertheless, if the telescope is reversible end for end as in the Wye level, the accuracy of the instrument can be tested from a single station, since a true staff reading can be obtained as the mean of four readings, obtained by rotating the telescope and bubble tube through 180° about the longitudinal axis, and also by reversing it end for end.

The ideal reversible level tube is one which in longitudinal section has perfect symmetry both about a horizontal centre line and about the vertical passing through the centres of curvature of the upper

and lower surfaces. These conditions are approached very nearly in certain makes of tube. A notable advance in the application of reversible level tubes, however, was made by Mr. T. F. Connolly,* who patented a type of level in which the bubble tube is so mounted as to be capable of adjustment to secure that it may be rotated about its longitudinal axis, or an axis parallel to it, without movement of the bubble along the tube. Although the bubble tube may not be perfectly symmetrical, the reversal of the bubble without movement defines the position of two parallel axes, which can be recorded by calibration marks on the tube. By means of such an adjusted bubble tube, the horizontality of the line of collimation of the telescope may be tested by means of two observations, with the telescope in the direct and reversed positions respectively.

Watts Self-Adjusting Level.—The Watts Self-Adjusting level, Fig. 97, is a good example of a simple modern reversible level. In the diagram, P and P' are reflecting prisms which show one end of the bubble and an index line as in Figs. 97*b* and *c*, the instrument when in use being set by means of the milled and graduated drum B so that the end of the bubble appears to coincide with the index line. R and R' are mirrors to reflect light through the level tube, these

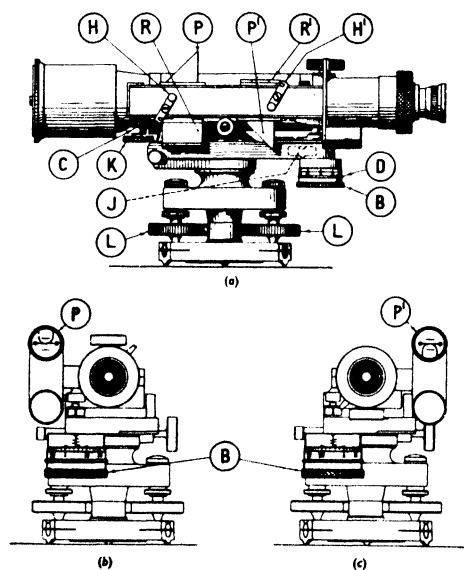


FIG. 97.—WATTS SELF-ADJUSTING LEVEL.
(By courtesy of Messrs. Hilger & Watts, Ltd.)

* "New Types of Levelling Instruments using Reversible Bubbles," *Trans. Optical Society*, Vol. XXV, and "Theory of the Adjustment of Levels," *Journal of Scientific Instruments*, Vol. I.

mirrors lying flat when not in use. In Figs. 97*a* and *b* the level tube is on the left of the instrument but the telescope can be reversed through 180° about its longitudinal axis so as to bring the bubble on the right of the instrument as in Fig 97*c*. The mean of readings taken to a staff with level tube left and level tube right and centered in each case gives the reading for a truly horizontal line of sight. When the instrument is set by turning the tilting screw B to this reading, the end of the bubble will no longer coincide with the index line and the adjustment consists in holding the line of sight fast to the mean staff reading while adjusting, by means of the milled screw K and the clamp screw C, the level tube so that the end of the bubble appears to coincide with the index mark. When this adjustment has been made, the line of sight is parallel to the axis of the bubble and the staff reading should be the same whether the level tube is to the right or left of the instrument. (See also Adjustment of the Tilting Level on page 128.)

The Zeiss Level.—The instrument introduced by the firm of Zeiss is also typical of reversible tilting levels. Different patterns of the

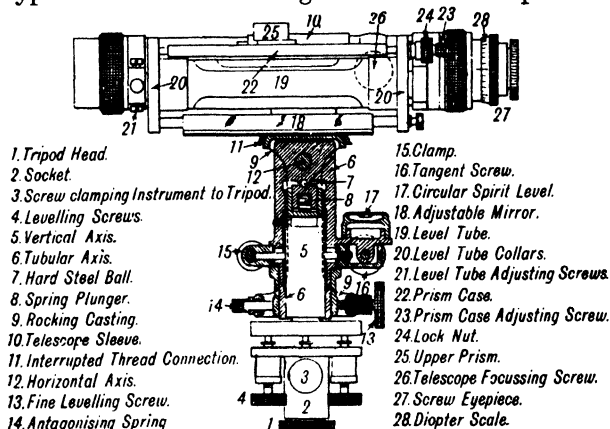


FIG. 98.—ZEISS LEVEL.

instrument are made; those designed for ordinary levelling are considered here, the precise instrument being treated in Vol. II, Chap. V.

Fig. 98 shows an early pattern in sectional elevation. Three levelling screws and a circular spirit level are provided for the rough levelling of the instrument. The vertical axis is long and is of cylindrical form, the weight of the upper portion of the instrument being carried on the hard steel ball 7. The vertical axis is surrounded by a tubular axis 6, through the upper portion of which passes the horizontal axis 12, about which the telescope can be rotated through a small vertical angle. This motion is obtained by mounting the sleeve carrying the telescope on the top of a casting 9, pivoted at 12. This casting is in the form of a hollow

cylinder with the periphery cut away on both the objective and eyepiece sides, and is shown dotted for the greater part of its length. The final levelling of the instrument is effected by means of the screw 13 and antagonising spring 14, which tilt the pivoted casting, telescope, and level tube about the horizontal axis.

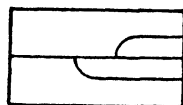
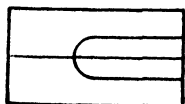
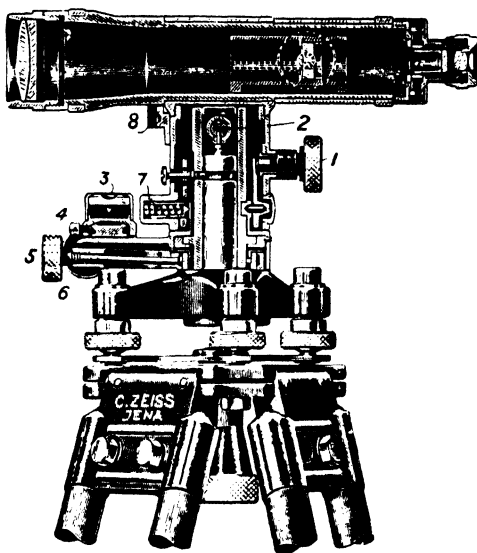
*a**b*

FIG. 99.

view of the bubble is presented to the observer. The bubble is illuminated by means of the mirror 18, and the ends of the bubble are reflected in an arrangement of prisms in the case 22, from which the images are reflected at the prism 25 into the eye. The level tube is not graduated, but when the bubble is not quite central the appearance is as in *a* (Fig. 99). By manipulating the screw 13, the central position is attained when the halves of the two ends appear coincident as at *b*.

For the purpose of testing the instrument, the level tube can be rotated along with the telescope through 180° about the telescope axis, so that, if the level tube is originally on the observer's left, it may be swung over to the right-hand side. For the same purpose, the

The level tube is ground to the appropriate curvature on both the upper and lower sides, and its metal case is cut away on top and bottom. The tube lies on one side of the telescope, and is mounted between two collars which are rigidly fixed to the telescope. An important feature of the instrument is the arrangement whereby a

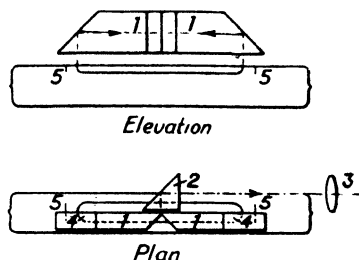


1. Tilting Screw.
2. Horizontal Axis.
3. Circular Spirit Level.
4. Adjusting Screw for Do
5. Clamp
6. Tangent Screw.
7. Counter Spring for Tilting Movement.
8. Stop for Telescope Reversal.

FIG. 100.—ZEISS LEVEL II.

line of sight may be reversed end for end in an unusual manner. The telescope, which has an internal focussing lens, is also fitted with a lens exactly the same as the objective, and placed in front of the eyepiece. Reversal is effected by removing the eyepiece and fixing it in front of the objective, the additional lens then acting as the objective. Prism 25 can be turned through 180° to face the observer.

Fig. 100 shows a later model. The principal difference from former patterns consists in the introduction of Mr. Connolly's type of bubble tube. By means of two reference lines on the tube the bubble reading attachment can be set accurately to define the two parallel bubble tube axes. The correct setting of the prism case is secured when the images of the reference lines are seen to coincide. The testing of the instrument can be effected at a single station by taking two readings on a fixed staff, bubble tube left and right, the mean being the true reading. The necessity for detaching and replacing the eyepiece does not therefore arise, and an ordinary internal focussing telescope, $8\frac{1}{4}$ in. long, is fitted. The accuracy with which the bubble can be centered is increased in this pattern by adding a lens to the prismatic reading system (Fig. 101) so that the bubble ends are seen with a two-fold magnification. It is worthy of notice that the Zeiss prism reader enables the bubble to be centered with a high degree of accuracy. Tests have shown that with a tube having a sensitiveness of 15 to 16 sec. per 2 mm. the bubble can be centered with a mean error of less than 0.5 sec.



1. Coincidence Prism.
2. Reader Prism.
3. Magnifier.
4. Parts of Bubble Reflected.
5. Reference Marks.

FIG. 101.—BUBBLE READER.

Other Forms of Tilting Level.—The patterns designed by different makers vary in detail. Commonly the fine levelling screw is placed vertically and operates directly upon the telescope barrel. It may be provided with a scale so that it is available for measuring or setting out gradients. In the Casella tilting level use is made of the Zeiss bubble reflector, but the image of the bubble ends is brought into the field of the telescope, the plane separating the halves of the bubble coinciding with that of the horizontal hair. A "Constant" level tube is employed in Watts' self-adjusting level, and a view of one end of the bubble is presented by a prism when the tube is on the observer's left, the other end being shown when it is on his right.

Accessories to Levels.—*Quick Levelling Head.*—This proves a very useful fitting for work in rough country, where it is often troublesome to set up the instrument approximately level. Various forms differ in detail, but the majority are constructed as a ball and socket joint, with the addition of a clamping device (Fig. 89). The tripod having been planted, the clamp is released, and the instrument is turned on the ball until approximately level. It is then clamped in that position, and the levelling is completed by means of the foot screws.

Compass.—Some levels, particularly older patterns, have a circular compass mounted on the stage. In general the extra weight is not justified, as this compass is seldom used.

Bubble Reflector.—In the majority of old patterns of level no provision was made for presenting a view of the bubble to the observer at the eyepiece. This is an essential feature of the tilting level, and it is now generally recognised as a very useful aid to accurate work with all kinds of level. In its simplest form, the reflector is simply a plane mirror, hinged as shown in Fig. 91.

Clamp and Tangent Screw.—These must be fitted to the vertical axis in levels with non-rigid telescopes and in instruments provided with points in the diaphragm instead of webs.

Ray Shade Clinometer.—See page 131.

Levelling Staffs.—The levelling staff or rod, the reading of which gives the vertical distance between the instrument line of sight and the point on which the staff is erected, is a wooden rod, 10 to 16 ft. in length. Patterns may be classed as (a) Self-reading, (b) Target. In the former, the graduation is such as to enable the leveller to observe the reading at which his line of collimation intersects the staff. The latter form is furnished with a sliding vane or target, the centre of which is moved into the line of sight by the staffman in response to signals. The position of the target is then recorded by the staffman.

The target rod is now very rarely used in Britain, but is still largely employed in America in ordinary levelling.

Self-reading Staffs.—The greatest accuracy is achieved by means of a staff in one length, not exceeding about 10 ft., but progress is expedited by using longer staffs, which for convenience of transportation consist of two or three jointed pieces, in the following forms :

- (a) The Sopwith or telescopic staff,
- (b) The separate-piece or Scotch staff,
- (c) The hinged staff.

The *Sopwith Staff* is illustrated in Fig. 102, which shows a 14-ft. staff consisting of two hollow slides and a solid top piece. When extended, the lengths are held by brass spring catches.

The *Scotch Staff* (Fig. 103) is formed of three separate lengths, which are fitted together by means of brass socketed joints and set-screws. It is commonly graduated on both front and back faces, which are hollowed to protect the marking from abrasion. This staff, although solid, is usually lighter than the last because of its smaller section.

The *Hinged Staff* is usually 10 ft. long, and folds to 5 ft. with the graduations inward. When open, it is kept straight by a strong hook across the hinge.



FIG. 102.
SOPWITH STAFF.

Relative Merits of Different Forms.—The Sopwith staff has to be carefully made so that the telescopic work will fit properly and not exhibit shakiness when extended. Trouble is often experienced in drawing out the slides when swollen with rain. An objection sometimes urged is that the top piece has only about half the breadth of the lowest piece, and, unless specially graduated, proves more troublesome to read at a distance. It is, however, the most commonly used form in this country, but is also the most expensive.

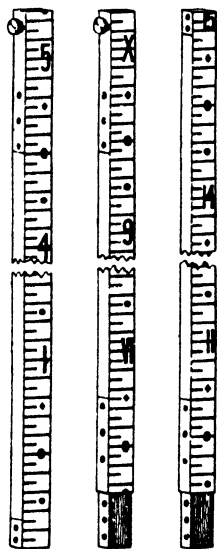


FIG. 103.
SCOTCH STAFF.

The Scotch staff may have the breadth maintained constant to the top, but some makers give a small taper. The smallness of the base compared with that of the Sopwith type is favourable to accurate holding. An objection lies in the possibility of a careless staffman losing a set-screw or, when the staff is not fully extended, leaving the top piece behind.

The hinged staff is less used because of its shorter length and its liability to develop shakiness.

Graduation.—The advantages of a decimal over a duodecimal subdivision of the foot are recognised in levelling, and staffs are graduated accordingly. The smallest division ranges from $\cdot 01$ to $\cdot 10$ ft. By means of the former, readings for special work can be observed to $\cdot 005$ ft. with short sights, but at long range close division is confusing. Numerous patterns have been proposed in which $\cdot 01$ ft. graduations are accompanied by bold marking of the tenths for facility of observation at long distances. Equally good results are obtained with a division to $\cdot 05$ ft., the position of the hair

being read by estimation to $\cdot 01$ ft., or even $\cdot 005$ ft., and this graduation remains clear at considerable distances.

Typical graduation patterns will be found illustrated in makers' catalogues. Before using a particular pattern for the first time, it should be carefully examined, and the significance of the markings understood.

Target Staffs.—Details differ in the several types of these. Fig. 104 shows a 2-ply New York pattern rod, 6·8 ft. long when closed, but extending to 12 ft. The graduation is carried to $\cdot 01$ ft., and the target is provided with a vernier whereby its position can be read to $\cdot 001$ ft. In this example, if the reading does not exceed 6·5 ft., the staffman slides the target along the rod until signalled that its centre is in the line of sight, clamps it, and reads the vernier. If, however, the reading exceeds that amount, the target is clamped at 6·5 ft., and the upper part of the rod, carrying the target, is slid upwards until the target is bisected by the line of sight. The reading is then obtained on a vernier at the back of the rod.

Some forms (notably the Philadelphia rod) have sufficiently bold graduation that they can also be used as self-reading rods. The pattern illustrated can be so used only for short sights.

Relative Merits of Self-reading and Target Staffs.—A comparison is greatly in favour of the self-reading form, which has a decided advantage in the rapidity with which observations may be taken. The process of adjusting a target is tedious, and the time occupied in setting and reading largely depends upon the capability of the staffman. Again, in using a self-reading rod, the responsibility for correct reading lies with the leveller, as it should do, and the staffman can concentrate his attention on holding the staff plumb. The duties of a target staffman are nearly as important as those of the leveller, and demand the services of a trained man.

The fineness of reading is greater in the target rod than in the self-reading, but the refinement is usually more apparent than real, as it largely depends upon the accuracy with which the bisection of the target by the line of sight can be estimated.

FIG. 104.
TARGET
STAFF.

The ultimate degree of accuracy attained is not materially different in the two types.

Reading the Staff.—The graduation of the staff being understood, the only difficulty experienced by the beginner in observing a

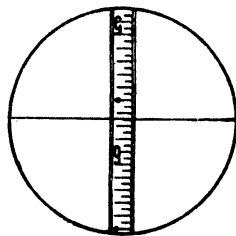


FIG. 105.

self-reading staff is due to its appearing inverted, as this necessitates reckoning downwards. Thus, in Fig. 105, the reading is 5.64.

Notes.—(1) When the staff is sighted at a short distance, it may happen that no foot graduations appear in the field of view. In such a case the staffman should be instructed to raise the staff slowly. The figure which appears at the top of the field represents the whole feet to be booked. This trouble is avoided if the staff exhibits the value of each foot at intermediate points on it.

(2) Readings may be taken against any part of the horizontal hair of a dumpy level, but it is safest to point the telescope so that the staff appears midway between the vertical hairs. In the wye level, however, the perpendicular cross-hairs may lie in any direction from their intersection, and observations can be made only with respect to the intersection.

Temporary Adjustments of the Level.—These are :

(1) Levelling up.

(2) Elimination of parallax (see page 28).

The procedure in levelling up is similar to that for the theodolite (page 85). With a four-screw head, the bubble tube must be brought over each pair of diagonally opposite foot screws alternately. With three screws, it should be placed parallel to two of them and levelled by that pair. The telescope is then swung through 90° , and the bubble is adjusted by the third screw. The levelling is completed by successive trials with the telescope in its alternate positions at right angles to each other.

Note.—To save time, set up the instrument as nearly level as can be judged before proceeding to use the levelling screws. A circular spirit level or cross bubble tube greatly facilitates the preliminary levelling. At the first manipulation of the screws do not be over particular with the centering of the bubble, but perfect it gradually at the successive wheelings of the telescope through 90° .

TESTING AND ADJUSTMENT OF THE LEVEL

In the examination of the level with reference to the quality of the non-adjustable parts, the important requirements are stability and a proper relationship between the sensitiveness of the level tube and the resolving power of the telescope. The tests are the same as for the theodolite (pages 89 and 92).

The number and nature of the adjustments of the level, the order in which they should be made, and the routine to be followed depend upon the type of instrument. It will suffice to describe here the procedure in the case of the commonest patterns of dumpy, wye, and tilting levels. This will serve as a sufficient guide to the methods to be followed for modified types of the standard instruments, the adjustments of which are usually published by the makers.

In instruments having a fixed telescope fitted with four adjusting screws at the diaphragm, the horizontality of the horizontal hair may be tested and, if necessary, adjusted, as on page 94, before proceeding to the other adjustments.

Adjustment of the Dumpy Level

The requirements of the dumpy level which can be established by adjustment are :

1. The line of collimation of the telescope should be parallel to the bubble tube axis.
2. The bubble tube axis should be perpendicular to the vertical axis.

The first of these is very much more important than the second, and, in adjusting an instrument, the latter should be attended to first. This order is essential in the case of instruments of the solid type (Fig. 90).

It is frequently regarded as essential that the line of collimation should coincide with the optical axis of the telescope, but it may be shown* that this condition is not essential to accurate work provided the objective does not move in focussing.

1. Adjustment of Perpendicularity between Vertical Axis and Level Tube Axis.—*Object.*—To ensure that, once levelled up, the bubble will remain central for all directions of pointing of the telescope. This will occur when the vertical axis is truly vertical.

Necessity.—This adjustment is made merely for convenience in using the level. Assuming that the instrument is otherwise correct, the essential requirement for accurate observation is that the bubble should be central at the instant of sighting. If the adjustment is out, so that the bubble moves towards one end on swinging the telescope to point towards the staff, it must be brought back to its central position by the levelling screws. Any change in the elevation of the line of sight thus produced is negligible.

Test.—1. Set up the instrument on firm ground, and level carefully in two positions at right angles to each other in the usual manner.

2. Swing the telescope through 180° . If the bubble remains central, the adjustment is correct.

Adjustment.—1. If not, bring the bubble half-way back by the adjusting screws 2, connecting the bubble tube to the telescope in the case of the solid dumpy (Fig. 90) or by the screws 11, 12 in the case of the stage type (Fig. 89).

2. Level up, and repeat the test and adjustment until correct.

2. Adjustment of the Line of Collimation, or the Collimation Adjustment.—*Object.*—To place the line of collimation and the bubble tube axis parallel.

Necessity.—If this condition obtains, the line of collimation is horizontal when the bubble is in the centre of its run. Since the whole function of a level is to furnish a horizontal line of collimation, this requirement is of prime importance, and merits frequent examination.

* See Rankine, *Civil Engineering*, Chap. IV.

INSTRUMENTS—CONSTRUCTION AND ADJUSTMENT 125

Test.—1. Peg two points, A and B (Fig. 106), 300 to 400 ft. apart, on fairly level ground. Set the level between them at C, so that the eyepiece will almost touch the face of a levelling staff held on A.

2. Level up carefully, and note the reading b of a staff held on B: then, by looking through the object glass, observe the reading a of the staff on A.

3. Transfer the instrument to D near B, level up, and read the staff on both pegs as before, obtaining the readings c and d . If

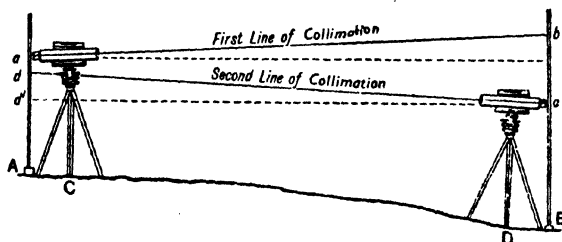


FIG. 106.

the difference of level between A and B, as deduced from readings a and b , equals that given by c and d , the result is their true difference of level, and the adjustment must be correct, since such equality could be secured only by a horizontal line of collimation.

Adjustment.—1. If not, compute the true difference of level between A and B. This is given by the mean of the two erroneous differences, for if D be the true difference of level, and e the error introduced in reading the far staff, then, for the case illustrated,

$$\begin{aligned} b-a &= D+e, \\ \text{and } c-d &= D-e, \\ \text{whence } D &= \frac{(b-a)+(c-d)}{2}. \end{aligned}$$

2. By applying D to the reading c , compute d' , the reading on A at the same level as c . The instrument being still at D, turn the top and bottom diaphragm adjusting screws b (Fig. 90) until the line of collimation is directed to d' , taking care that the bubble is in the centre of its run. The line of collimation is now horizontal. In the case of the stage dumpy (Fig. 89) it is more convenient to direct the line of collimation to d' by turning the levelling screws. The bubble is then returned to the centre of its run by means of the adjusting screws 16 connecting the level tube to the telescope, but in this method the present adjustment must be the first made.

Notes.—(1) It is presumed that altering the line of collimation to read d' instead of d will not appreciably affect the reading c . It is unlikely that the

adjustment will be so much in error that change in c could be observed, but, if so, the error is made negligible by repeating the adjustment.

(2) If A and B are nearly at the same level, the results of the observations from C and D may show a difference of sign, i.e. one may indicate a rise, and the other a fall, from one peg to the other. The true difference of level is always half the algebraic sum of the two erroneous differences, and therefore has the sign of the greater difference.

(3) The method given is designed to eliminate possible error in focussing due to droop of the slide carrying the diaphragm and eyepiece in the case of an external focussing telescope. The hairs are not visible on looking through the object glass, but the reading can be estimated with considerable precision.

(4) On completion of the adjustment, a test may be made for droop by removing the instrument a little farther from B so that the staff on B may be read in the ordinary manner although the sight is short. If the difference between the readings is not now the correct difference of level between A and B, the error must be occasioned by the change of focus required. The defect may be remedied in certain instruments by means of screws passing through the outer tube and bearing upon the slide, but more commonly these are omitted, in which case the correction is best undertaken by the maker.

Alternative Method.—The possible effect of droop is neglected in the following very commonly used method, which is entirely suitable for the case of internal focussing instruments.

Test.—1. Drive two pegs, A and B, as before. Set up the instrument at C, exactly midway between them (Fig. 107), level up

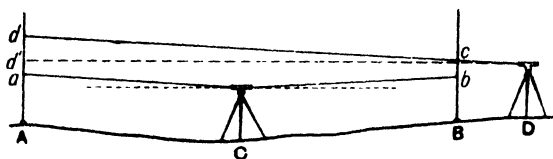


FIG. 107.

carefully, and read a staff on A and on B. The difference of the readings a and b is the true difference of level, since the error affects both readings in the same way and to the same extent.

2. Set the instrument near one of the pegs, and preferably beyond it, as at D, level up, and note the readings c and d of the staff on the pegs. If the difference between c and d equals the true difference of level, the adjustment is correct.

Adjustment.—If not, by applying the true difference of level to the reading c , deduce d' , the reading of A which would be observed by a horizontal sight from D, and by means of the diaphragm adjusting screws 6 (Fig. 90) direct the line of collimation to d' taking care that the bubble is central. In the case of the stage dumpy (Fig. 89) use the levelling screws for setting the line of sight on d' , and bring the bubble central by the adjusting screws 16, as before.

Notes.—(1) It is quite possible in this case that the reading c will be perceptibly changed in the course of the adjustment owing to the greater distance between B and D. The adjustment should therefore be perfected gradually by repeating the observations from D once or twice.

(2) Swinging the telescope, with the bubble maintained in the centre of its run, causes an unadjusted line of collimation to generate a cone, the axis of which is the vertical axis of rotation. The true difference of level between two points may therefore always be obtained by placing the instrument equidistant from them. This constitutes one of the most important precautions in levelling practice.

Adjustment of the Wye Level

While the ultimate aim in adjusting the wye level is the fulfilment of the two geometrical relationships given for the dumpy level, the arrangement of the wye instrument alters the procedure, and three adjustments are required. The relationships to be established are :

1. The line of collimation should coincide with the axis of the telescope collars, the latter axis being, in a well-made instrument, coincident with the optical axis.

2. The line of collimation should be parallel to the bubble tube axis.

3. The bubble tube axis should be perpendicular to the vertical axis.

1. Adjustment of the Line of Collimation.—*Object.*—To make the line of collimation coincide with the axis of the telescope collars.

Necessity.—The fulfilment of this condition is highly important ; otherwise, unless the telescope can be clipped in one position, rotation about its axis would cause the line of collimation to generate a cone, and the fundamental requirement of parallelism between the bubble axis and the line of collimation would obtain only for a particular position of the telescope in the wyes.

Test and Adjustment.—As in adjustment 2 of the wye theodolite (page 100).

2. Adjustment of the Level Tube.—*Object.*—To make the level tube axis parallel to the line of collimation. The manner in which the telescope is mounted renders it necessary to effect this adjustment in two steps : *A*, to bring the bubble tube axis into the same plane as the line of collimation ; and *B*, to place them parallel.

Necessity.—As for the dumpy level.

A.—Test.—1. Level up carefully.

2. Rotate the telescope in the wyes through a small angle. If the bubble remains central, the adjustment is correct.

Adjustment.—1. If not, bring the bubble central by the adjusting screw 8 (Fig. 94) controlling the level tube laterally.

2. Repeat the test and, if necessary, the adjustment.

Note.—No reversal being performed in the test, the apparent error is the actual error.

B.—Test.—1. Set the telescope parallel to two levelling screws, open the wyes, and level up carefully.

2. Lift the telescope gently from the wyes, and replace it end for end. If the bubble remains central in the reversed position, the adjustment is correct.

Adjustment.—1. If not, move the bubble half-way towards the centre by the screw 7 (Fig. 94) at one end of the tube.

2. Repeat test and adjustment until correct.

Notes.—(1) Since the reversal is made with respect to the wyes, the adjustment really secures parallelism between the level tube axis and the line joining the bottoms of the wyes. If, however, the telescope collars are exactly similar, and adjustment 1 is correct, the bubble tube axis will have been placed parallel to the line of collimation. The error arising from inequality of the collars will probably be negligible for ordinary levelling.

(2) The two-peg method used in the same adjustment of the dumpy level may be adopted for step *B*. Although less convenient, it has the merit of being unaffected by collar inequality.

3. Adjustment of Perpendicularity between Vertical Axis and Level Tube Axis.—As for the stage dumpy level, adjustment 1 (page 124).

Adjustment of the Tilting Level

In the adjustment of the tilting level it is only necessary to secure that the line of collimation is horizontal when the bubble is centered. In tilting levels of the non-reversible type, the test is performed as for a dumpy level by the method of page 126 (Fig. 107). If it is found that the difference of the staff readings observed from *D* does not represent the true difference of level of *A* and *B*, the reading *d'* is computed as before, and the line of sight is directed upon *d'* by means of the tilting screw, the bubble being ignored. The adjustment is effected by returning the bubble to the centre of its run by means of the bubble tube adjusting screws.

Instruments of the reversible type possess the advantage that the test can be rapidly performed from one position of the instrument, as follows :

Test.—1. With the level tube, say, on the left-hand side of the telescope, sight a staff held upon a firm point. Centre the bubble accurately, and note the reading.

2. Rotate the telescope about its longitudinal axis through 180° , so that the level tube is brought to the other side, and again read the staff. If the readings agree, and the bubble tube is of the type having its upper and lower axes parallel, the line of collimation must be horizontal, and the adjustment is correct.

Adjustment.—1. If not, direct the line of sight towards the mean reading by operating the tilting screw.

2. In the case of an instrument having a simple bubble reflector,

centre the bubble by means of the adjusting screw connecting the bubble tube to one of its supports. If a Zeiss prismatic reader is fitted, adjust by moving the prism case longitudinally until apparent coincidence of the bubble ends is obtained. In the instrument shown in Fig. 98, screw 23 is provided for this purpose, and screws 21 are available if the error is more than can be eliminated by adjustment of the prism case.

3. Repeat the test and adjustment until no error is shown on reversal.

Gauss Method of Collimation Adjustment.—This method is not a field one but is very generally used in workshops and in testing laboratories. It involves the use of a special instrument, called a collimator, which is simply a large sensitive wye level with a high class objective. This instrument is very carefully adjusted for collimation by the method ordinarily used for the adjustment of wye levels, and, when tests are being made, the focus of the telescope is accurately adjusted for infinity, so that, after the instrument is carefully levelled, rays of light from the cross-hairs leave the objective as a beam of horizontal parallel rays. The instrument to be tested is set with its telescope at approximately the same height as the telescope of the collimator and with the objectives of each instrument facing each other. When the collimator is illuminated from the eyepiece end, an image of its cross-hairs appears in the field of the instrument to be tested, and the focus of the latter is adjusted to eliminate parallax between the images of the two sets of cross-hairs. The adjustment consists in bringing the images of corresponding hairs into coincidence when the bubbles of both instruments are at the centre of their run.

HAND LEVELS AND CLINOMETERS

These levelling instruments are designed with a view to lightness and compactness rather than a high degree of precision, since the absence of a steady support does not allow of the employment of a sensitive bubble tube. They are convenient for rapid work, especially where serious error cannot be accumulated, as in cross sectioning. The clinometer is simply a hand level adapted for measuring vertical angles, and, as it can be employed as a hand level, is the more useful instrument.

The Hand Level.—The hand level (Fig. 108) consists of a tube, about 4 in. long, circular or square in section, and having a small bubble tube mounted on top. A line of sight parallel to the axis of the bubble tube is defined by a pin hole at the eye end and by a cross bar, or one edge of a rectangular opening, at the

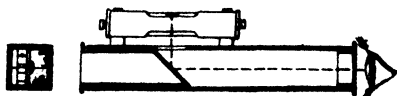


FIG. 108.—HAND LEVEL.

other. To afford the observer a view of the bubble to enable him to hold the instrument level, the metal body is cut away under the level tube, and a reflecting surface is inserted in the tube at 45° to its axis. This reflector extends half-way across the tube, and distant objects can be viewed through the other half. When the centre of the bubble appears opposite the cross-bar sight, or lies on a line ruled on the reflector, the instrument is being held level, and all points intersected by the line of sight are at the same level as the eye. Telescopic forms of the instrument are also available.

If provision is made for adjustment of the bubble tube relatively to the line of sight, the test is carried out by the two-peg method as for the dumpy level.

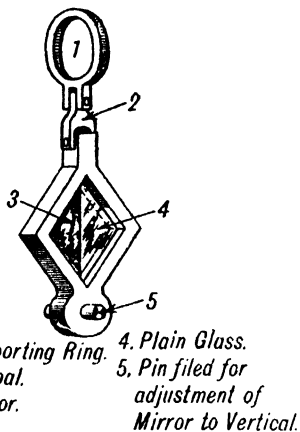


FIG. 109.—BUREL HAND LEVEL.

The Burel Hand Level.—This form of hand level is based on the fact that a ray of light which is reflected back from a vertical mirror along the path of incidence must be horizontal. The instrument (Fig. 109) consists of a glass mirror, the frame of which is suspended in gimbals so that the mirror hangs vertically. The mirror extends half-way across the frame, and the other half is plain glass, the edge of the mirror forming a

vertical or, in some patterns, a horizontal line. To use the instrument, it is suspended from the thumb and held at arm's length in such a position that the observer sees the reflected image of his eye at the edge of the mirror. Distant objects appearing through the plain glass opposite this image are at the same level as the observer's eye. The instrument proves very rapid in practice, but is unsatisfactory in windy weather.

The Abney Clinometer.—This instrument (Fig. 110), usually known as the Abney level, is deservedly the most popular instrument of its class, and is simply the hand level of Fig. 108 adapted for measuring vertical angles. The bubble tube is pivoted, and carries an index arm perpendicular to its axis. Rotation of the milled wheel moves the index over a graduated arc attached to the sighting tube, and, when the bubble is centered, the reading on the arc represents the inclination of the line of sight above or below the horizontal. A small clamp is sometimes fitted to the index arm, and is useful for fixing the index to zero, so that the clinometer can be used as a hand level. The instrument is also made in a telescopic form.

In using the Abney, the line of sight is directed towards the object, and the milled wheel is turned until the reflected image of the bubble

appears in the plane of sight. The required angle is then read on the arc. To test and adjust the instrument, the index is set to zero, and the two-peg method is applied. Alternatively, two points at different elevations are selected, and the vertical angle between

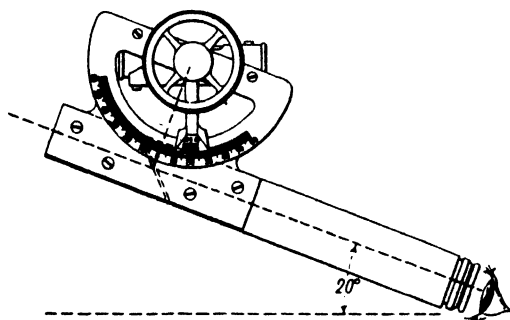


FIG. 110.—ABNEY CLINOMETER.

them is observed from both. The angle of elevation observed from the lower station should equal the angle of depression taken at the upper. If these differ, their mean is the correct value of the inclination, and the instrument is made to record this by means of the adjusting screws controlling the level tube.

The De Lisle Clinometer.—This instrument is a modified Burel level in which the mirror can be inclined to the vertical by means of a weighted arm. The inclination to the horizontal of the line from the eye to the point at which it appears in the mirror equals the inclination of the mirror to the vertical. To measure a vertical angle, the mirror is tilted until the image of the eye appears opposite the signal, and the angle is read by the position of the arm on a graduated arc.

The Ray Shade Clinometer.—The ray shade of the level has been neatly adapted for rough levelling and clinometric observations at right angles to the telescopic axis. The shade is provided with two narrow slits (Fig. 90) diametrically opposite each other. The centre line of one coincides with the zero of a scale graduated to show the inclination of the line of sight through the slits. When the instrument is levelled up, and the zero of the scale placed opposite an index mark on the telescope barrel, the plane of the slits is horizontal. To measure an inclination from the horizontal, it is only necessary to turn the ray shade until the line of sight intersects the object, when the angle is shown against the index.

The Foot-rule Clinometer.—This consists simply of a stout 12-in. boxwood rule fitted with a small bubble tube on each part, a

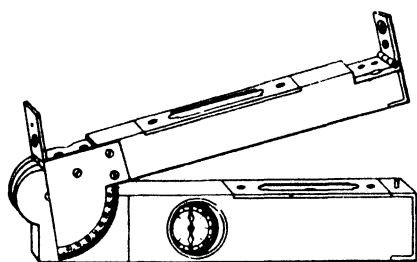


FIG. 111.—FOOTRULE CLINOMETER.

graduated arc, and a pair of sights (Fig. 111). In sighting, the instrument is held against a rod as firmly as possible with the bubble central in the lower tube, and the rule is opened until the line of sight lies at the required inclination, which is then read on the arc. In a common application of the clinometer the sights are not used, and the instrument

is placed on a straight-edge laid on the slope to be measured. In this case the rule is opened until the upper part is level.

Miscellaneous Levelling Instruments.—The gradiometer is an instrument designed to facilitate the measurement or setting out of long uniform gradients. It consists of a level in which the telescope is so mounted that it may be set at various inclinations to the horizontal, the gradient being registered on a drum. When the index is set to zero, the instrument serves as an ordinary level.

For levelling over a distance of a few feet, the commonest apparatus employed is a straight-edge laid horizontal by means of the familiar mechanic's level, which is simply a bubble tube mounted on a flat base parallel to the axis. Levels can also be transferred for short distances without a spirit level by using a frame having a straight edge square to a plumb line. In an emergency, a horizontal line of sight can be obtained by placing in a pail of water a wooden float with two equal upright pieces, the tops of which can be sighted across. In the water level proper, the horizontal is defined by the water surface in the legs of a U-tube or in a pair of glass tubes the lower ends of which are connected by rubber tubing. The latter apparatus is of considerable utility and accuracy.

PRECAUTIONS IN USING INSTRUMENTS

Setting Up.—(a) In lifting the instrument from the box, handle it in such a way as to minimise the possibility of strain. It must not be lifted by the telescope, the standards, or axes, but should be supported by the hand placed under the levelling head.

(b) Screw the instrument firmly home on the tripod head, so that the connection may not slacken during observations.

(c) If the instrument has three levelling screws held by a locking plate, see that the plate is gripping the screws and is properly locked, otherwise the instrument will fall off the tripod when lifted.

(d) Do not force the levelling screws of a four-screw instrument.

(e) After centering a theodolite by means of a shifting head, do not fix the locking nut of the head too tightly. Certain forms are

easily jammed, and it is well to make trial of the least force necessary to prevent slip, and, in practice, to give only a slightly greater grip than this.

Carrying.—(a) Support the instrument against the shoulder as uprightly as possible. In the case of the theodolite, place the telescope vertical with its clamp slack, and release the lower clamp.

(b) In situations with limited headroom carry the instrument in front, and see that it does not strike branches, etc. In crossing a fence or any awkward place, it is best for one man to go over first and have the instrument handed to him.

(c) If the compass has been in use, throw the needle off its pivot before lifting the instrument.

In Wind.—(a) Spread the tripod legs well apart, and thrust them firmly into the ground.

(b) On a pavement or other smooth surface, if sufficient spread is given to prevent overturning by wind, the instrument may collapse by the feet sliding outwards: they should therefore be inserted in joints or cracks.

(c) Do not leave the instrument in high wind, even although the above precautions have been taken.

In Rain.—(a) Protect the objective as far as possible by extending the sun shade. Use a brush for absorbing moisture from the eye lens. If a brush is not available, a corner of the handkerchief may be used, but the lens should not be rubbed.

(b) If work is stopped temporarily, close the sun shade by the shutter, and, if the eyepiece has no cap, make a paper cover and attach it by a rubber band. The telescope of a theodolite should be set vertically, with the eyepiece down. In wet climates a waterproof hood to slip over the instrument is useful.

(c) If water enters the telescope tube, it will cause dewing of the lenses, and observations are impossible. This cannot be remedied at once in the field, as the hairs must never be exposed out of doors. The moisture will gradually evaporate if the instrument is left in the sun or in a moderately warm room: in the latter case, drying is hastened by removing the eyepiece.

(d) Dry all exposed parts thoroughly before putting away the instrument. Use a soft piece of linen in wiping graduated circles.

General.—(a) Do not force screws, or apply clamps too tightly.

(b) A camel-hair brush is best for dusting instruments. Levelling and tangent screws are cleaned with a nail brush, and a little oil should be applied, and finally wiped off. Oil should not be left on exposed parts as it collects dust. Vaseline or watch oil is used to lubricate axes, but this need seldom be renewed. Unless the instrument develops stiffness, it should not be taken apart.

(c) Avoid fingering silver graduated surfaces. When tarnished,

apply a very little vaseline, and where necessary rub it with the finger, and wipe clean with a soft rag. On no account use plate polish.

(d) For care of telescope, see page 33.

Packing.—(a) There is only one position in which an instrument will fit into its box, and the difficulty of packing may be lessened if a note of the correct positions of the various parts is written in the box. The plates of the levelling head must be left approximately parallel, and clamps should be slackened.

(b) The instrument should pack without forcing, but it may happen that difficulty is experienced owing to shrinkage or warping of the wood of the box. The offending part can be discovered by chalking those portions of the instrument held by the bearing pieces inside the lid, and may be cut down a little.

(c) The cap and ring of the tripod should be kept in the box while the instrument is in use.

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CHAPTER II

LINEAR MEASUREMENTS

THE methods of making linear measurements, or measurement of length or distance, required in surveying vary greatly with the standard of accuracy required. For some classes of work an error of one or two feet in a hundred feet, or even more, does not matter ; but, in others, an error of one foot in a mile would be considered excessive, while, for the very highest class of linear measurement done in survey work—that is in the measurement of a base line for geodetic triangulation—the discrepancy between two separate measures of the whole line will rarely exceed one part in two or three million.* Corresponding to these different standards of accuracy, the methods and apparatus used vary from those of the roughest possible description to others carried out with apparatus of great refinement and delicacy. In ordinary engineering work, and in simple land surveying, great accuracy in linear measurement is seldom necessary, and quite rough and ready methods may often be used. In this chapter we will deal first of all with these less accurate methods and then proceed to consider others from which a fairly high degree of accuracy may reasonably be expected ; but, for an account of the methods and apparatus used in base line measurement, reference should be made to the second volume of this book.

Cases where quite rough methods of linear measurement are permissible are the survey of ill-defined detail, such as the edge of a marsh, or the filling in of more important detail when a sound system of control points is already available. In this class of work, the use of the ordinary surveyor's wire chain is justified for the longer lines and a simple linen or "metallic" tape for the measurement of short "offset" lines. At the other extreme are such cases as laying out tunnels, where the survey is often of geodetic accuracy, or measurements required for the design or lay-out of bridges over wide rivers, where a short base line or traverse may have to be measured as part of a small scheme of triangulation.

There are two main methods of measuring distance, the one

* This figure represents the difference between two field measurements, but it does not represent the true "probable error." When all sources of error, such as errors in temperature, slope, standardisations, etc., are taken into account, the real probable error of measurement of a geodetic base line is seldom much less than one part in five hundred thousand, and this may be considered to be about the standard of accuracy ordinarily attained in such work.

direct and the other by optical means. The direct methods include the use of the wire chain or of the steel band, while the optical methods include readings on an ordinary range-finder and those to which the name "tacheometry" has been given. The accuracy of the ordinary tacheometrical methods seldom exceeds one part in five hundred to one part in a thousand, but with the modern double-image tacheometers described in pages 613 to 621 an accuracy of about one part in five thousand to one in ten thousand is obtainable. Where great accuracy is not required, optical methods are often much quicker than an ordinary chain or steel band and may save a good deal of clearing in bush or forest country. Nowadays, most theodolites and levels are fitted with "stadia hairs", which enable simple optical methods to be used when desired.

In this chapter we will consider the direct methods only, while tacheometrical methods will be described in Chap. XI.

RANGING OR SETTING OUT CHAINAGE LINES

In measuring the lengths of lines it is important that the chain or band should follow, as far as possible, the straight line between the terminal points, and that deviations from that straight line should be as small as possible. If the line is short, or the distant end easily visible from every point along it, it is easy to maintain direction; but, if the line is long, or the station at the distant end of it not visible from every point, it may be necessary to put some intermediate poles or ranging rods in positions where they will assist the rear chainman to control direction and prevent the path actually followed by the chain from deviating to any considerable extent from the line that it is supposed to follow. This "ranging," as it is called, should generally be done before measurement commences; and, for most purposes, it can be done by eye without the use of any directing instrument, though, in the case of the more precise type of traverse, or in such work as laying out long "straights" in railway surveys and construction, it will nearly always be advisable to range and set out the line by using a theodolite to control direction.

Ranging by eye.—The ranging of a line by eye is performed by the surveyor and an assistant. The former remains at one station, while the latter proceeds the required distance along the line and puts himself approximately in alignment. He faces the surveyor, and holds a pole vertically and nearly at arm's length, so that his body will not obstruct the surveyor's view. The latter directs him to right or left until the pole appears in line with the remote station pole. When signalled to mark, the assistant fixes his pole and examines it for verticality. He should then wait for a second signal as to whether the surveyor is satisfied with the fixing of the pole.

Notes.—(1) To obviate errors due to non-verticality of the poles, it is most important that, where possible, ranging should be performed with reference to their lower ends. The surveyor should therefore bring his eye sufficiently low by stooping.

(2) For good work he should station himself a few yards behind his pole, and should sight with one eye along the edges of the poles. Alternatively, he may bring the mid-point between his eyes into line by finding that position from which, by opening and closing each eye alternately, his pole appears to move equally on either side of the farthest pole. The alignment of intermediate poles is then easily judged by sighting them with both eyes.

(3) The assistant should align himself roughly, and should not have to be repeatedly warned that his pole is not plumb. It should be held lightly by the forefinger and thumb so that it hangs vertically with the shoe an inch or two off the ground. On fixing, it may be tested by a plumb line or by dropping a pebble alongside.

(4) In guiding the assistant into line, the surveyor should use a pre-arranged code of signals, such as :—

Rapid sweeps with right (left) hand—Move considerably to right (left).

Right (left) arm extended—Continue to move to right (left).

Slow sweeps with right (left) hand—Move slowly to right (left).

Right (left) hand up and moved to left (right), left (right) hand down and moved to right (left)—Plumb pole in direction indicated.

Both hands above head—Correct.

Such signals should be made to be seen clearly. The arms should be extended clear of the body ; signals to move to the left should not be made with the right hand. When they have to be read from a considerable distance, signals should be made with a handkerchief.

Ranging by Theodolite.—In ranging a line by theodolite, the instrument is set up and adjusted over one station and sighted so that the vertical cross-hair intersects the distant station. The vertical axis is now firmly clamped, and, if necessary, the cross-hair brought on to the mark by the lower tangent screw, which is not touched after this setting has been made. Intermediate points can then be set out by signalling to an assistant holding the ranging rod until the rod is seen behind, and apparently bisected by, the vertical cross-hair.

If a straight line has to be prolonged, and no forward station can be seen to sight on, the instrument is set up, centered, and levelled over a peg at the forward end of the line, and the telescope directed to the back station. Much the same procedure is now adopted as was done in adjusting the vertical cross-hair for collimation (page 95). Referring to Fig. 81, let A be the back station. After sighting A and clamping the lower plates, transit the telescope and line out a wide-topped peg at B, the point where the next forward station is required, and get the assistant there to put a tack in the peg to mark the exact point where the vertical cross-hair intersects it. Swing the instrument through 180° , again sight A, transit the telescope, and line in a tack in the peg at C (Fig. 81*b*). Unless the instrument is in perfect adjustment for collimation, which it very seldom is, the tacks at B and C will not coincide. Take the point A' half-way between B and C and put a tack there. Then OA' is the prolongation of the line AO.

If the collimation error of the instrument is rather large it may

be impossible to get both marks on the peg at B. In that case it will be necessary to drive a separate peg at C and then either nail a slat across both pegs or put another one in between them to take the mark A. If an extra peg has to be driven, the sights should be repeated in case driving one peg has disturbed the position of another.

In all cases where straight lines are set out by theodolite the procedure given above should be followed strictly unless the surveyor is absolutely certain that the collimation error of his instrument happens to be so small as to be negligible. If an appreciable collimation error exists, and steps are not taken to eliminate the effects of it, the line traced out on the ground will consist of a series of chords to a flat curve and will thus not be a straight line.

LINEAR MEASUREMENTS WITH THE CHAIN

Although it can only be expected to give comparatively rough results, and should only be used when any high degree of accuracy is not essential, the ordinary wire chain, already described on page 47, is nevertheless a most useful article in general surveying and engineering. Compared with the long steel band or tape, it is much stronger and less liable to break, and, being very much cheaper in the first instance, it is less expensive to replace. It will stand up to much rougher treatment, and, in many respects, is easier—especially for inexperienced labourers—to handle, although in bush country this may not always be so, because there is a tendency for the rings joining the links, and those to which the tally marks are attached, to catch on odd twigs or low-cut stumps. Unfortunately, its relative strength, compared with the steel band, may be a defect rather than an advantage, as it may easily sustain sudden jerks or jars which would cause the steel band to break but which, in the case of the chain, will only either open out and lengthen the connecting links or will cause lengthening of the individual links themselves. Thus, the chain may easily have its length altered quite appreciably without this being at once apparent. On the other hand, when a chain breaks, it usually does so at a ring or one end of a link, and a new ring or a new link can easily be inserted. Most chains, however, increase in length with use, as the material from which they are made seems to be comparatively soft, and this lengthening may easily amount in time to several inches or even to a foot. Hence, all chains should be tested regularly against some sort of standard of length.

As regards the accuracy to be expected from work with the chain, no hard and fast rule can possibly be formulated. With a new chain, carefully tested under conditions of temperature, pull, etc. similar to those under which it will be used, an accuracy of about one part in two thousand, or 1 : 2,000, can sometimes be attained

on flat ground and with careful work. As a general rule, this figure will not be reached, and it is probably safer to reckon that, under average conditions, an accuracy of about 1 : 500 is more likely than not to be the limit reached. Even that limit postulates a chain in reasonably good condition, and, with old chains, in which some of the links and rings have lengthened, the overall length is quite likely to be in error by as much as 1 : 200 to 1 : 100. For these and for other reasons it is inadvisable to attempt to lay down the exact conditions when the use of the chain is permissible and when it must be replaced by the steel band, and this point must be decided by the engineer or surveyor himself, bearing in mind the nature of the work he is called upon to do and the accuracy required.

Field Party.—The party consists of :

The Surveyor, who determines what measurements are to be taken, directs their carrying out, and records the results in his field book.

Two Assistants or Chainmen, who make the measurements.

Except in the case of a survey which may be completed in an hour or two, it is a mistake for the surveyor to work with only one assistant, as he must then take part in the chaining, etc., time is wasted, and it is difficult to keep the notes clean.

Equipment.—*Essential.*—One chain, 66 ft. or 100 ft., with ten arrows ; one linen or metallic tape, 50 ft. or 66 ft. ; six to twelve ranging poles, according to size of survey and character of ground ; field book, pencil, and rubber.

Optional.—Cross staff, or optical square, or box sextant for laying out offset lines ; line ranger ; clinometer ; small compass ; plumb line ; offset rod ; pegs ; flags for attaching to poles ; field glass.

Chaining the Line.—The word “chaining” denotes the measurement of a distance whether it is executed by the chain or the steel band. No explanation is required of the case where the distance is shorter than the length of the chain, but otherwise the chain must be stretched successively in the correct line, and it is important to realise the need for system in this apparently very simple operation if speed is to be acquired and serious error avoided.

Throwing out the Chain.—With both handles in his left hand, the chainman throws out the chain with his right, and, assisted by the second chainman, proceeds to free it from twists and knots. Otherwise, one man may retain the handles, while the other walks forward with the remainder of the chain, paying it out on to the ground. When the chain has been roughly straightened, and examined for bent links and badly opened joints, the work of laying it down successively may proceed.

Lining and Marking.—The more experienced of the two chainmen remains at the zero end or rear of the chain, and is known as the

follower; the other, called the leader, takes the forward handle. Before starting, the leader must provide himself with the ten arrows with which to mark the successive positions of his end of the chain, and he and the follower should each have a ranging pole.

While the follower holds the rear handle at the terminal point, the leader stretches the chain as nearly in the line as he can judge, and, standing erect, holds his pole for alignment by the follower. It is desired to range a point a little short of the chain length from the follower; the end of the chain being slack and held in the leader's left hand, the pole may therefore be placed about opposite the handle. When the pole has been lined, the position it occupied is marked by the hole made, or by a scratch, and the pole is removed, but in long grass or heath it should be left in. The follower now holds his handle exactly at the station point, while the leader proceeds to stretch the chain in a straight line over his mark or against the pole. This is accomplished by transmitting a series of gentle undulations along it by shaking the handle up and down, and at the same time bringing it gradually into line. Only sufficient tension to keep the chain straight is required. While straightening the chain, the leader should be stooping, and holding an arrow hard against the end of the handle and in the same hand, so that he can thrust the arrow vertically into the ground immediately the follower instructs him to mark.

To repeat the procedure for the next chain length, the leader, taking his pole and remaining arrows, walks forward and pulls the chain after him, having first swung it a little out of the line so that the arrow which has been placed will not be disturbed. He should count his paces so that he may know to stop and turn round when the follower, carrying the rear handle, has reached the arrow. The follower ranges the leader by planting his pole behind the arrow, and, having held his handle at the arrow (preferably flush with that side which was against the leader's handle), and got the second length marked by the leader, removes the arrow. The number of arrows in the possession of the follower at any time thus shows the number of completed chain lengths.

Transfer of Arrows.—At the end of the tenth chain length the leader has inserted his last arrow. When the chain is next pulled forward, the follower removes the tenth arrow, and marks the place by pushing a nail or pencil into the ground. The two men then meet, and the ten arrows are handed over to the leader, who uses them over again. In a long line a careful record must be kept of the number of such transfers. Occasionally eleven arrows are carried, the additional one being used before the transfer to mark the forward end of the eleventh length.

End of Line.—At the end of a line there will usually be a part chain length to be measured. The leader pulls his end of the chain beyond the station, and then straightens the chain up to the station pole, and reads.

Making up the Chain.—The chain must be coiled up in a particular manner to form a compact bundle which can be tightly held by a strap. Having roughly doubled the chain by pulling it by the middle, the chainman holds it at the 50 tab in the left hand, and with the right grasps it at the joints corresponding to 48 and 52. He passes the four links to the left hand, doubling them as he does so to form a length of one link fourfold. He continues with the remainder, laying each four links obliquely across those in his hand. When finished, the bundle should have an hour-glass shape, all the links bearing on each other at the waist. The strap is passed round the narrowest part and through the handles.

Notes.—(1) Much time is saved if the leader can put himself approximately in line. If one or two intermediate poles have previously been ranged in, he can do so accurately, but should always allow the follower to satisfy himself before marking. If no intermediate poles have been set, he should, when first aligned by the follower, sight past the latter's pole and "take a back object," i.e. note any feature of the landscape which lies approximately in the line produced. He can then place himself nearly in line at every chain length by getting the distant point and the rear pole in range. A forward object in line with the other station may also be taken.

(2) Some surveyors do not use poles for the alignment of the chain, the follower sighting the arrow held by the leader against his handle. This is quite satisfactory where the ground is smooth, but in the majority of cases it is unwise to dispense with poles.

(3) The counting of his paces by the leader when pulling the chain forward should not be omitted, as it may save the follower a search for the arrow in long grass. The leader should not be stopped suddenly by the follower pulling the chain, as the jerk contributes to lengthen the chain.

(4) It requires a little practice to be able to stretch the chain quickly. Beginners are apt to shake and tug violently, making it very difficult for the follower to hold at his arrow. The tension should be only sufficient to make the chain lie straight.

(5) When chaining on a hard surface, such as that of a road or street, the leader should mark each chain length by a scratch made with an arrow or by a chalk mark, and should lay the arrow on the ground. It is advisable to write the chainage at each mark.

Chaining on Sloping Ground.—If the ground over which a measurement has to be made is not level, means must be taken to obtain the *horizontal* length of the line. This may be done directly by "stepping" the measurement in horizontal lengths, or indirectly by deducing the horizontal equivalent of the distance along the slope.

By Stepping.—**Chaining Downhill.**—In this case the follower's end of the chain is held on the ground, and his routine is as before. The leader must hold his end, either of the whole chain or of a suitable portion of it, above the ground so that the chain is stretched horizontally in the air (Fig. 112). It is ranged, and the end of the suspended length transferred to the ground by means of a plumb line held by the leader.



FIG. 112. —STEPPING.

Chaining Uphill.—It is more difficult to obtain good results in this case, as the follower's handle is now off the ground, and he must simultaneously plumb the end over the mark and range the leader. It is therefore a great convenience to have sufficient poles in the line that the leader can align himself.

Notes.—(1) The length of steps permissible, to yield material accuracy, decreases with increase of the slope of the ground and the weight of the chain or band. For convenience of manipulation, the suspended end should not be more than about 6 ft. off the ground. The error introduced by sag is proportional to $\left(\frac{\text{weight of chain}}{\text{pull}}\right)^2$, so that a light steel band is best for slope work.

In place of stepping with the chain, a good method is to align and lay the chain on the ground, and then perform the stepping by a linen or steel tape. This saves the ranging of each step, but, if a badly worn linen tape is used, it may be necessary to correct for the error of the tape.

(2) The transfer of the horizontal lengths to the ground may be performed by a pole or by dropping an arrow, ring down, but the plumb line method is the most accurate.

(3) It is very difficult for the chainmen to judge when the tape or chain is horizontal. The surveyor should therefore stand clear to one side, and direct them. Even from such a position, the inexperienced will think the chain horizontal when it really slopes downhill. That it should make a right angle with the suspended plumb line is a good guide.

(4) Great care is necessary to keep a correct record of the steps. Those in a series should, as far as possible, have the same length, preferably 10, 20, 25, 50, or 100 ft. or links. Caution must be exercised if the chaining arrows are used to mark the ends of short steps. It is better to insert arrows only at the ends of chain lengths and to use cleft twigs or nails at the intermediate points.

By Measuring along Slope.—In this method, chaining is performed along the surface of the ground, and the various slopes encountered are measured by clinometer (page 130). If the measurement is composed of a series of lengths l_1, l_2 , etc., inclined at θ_1, θ_2 , etc., to the horizontal, then

the required horizontal distance = $l_1 \cos \theta_1 + l_2 \cos \theta_2 + \text{etc.}$

If only the total distance is required, the calculation may be made on completion of the chaining, but, when numerous intermediate points have to be located, it is better to make a correction in the field at every chain length. The chain having been stretched in the position AB (Fig. 113), the leading arrow is shifted from B forward to B', where $BB' = 100 (\sec \theta - 1)$ ft. or links, and the next chain length starts from B'. On moderate slopes the chainage of intermediate points may be read directly from the chain with sufficient accuracy for ordinary offsetting. In applying the method, a table should be prepared of the values of BB' for various slopes. (See page 159.)

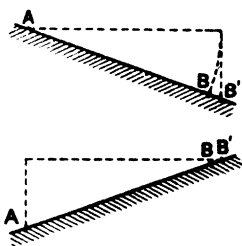


FIG. 113.

Relative Merits of Stepping and Measurement on Slope.—Measurement on the ground yields better results than stepping, but is

somewhat tedious except on ground characterised by long gentle slopes. On short slopes of varying degree the method of stepping is quicker, and is that more generally used in ordinary work. Stepping is useless on very flat slopes, as the sag error may exceed that introduced by assuming the inclined and horizontal lengths equal. At the same time, there is little point either in stepping or in measuring slopes when the chain is used and the slope of the ground is less than about 3° , or, say, 1 in 20, as the accuracy of the method will seldom justify such refinements; but, on gradients exceeding 1 in 20, it is as well to correct for slope, either by stepping or by measuring the actual slope. The slope correction for this gradient, for a length of 100 ft., is 0.13 ft., or 1 in 800 approximately.

ERRORS IN ORDINARY CHAINING

An examination of the nature and effects of the various sources of error in ordinary chaining is necessary for the due appreciation of the relative importance of the precautions to be observed against them. The difference between cumulative and compensating, and positive and negative errors (page 7) should be kept in mind. Errors and mistakes arise from :

1. Erroneous Length of Chain.
2. Bad Ranging.
3. Bad Straightening.
4. Non-horizontality.
5. Sag.
6. Careless Holding and Marking.
7. Variation of Temperature.
8. Variation of Pull.
9. Displacement of Arrows.
10. Miscounting Chain Lengths.
11. Misreading.
12. Erroneous Booking.

1. Erroneous Length of Chain.—Cumulative, + or — This is the most serious source of error in using the wire chain because of its liability to stretch. It should be tested before commencing work, and on important surveys should be compared with a field standard every day or two, while knotting of the rings should be carefully avoided. Steel bands for ordinary work need be tested only occasionally. Measurements obtained with a chain or band which has been found in error can be corrected as follows :

$$\text{Correct distance} = \frac{\text{Erroneous length of chain}}{\text{Nominal correct length of chain}} \times \text{observed distance.}$$

If the chain is too long, the error is negative (the chain not going sufficiently often into the line), while the opposite effect is produced if it is short.

2. Bad Ranging.—Cumulative, +. In ordinary work this pro-

duces a relatively small error. If a chain length diverges a perpendicular distance d from the correct line, the error in length is practically $d^2/200$ ft. or links. Very refined ranging is unnecessary if only distance is required, but greater caution is called for if off-setting is being performed, as the offsets are thrown in error by the full amount of the divergence.

3. Bad Straightening.—Cumulative, +. The effect produced by the chain lying in an irregular horizontal curve is similar to the last, but is more productive of error as the deviation is not so easily seen.

4. Non-horizontality.—Cumulative, +. This error is common in stepping, but also arises from disregarding flat slopes. It is the second error above, reproduced on the vertical instead of the horizontal plane, but is much more important in practice.

5. Sag.—Cumulative, +. When the chain is stretched in the air either in stepping or in measuring over small undulations or obstructions, it must necessarily sag, and the distance between the ends is less than that read. The error is reduced by suspending short lengths only and pulling firmly.

6. Careless Holding and Marking.—Compensating, \pm . The follower may hold his handle to one or the other side of the arrow, and the leader may not thrust his arrow vertically into the ground or exactly at the end of the chain. The possibility of inaccurate marking is much increased when plumbing. The error of marking developed by inexperienced chainmen is often of a cumulative character, but with ordinary care the distance, as marked, may be greater or less than a chain length, and such errors tend to compensate. If an error of $\pm e$ is made in marking each of N chain lengths, the probable uncompensated error in the length of the line from this cause is $\pm e\sqrt{N}$.

7. Variation of Temperature.—Cumulative, + or —. The effect of temperature is negligible in ordinary chaining, but must be allowed for in careful work with the steel band. If the band is used at a temperature different from that at which it was compared with the standard, the necessary correction can be computed from the known temperature difference and the coefficient of expansion of the material. (See page 160.)

8. Variation of Pull.—Compensating, \pm . Variation in length produced by varying tension is also unimportant in ordinary work. The pull should be maintained at that applied in standardising the chain, but, unless a spring balance is used, it will probably vary on either side, and the errors tend to compensate. The chainmen may, however, persistently apply too great or too small a tension, when the error becomes cumulative.

The following mistakes produce quite irregular effects.

9. **Displacement of Arrows.**—If an arrow has been knocked out of the ground or disturbed by any means, it may be replaced wrongly. Such an accident is easily avoided, but, in view of its possibility, the leader may mark the end of each chain length by a scratch on the ground as well as by the arrow.

10. **Miscounting Chain Lengths.**—This serious blunder should be guarded against by seeing that the leader has the full number of arrows on starting, and by both men counting them at each transfer.

11. **Misreading.**—The most likely mistake in reading the chain is to confuse the 40 with the 60 tally. The position of the 50 tab should be noticed. In using a fully graduated band or tape, it is not uncommon for beginners to misread the feet by concentrating their attention on the inches. The surveyor should himself verify readings where possible.

12. **Erroneous Booking.**—The possibility of booking figures wrongly is obviated by the chainman calling out the measurements loudly and distinctly, the surveyor repeating them as he makes the entry.

Relative Importance of Sources of Error.—It is instructive to compare the conditions under which each source of error, taken by itself, may give rise to a definite amount of error. Supposing the 100-ft. chain or band to be used, an error of about 1 in 1,000 is produced by each of the following conditions :

1. Length of chain 0.1 ft. or about $1\frac{1}{2}$ inches wrong.
2. Divergence in direction per chain length of 4.8 ft.
3. Middle of chain 2.25 ft. off the line, the half lengths being straight.
4. One end of chain 4.5 ft. higher than the other.
5. Sag per chain length of 2 ft.
6. Each marking ± 0.73 ft. or $8\frac{3}{4}$ inches out (taking the line as a mile long).
7. A variation of temperature of 50° F. from standard leads to an error of about 1 in 3,000. Hence temperature errors with the wire chain are negligible.
8. A variation of pull from standard of 125 lb. and upwards, depending upon cross section of chain. A pull of this sort could only happen through accident.

These figures emphasise the futility of elaborate refinement in ranging and marking unless the chain is correct or its error is known. Speed in chaining, as in other surveying operations, is much increased by consistency between the precautions against error and the probability and seriousness of the errors.

LINEAR MEASUREMENTS WITH THE LONG STEEL BAND OR TAPE

The invention of the long steel band or tape has introduced the possibility of greatly increased accuracy in surveying, without this involving any undue waste of labour or of time. These bands can now be obtained in different lengths and widths, the lengths varying from 50 ft. to 1,000 ft. and the widths from $\frac{1}{8}$ in. to $\frac{3}{4}$ in. For the measurement of long lines and traverses, and for laying out railways and roads, a tape 300 ft. long and $\frac{1}{8}$ in. wide, and weighing about 9 to 14 ozs. per 100-ft. length, has been found to be a convenient size, though some surveyors say that they prefer to use a tape 500 ft. long. Where only comparatively short lengths have to be measured, and for much purely engineering work, a tape 100 ft. long and $\frac{1}{8}$ in. wide will probably be found as convenient a size as any.

The advantage of the long tape is, of course, that the longer length means fewer applications of the tape to measure a given distance than would be the case if a short length is used. Very long tapes, however, are inconvenient to handle, more especially when working over a line cut in bush or forest, and are more liable to accidents than short tapes. Moreover, as these long tapes generally have to be of light section, they should not be dragged along the ground, but should be held clear of obstructions when being moved forward, and this means extra labourers to support them at intermediate points when they are being carried or used. Hence, if labour is scarce or expensive, this may well be a deciding factor against using them.

In some types of tapes the over-all length is that between the outsides of the brass handles at the ends, but, for the better classes of work, it is far better to use a tape in which the zero and end marks are on the tape itself, either in the form of lines etched on the metal or on small sleeves or studs fastened to it. When ordering a tape, it is always necessary to specify very clearly which type is required.

Graduations, marking intermediate lengths, are usually either etched on the metal or consist of small brass studs let into it at the required intervals. Etched graduations sometimes tend to get rusted up, and, when the tape is dirty, may not be easy to see or read in bad light. Studs are easy to see, but tend to weaken the tape since the pins to which they are connected pass through the latter, and thus reduce its cross-sectional area. Hence, if a tape carrying studs breaks, it usually does so at one of the studs.

Tapes may be graduated in different ways. Those intended to be used solely as standards of length may be divided at the 100-ft. marks only. For ordinary purposes, the most generally useful system of graduation consists of numbered marks at every 10 ft.,

with small studs at every foot and the first and last foot subdivided into tenths by still smaller studs. Some people also like to have *a 1-ft. length outside both the zero and end divisions graduated to tenths of a foot*, so that the total length of the graduated part of the tape is 2 ft. longer than the nominal length. This extra 2 ft. is often useful when careful work is being done and pegs or marks have been put in beforehand at every tape length. In such a case, the peg may come just outside the zero or end mark, and the difference is then read off very easily on the extra graduated foot at the end. This system, however, is not usual.

There are two ways in which the tape may be used. The first, and more common, method is to lay it flat on the ground like an ordinary wire chain, and to use it in a manner similar to that in which a chain is used. The second method is to work with it in "catenary"—that is, suspended in the air in such a manner that it hangs, clear of the ground, in a natural curve, similar to the curve in which a telegraph wire hangs. On very flat and even ground the first method is quicker and more convenient than the second, but, unless very special precautions are taken, it is not so accurate. When, however, work is being done over rough ground and a reasonable degree of accuracy is desired, the second method is generally the more economical in the end. This is particularly the case when the survey is along lines cut through bush or forest. When cutting lines of this kind it is easiest to cut trees and bushes about 18 to 24 in. above ground level, but, to cut lower and clear everything below that height is troublesome and adds considerably to the time, labour, and cost of clearing. If a tape is to be used along the ground, all, or nearly all, of this low stuff has to be cleared away, as otherwise it is difficult to stretch the tape out straight without catching or kinking it on ground obstacles. Hence, by suspending the tape in such a way that the lowest point of it is about 3 ft. above ground level, all this additional cutting and clearing is avoided, and the time lost by using a somewhat slower method of taping is more than counterbalanced by the saving in money on cutting and clearing.

USING THE TAPE ON THE FLAT

When using the tape on the flat, varying degrees of accuracy may be obtained by slight variations in method, and, as time and costs can be reduced by reducing the standard of accuracy aimed at, the surveyor can exercise considerable discretion in adjusting his methods to what he considers to be the necessities of the case with which he is dealing. Thus, the tape can be used in exactly the same manner as the chain, without observing temperatures or taking special precautions to maintain constancy of pull, and, when so used, it should yield slightly better results than the chain. For a

better class of work, precautions may be taken, by means of a small spring balance attached to the tape, to ensure that a constant stated tension is applied while the tape is being used. Again, even when care is taken to keep a uniform pull, it may be decided that no special observations need be taken, or allowance made, for temperature. On the other hand, the thermometer may be read at each set up of the tape, and a correction made for the mean temperature recorded for the whole of the one line or for the whole day, or a mean temperature may simply be assumed, either for the day or season, and the correction made on the assumption that this will be the prevailing temperature during the progress of the survey. Hence, considerable latitude in the choice of details of method is available, but, when the standard of accuracy required has been decided, the particular means to be adopted to reach this standard can best be selected by a careful consideration of the different sources of error and the effect of each of them on the final result. For this, the detailed analysis given on pages 166 to 174 can usefully be consulted.

Unrolling and Rolling up the Tape.—The end handle of the tape is usually attached to the drum or casing by a strap passing through a small slot in the latter. The strap is undone, and the handle taken by the leader or front chainman, the follower holding the drum about waist level, with its circular sides in a vertical position. The follower now walks slowly backwards, letting the tape unroll from the drum. While doing this, he keeps his hand on the winding knob of the drum, if it has one, and sees that the tape does not run out too quickly or in a jerky manner. If he feels or sees any indication of a jerk, he should yield to it so as to prevent undue stresses being imposed on the tape. When the latter is very nearly fully drawn out the follower stops and then proceeds to detach the rear handle from the drum.

It is inadvisable to let the tape be drawn off the drum by the leader moving forward, and the follower remaining stationary with the drum, as this tends to cause sudden jerks being applied.

When re-winding at the end of a day's work, the tape is laid straight out on the ground, and the handle of the rear end fastened to the axis of the drum. The latter is then wound slowly so as to bring the tape on to it fairly tightly, but not too tightly, and without jerks. During winding, the tape should not be drawn forward along the ground, but the drum should be carried along slowly, winding taking place all the time, until the other handle is reached. This is then wound on and fastened to the drum by the leather strap.

Tapes, when not in use, should be carefully greased with non-corrosive grease and this should be well cleaned off before use.

Attaching the Spring Balance.—Most instrument makers supply special spring balances, of the barrel type, for applying and controlling the pull used with steel tapes. Some tapes have small lugs

fitted at the side of the handle to which a special fork can be attached, a fork of this description being shown in Fig. 114. Small

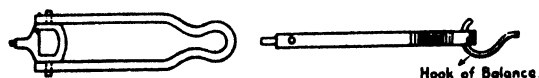


FIG. 114

holes in the ends of the prongs slip over the lugs at the side of the handle, and the hook of the spring balance is passed through the loop at the end of the fork.

If there are no lugs at the sides of the handles, or if a fork is not available, the spring balance can be attached by a strong string or a leather thong passing through the hook of the balance and the metal ring or loop attaching the handle to the tape. As a general rule, this is not so convenient as the fork because the handle hangs loose and is apt to catch on stones or twigs on the ground. In any event, the hook of the balance should not be attached direct to the end of the handle, as, if this is done, the tension may be applied anything but axially, and hence the actual tension in the tape may be very different to the pull recorded on the balance.

Sometimes there is a small hole in the back of the handle through which the hook of the balance can be passed, or, if there is not one already and the material in the handle is thick enough, one can easily be drilled. In this, also, as in all other ways of attaching the balance, care must be taken to see that the pull is being applied really tangentially.

The pull to be used should depend on that at which the tape was standardised, which should always be known, and the same pull should be applied during the field work. For a tape $\frac{1}{4}$ in. wide, used for ground taping, the usual pull is 10 or 15 lb. If a pull different to that used in standardisation is employed, a correction should be made. The formula to be used in computing this correction is given on page 163.

Applying the Tension.—At the rear end, the tape can be held by an arrow passed through the handle, the point being stuck in the ground and the top held in the hand, so that, when the arrow is pressed against the inside of the back of the handle it acts as a lever. If, however, there is too long a length between the zero mark on the tape and the end of the handle the chainman may have difficulty in holding the tape and at the same time seeing that the zero mark on it is against the rear station mark or arrow. In this case he can use a ranging or other pole as a lever to hold the end of the handle, and, in this position, he can stand up to hold the pole and so get a better view of the zero and end or station marks.

At the forward end, the end of the balance should be fastened, either by a strong string or leather thong, to a point near the bottom

of a pole, which can be pressed into, or against, the ground and used as a lever. After he has obtained line from the rear chainman, the leader chainman holds the pole at the top and presses on it until he sees the correct pull registered on the balance. During this process the end of the tape will have to be moved forwards or backwards until the rear chainman signals that the marks at his end coincide. It will be difficult, with some kinds of tape, for the man maintaining the tension at the forward end to hold the proper pull and at the same time put in an arrow against the forward mark, or to take a reading there. Consequently, this may have to be done by the surveyor himself or by an extra assistant. Indeed, if sufficient labourers or assistants are available, it will be well to have a man or boy at the rear end of the tape to give the signal when the marks are in coincidence, as the man holding the end of the tape already has his hands fairly full in holding the tape steady. However, this is not absolutely necessary.

Observing the Slope.—After the measurement of one bay is complete and before the tape is moved forward, the slope should be observed. This may be done by the Abney level or clinometer, which can be read by the rear chainman. To do this, the leader chainman carries a mark on his ranging pole placed at the same height above the bottom of the pole as the rear chainman's eyes are above the ground. Thus, the gradient between the clinometer and the mark on the pole will be the same as the gradient of the ground and no corrections for differences in height between eye and signal are necessary. Hence, the leader chainman holds his ranging pole vertical until the rear chainman sights the mark on it through the clinometer. The rear chainman then reads the instrument and calls out the reading to the surveyor, who, to prevent mistakes in hearing or booking, repeats the figures in a voice loud enough for the rear chainman to hear.

If the chainmen are wholly illiterate the surveyor may have to read the clinometer himself. As he is at the forward end of the tape, he sights back to a mark held by the rear chainman. Unless it is desired to carry heights forward along the line, there is no need to distinguish between angles of depression or elevation.

If there is a decided change of gradient at any point between the ends of the tape, both gradients should be measured and the length of each noted.

Observations for Temperature.—For temperature observations, various types of thermometer are available, but probably the best for field use is one contained in a metal tube or casing, in which there is a narrow slot running down nearly the whole length of it, so that mercury and stem can be seen, and the graduations read, through the slot. If possible, the casing should be of very similar material to that of which the tape is made, and the colour, polish, and brightness

of the surface should be as near as possible to those of the tape itself. When readings are being taken, the thermometer should be held very slightly inclined to the horizontal, but with the bulb end slightly below the level of the top of the stem, the instrument being held as close as possible to the tape.* Some makers make thermometers which can be clipped on the tape in such a position that they lie parallel to it. These are quite suitable for use in surface taping, provided the tape does not throw a shadow on the stem, but are not so suitable for "catenary taping."

The thermometer can be read by the surveyor himself, but the number of times when he will read it will depend on the accuracy desired. In some cases, it will be read at each set up of the tape and a correction worked out for the whole line, using the mean temperature observed during the measurement of that line. In other cases, temperatures may only be read two or three times during the day and corrections computed on the basis of the mean temperature for the day.

Measurement of Odd Length Bays.—When the length of a line exceeds a whole number of tape lengths it is necessary to measure an odd length which is only a fractional part of the whole length of the tape. For this, the graduations on the tape are used, the tape being carried forward until the forward chainman comes opposite the forward mark. The tape is then stretched and adjusted until a convenient foot graduation comes against the rear mark, the front chainman signalling to the rear one to move the tape forwards or backwards until the front mark falls somewhere within the first and fully graduated foot of the tape. The front chainman then observes the reading at his end, either direct to tenths of a foot or by estimation to hundredths if fine work is being done, while the surveyor or the chainman at the rear end notes which foot mark comes against the station mark.

If the tape has not got an extra foot graduated outside the zero mark, care will have to be taken to record the right reading. Thus, if the front mark falls between the graduations 0 and 1 ft., decimal parts should be read forwards from the 1-ft. mark and 1 ft. subtracted from the number of the foot graduation appearing opposite the rear mark, or the decimals may be read backwards from the zero mark and the reading subtracted from the length recorded against the rear mark.

In stretching the tape when measuring odd length bays it is best to apply the pull at both ends, just as if a complete bay were being

* The reason for having the surface of the casing as similar as possible in colour, polish and brightness to the surface of the tape is that the two surfaces will then reproduce similar conditions for the absorption, reflection and radiation of heat. The reason for holding the bulb slightly lower than the top of the stem is to prevent the occurrence of broken threads of mercury. For an account of some experiments on temperature errors in taping see *Report of Proceedings. Conference of Empire Survey Officers*, 1935, pages 295 to 325

measured. Sometimes, however, it is inconvenient or impossible to do this, and, in that event, the pull has to be applied at some intermediate point of the tape. This can be done by means of special clips which are sold by the makers for the purpose. One form (Fig. 115*a*) of these clips consists of two thin sheets of metal,

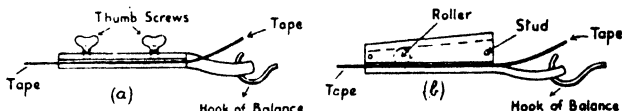


FIG. 115.

separated by a narrow slot, and carrying a handle at the end. The tape is passed into the slot and clamped and held in position by two thumb screws carried by the top leaf of the clamp, while the hook of the spring balance is passed through a hole in the handle of the clip. In another form, called the Littlejohn Roller Grip (Fig. 115*b*), the tape is held by a steel ball or roller carried in a small brass box, the top of which tapers towards one end. This box is of square cross-section and running along the bottom of one side is a narrow slot, through which the tape can be passed so as to lie underneath the ball or roller. When the pull is applied by attaching the spring balance to the handle at the end of the grip, the ball or roller is forced against the inclined top of the box and holds the tape firmly.

Check Taping and Survey of Detail.—There are considerable possibilities of making mistakes while taping and these may range from the omission to book a whole tape length to an erroneous reading of the feet or decimals at the odd length bay. Consequently, it is usually advisable to arrange for lines to be taped twice. The object of the check taping will be to detect gross errors of various kinds, but not small errors due to misreading tenths or hundredths, which, when they only occur very occasionally and are not repeated time after time, do not matter very greatly. Hence, check taping may be done by less accurate methods than those used for the main taping, which is the accepted one for use in the computations. For instance, during check taping it will not usually be necessary to record temperatures or to allow for differences in temperature, and, in some cases, it may not be considered worth while either to use a spring balance to control pull or to record slopes. Also, if a certain amount of detail has to be surveyed, it is as well to do this during the course of the check taping so as to leave everyone concerned free to concentrate on the more accurate taping when this is being done.

When both main and check taping are completed, the books containing the results should be carefully compared as soon as possible to see if any really serious discrepancies, calling for re-measurement, exist. This seems such an obvious thing to do that it appears

to be utterly absurd to mention it. Yet the writer has seen not one case, but several, of mistakes having been made and not noticed until the survey was complete, and the party moved elsewhere, simply because the surveyor did not take the trouble to compare his check chaining books with those containing the results of his main chainage.

Booking the Results.—Fig. 116 shows a page from a field book to show how results are booked. In this case a 300-ft. tape,

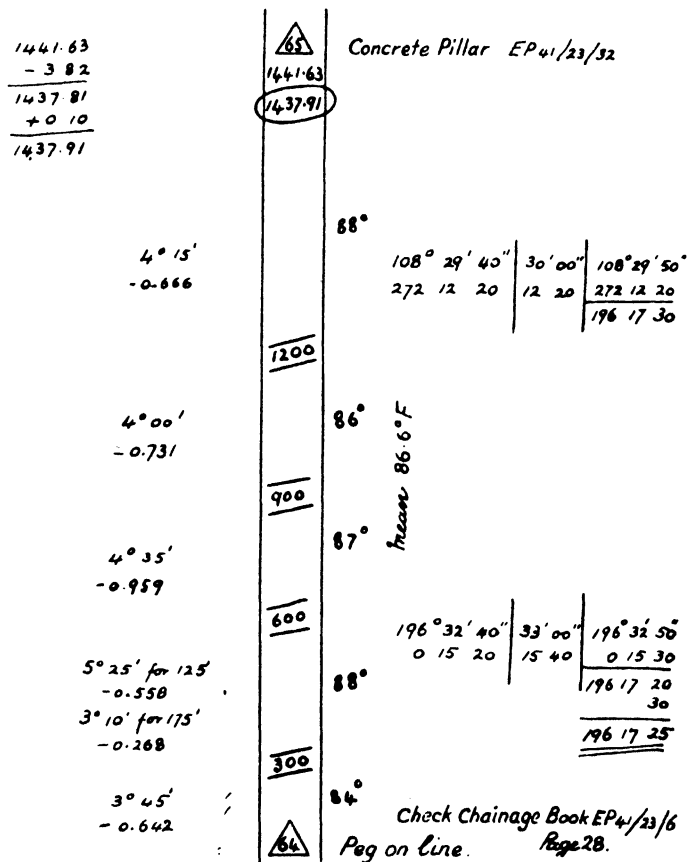


FIG. 116.

was of standard length at 76.8° F. and at a pull of 15 lb., was used. Bookings start from the bottom of the page and run upwards. Slopes are recorded on the left and temperatures at the right. The actual length measured on the ground was 1441.63 feet. Between the 300 and 600-ft. marks there was a decided change of

slope which occurred at chainage 475. This resulted in a length of 175 feet at a slope of $3^{\circ} 10'$ and 125 ft. at $5^{\circ} 25'$. The correction for each length is worked out and entered, as shown, under the figures representing the angle of slope, and all corrections are summarised and entered under the un-corrected length at the top left-hand side of the page, -3.82 being the correction for slope and $+0.10$ that for temperature. The corrected length is 1437.91 and this is entered in a ring under the figure 1441.63 at the top. This line was a line in a traverse, so note the reference to another field book in the bottom right-hand corner. The figures on the extreme right relate to the measurement of the included angle at station 64 and are explained on page 214.

Calling the Reading.—In all cases, where some other person takes the actual readings, the person recording them should always repeat the figures so that the person taking the readings can hear them and correct them if the recorder has not called them out correctly. This is to prevent mistakes, due to the person booking not hearing the figures properly, being made, and this procedure should always be adopted in every class of survey work, including computing work, when one person calls out figures to another.

USING THE BAND OR TAPE IN "CATENARY"

The catenary is the curve assumed by a perfectly flexible string (*i.e.* one in which there is no shear or bending moment) of uniform weight per unit length when hanging under its own weight and evenly supported at both ends. Hence, the term "catenary taping" may be applied to the case where a thin flexible tape, suspended in a natural curve clear of the ground surface, is used to measure distance.

In the method now to be described it will be noted that the theodolite is set up at the end of every other tape length, and, from this, it might be thought that the method is very much slower than ordinary ground taping. It undoubtedly is slower and less economical if the line lies in unbroken, flat, open country, or along a level street; but it is usually more accurate than surface taping, because, for one thing, there is more control over errors arising from uncertainties about slope, alignment, and temperature. Where the method scores heavily is in bush and forest country, as, not only is there the great saving in ground clearing, referred to on page 148, but, also, the surveyor or an assistant has not to run constantly up and down the line, preventing the tape catching on obstacles near the ground and freeing it and straightening it when it does, as he has to do if the tape is used on the ground and reasonable accuracy is expected. Indeed, some surveyors who have got more or less expert in catenary taping prefer to use this method even in flat open country and say that they can make almost as

good progress with it as they can with the ordinary method, and it gives them more confidence in the final accuracy of their work.

General Description of Method.—When using this method it is best to set out the line beforehand and to use the results of the taping employed during setting out as a check taping. However, this is not essential and the surveyor, if he chooses, can set out the line as he goes. The main thing is to have good stout pegs or posts driven on line at the points where each tape length will end. These posts should stand about 30 in. above ground level and the centre should be marked by stout wire nails, 3 in. long, of which about 1 in. projects above the top of the peg or post. These nails should be lined in by theodolite.

The general way of working will be understood from Fig. 117. The theodolite is set up at A, the beginning of the line, and, if a post or nail has not been placed at B, one whole tape length away, one should now be put in. The tape is then carried forward until the end mark is opposite B. The spring balance is attached at the instrument end of the tape, the other end of the balance being fastened by a looped string or leather thong to a long pole, *a*, which can be used as a lever to hold the pull steady. The other end of the tape near the post at B is fastened by a string or leather thong to a pole *d*, which is also used as a lever to hold the end mark of the

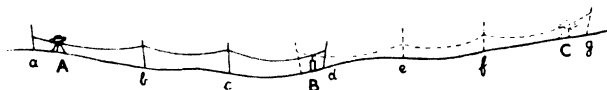


FIG. 117.

tape steady against the centre of the nail at B. If a tape over 100 ft. in length is used, it may be necessary to support it at intermediate points, as otherwise the sag may become unmanageable. A tape 300 ft. long can be supported either at the centre in two equal spans of 150 ft. each or in three equal spans of 100 ft. each, as shown at *b* and *c* in the figure. These supports can be short poles with a hook or fork at the top to take the tape and can be held against another pole, along which they can be slid up and down. This enables greater adjustment in height to be made than if a single long pole is used.

When everything is ready, the men at *a* and *d* get ready to take the pull while those at *b* and *c* put the tape in the hooks or forks on their poles. The surveyor sights his instrument on the nail at B and tightens all clamps on the instrument, at the same time giving the signal to his assistants to raise the tape and take the pull. The man at *d* sees that the mark at his end is against the nail at B, while the man at *a* sees that the pull is correct and the tape lying, so lightly as barely to touch it, against the trunnion axis of the theodolite. Meantime the surveyor lines in the men at

b and c and gets them to raise or lower their poles until the hooks are in the line of sight joining the instrument to the nail at B . As soon as everything is steady, the man at B signals that the mark there is against the nail, the man at a holds the pull required, and the surveyor measures, usually with a steel scale divided into tenths and hundredths of a foot, the small difference between the end of the tape at A and the centre of the trunnion axis of the theodolite. This completes one measurement, but, if it is thought desirable to do so, the tape is lowered, then brought into position again, and another reading taken. Before moving the theodolite, the surveyor observes the angle of slope from the instrument to the top of the nail and reads the thermometer. Also, if the line has not previously been laid out it will be necessary for him, as soon as the tape is carried forward one tape length and before he moves the instrument, to have a peg put in at C , one tape length from B , and line in a nail in it.

The theodolite is now moved forward and set up at C and the previous operation repeated, the pull again being applied at the theodolite end of the tape. Then, when the measurement of the bay CB is completed, the instrument is sighted forward and the next forward bay measured from C .

When carrying the tape forward, the usual care should be taken to prevent sudden jerks or jars and, for this, the men at b and c can use hooks attached to strings, or else forked sticks, to support it.

It will be noted that measurements are taken to the trunnion axis of the theodolite and this causes a small error in alignment. If d is the distance from the centre to the end of the trunnion axis and l is the length of the tape, a being the angle subtended by d at the other end of the tape, then the correction is $l(1 - \cos a)$. When a is small, as it always is, $a = \frac{d}{l}$ and the correction is equal to $\frac{d^2}{2l}$.

If d is 3 in. and l is 300 ft., this works out at 0.0001 ft., which is negligible. If necessary, however, a correction of this kind can be worked out for any given case and then included in the standardisation correction.

It is important, when using this method of taping, to see that the theodolite is directed properly at the nail in the peg at the other end of the tape at every set up. Thus, in Fig. 118, ab is the correct position of the trunnion axis at right angles to the line joining the theodolite to the peg at the other end of the tape, and the reading is taken when the axis is in the position cd . In this case there will be an error in reading equal to ce in magnitude.

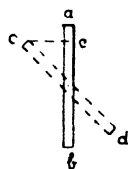


FIG. 118.

Measurement of Odd Length Bays.—When the end of the line is reached there will generally be a short length less than a whole tape length to be measured. This can be done by attaching one of

the special clips, already described on page 153, to a convenient point on the tape behind the instrument, the pull being applied *through the clip, and the surveyor measures the small interval between the foot mark near his instrument and the trunnion axis of the theodolite.* This odd length will mean that there is one span of shorter length than the ordinary span, and a special sag correction will have to be worked out for this. It is easy to apply the correction if a special table, which can easily be worked out from the table given on page 162, is computed for the tape and tension normally used, this table giving the sag correction for odd spans of 30, 40, 50, . . . ft.

Booking the Results.—Results can be booked in a manner similar to that used for ordinary surface taping. The small end differences read at the theodolite end of the tape are best written in the central column of the page, under the entry relating to that particular bay which gives the total chainage from the beginning of the line. The algebraic sum of these differences is then taken and added to the nominal length of the line. These end differences are positive if the trunnion axis of the theodolite lies between the end mark of the tape and the spring balance, and negative if the end mark lies between the trunnion axis and the spring balance. Each book should contain a very clear statement at the beginning to the effect that the measurements were made in catenary, and it should give the pull used, the total length of tape between end marks and the manner of support. It will sometimes happen that, owing to some obstacle or for some other reason, the tape cannot be supported exactly at the usual point of support, and, in such cases, two unequal spans, differing from the normal, will result. Here it will be necessary to make a special note in the field book, against the particulars relating to that particular span, and in such a form that it cannot be overlooked or a mistake made.

CORRECTIONS TO BE APPLIED TO MEASURED LENGTHS

Every measured length has to have various corrections applied to reduce it to the true distance. These corrections are :

- (1) Correction for standardisation.
- (2) Correction for slope.
- (3) Correction for temperature.

And, in certain cases :

- (4) Correction for sag.
- (5) Correction for pull, if a pull is used differing from that at which the tape was standardised.
- (6) Correction for height above sea level.

In many cases these corrections are small in magnitude and can be worked out by slide rule or by four-figure logarithms.

1. Correction for Standardisation.—It is very rare for the distance between the zero and end marks of a tape to be exactly equal to that which it is supposed to be under given conditions of temperature and pull, and, in consequence, it is desirable, and in precise work essential, to know the amount by which the tape is longer or shorter than its nominal length. A tape longer than normal will result in a measured length being too short so that the correction is additive, while the correction for a tape that is shorter than normal will be subtractive. Thus, suppose that the length measured is 3486·94 ft. and the true length of the tape at 80° F. is 300·023 ft. Expressing the standardisation error of the tape as a fraction of 100 ft., the error is 0·0077 ft. per 100 ft. Hence, for 3487 ft. it will be $0·0077 \times 34·87 = 0·27$ ft., and the corrected length of the line will be $3486·94 + 0·27$, or 3487·21 ft.

It is usually best to avoid using a special standardisation correction by combining it with the temperature correction. This can be done by working out the temperature at which the tape is of nominal length. Thus, in the above case, a decrease of temperature of 12·3° F. will cause the tape to shrink in length by 0·023 ft. Hence it will be of nominal length at $80^\circ - 12·3^\circ = 67·7^\circ$ F., and this will be the temperature used as normal when working out temperature correction, no special standardisation correction now being necessary.

2. Correction for Slope.—This is due to the length on the incline not being the same as the distance projected on the horizontal plane. The correction is given by $l(1 - \cos \theta)$, or l versine θ , where θ is the angle of slope and l is the length of the line. If no special slope tables are available, the correction can easily be worked out from a table of natural versines, such as that given in “Chambers’s Seven-Figure Mathematical Tables.” Thus, the natural versine for an angle of $3^\circ 20'$ is 0·00169, so that the correction for a length of 300 ft. on this slope is $0·00169 \times 300 = 0·51$ ft.

The correction for slope is always subtractive—that is, it is to be subtracted from the measured distance—and it should normally be applied before the standardisation and temperature corrections, so that the length to be corrected for standardisation and temperature is the reduced horizontal distance.

If the slope has been measured as a difference in height, the correction is $\frac{h^2}{2l} + \frac{h^4}{8l^3}$, where h is the difference in height between the ends of a line of length l . The second term is inappreciable when h is less than about 14 ft. in 100 ft. or for a slope of about 8° . It can therefore be neglected in all cases that are usually met with in ordinary practice.

3. Correction for Temperature.—Increasing the temperature of a steel band causes it to increase in length, while decreasing the temperature causes it to decrease in length. The increase or decrease is given by the formula :

$$\Delta l = l \times c \times t,$$

where l is the length of the tape, t the increase or decrease in temperature above its normal value, and c is the coefficient of thermal expansion. For steel tapes the value of c varies from about 0.000 0059 to about 0.000 0067 per 1° F., but throughout this volume we will assume a value of 0.000 0062 per 1° F.

Thus, take the case of a line 4,987.24 ft. long, measured with a 300-ft. tape which is of correct length at 96.4° F., and suppose the mean temperature during measurement was 76.8° F. Then $t = 19.6^\circ$. Hence, $\Delta l = 4,987 \times 0.000\ 0062 \times 19.6 = 0.61$ ft. As the temperature was below that at which the tape was standard, the tape was too short and the measured length was therefore too long. Hence, the correction is subtractive and the correct length of the line is $4,987.24 - 0.61 = 4,986.63$ ft.

The following table gives the correction for lengths from 100 ft. to 1,000 ft., and values of t from 10° to 50° F.

TABLE OF TEMPERATURE CORRECTIONS.

(Coefficient of Expansion = 0.000 0062 per 1° Fahr.)

t° Fahr.	Lengths in Feet									
	100	200	300	400	500	600	700	800	900	1000
10	0.006	0.012	0.019	0.025	0.031	0.037	0.043	0.049	0.056	0.062
20	0.012	0.024	0.037	0.050	0.062	0.074	0.087	0.099	0.112	0.124
30	0.019	0.037	0.056	0.074	0.093	0.112	0.130	0.149	0.167	0.186
40	0.025	0.050	0.074	0.099	0.124	0.149	0.174	0.198	0.223	0.248
50	0.031	0.062	0.093	0.124	0.155	0.186	0.217	0.248	0.279	0.310

4. Sag Correction.—If a tape is graduated and of standard length on the flat, the horizontal distance between the end points, when it is suspended in a horizontal catenary, will be less than the distance measured along the tape itself since the horizontal distance is a straight line and the length along the tape follows a curve. The difference between the horizontal distance and the length measured along the curve is called the “correction for sag” or, more shortly, the “sag correction.”

Let

w = Weight of tape in *lbs. per foot run*.

l = Length of tape in feet between marks.

F = Pull applied at ends of tape in *lbs.*

C = Sag correction in feet.

Then

$$C = \frac{w^2 l^3}{24F^2}$$

When the tape has been standardised on the flat, the horizontal distance between the end marks when it is used in catenary is obtained by subtracting the sag correction from the standardised length on the flat. Similarly, if the tape was standardised in catenary, the true length on the flat is obtained by adding the catenary correction.

For odd lengths, or lengths differing from that in which the tape was standardised, the correction takes a slightly different form according as to whether the tape was standardised on the flat or in catenary. If it was standardised on the flat, the correction for the particular span involved is simply subtracted from that span to give the equivalent horizontal distance. If it was standardised in catenary, let

x = The length of the span involved.

L = The length of the span used for standardisation.

C_L = The sag correction for L feet.

C_x = The sag correction for x feet.

Then :

$$\text{Correction to be applied} = \frac{x}{L} \cdot C_L - C_x.$$

This correction is positive and must be added if x is less than L ; it is negative and must be subtracted if x is greater than L . The result is added to the nominal length of x , corrected for its share of the standardisation correction.

An alternative way, which is easy to remember as it avoids using any formula, although it really amounts to the same thing, is to proceed as follows : (1) Correct L for standardisation. (2) Add the sag correction for L . The result now is the length of L on the flat, corrected for standardisation. (3) Work out true length of x on the flat. (4) Apply sag correction for x . The result now is the length of x in catenary and corrected for standardisation.

As an example, take a tape of nominal length 100 ft., weight 13.5 ozs per 100 ft., used under a pull of 15 lb. Standardisation correction for 100 ft. when suspended in catenary = -0.073 . Find the length of a span of 70 ft. when used under the same tension.

$$100 \text{ ft. corrected for standardisation} = 99.927$$

$$\text{Sag correction for 100 ft.} = 0.013$$

$$\text{True length of 100-ft. mark on flat} = 99.940$$

$$\text{True length of 70-ft. mark on flat} = 69.958$$

$$\text{Sag correction for 70 ft.} = -0.005$$

$$\text{True length of 70-ft. mark in catenary} = 69.953$$

The following table gives the sag correction for various spans for a tape weighing 10 ozs. per 100 ft. ($w = 1\frac{1}{16}$ lb. per foot)

and a pull of 10 lb. Then the sag correction for any other weight of tape, used at any other pull, can be obtained by multiplying the figures given in the table by $\frac{W^2}{F^2}$, where W is the weight of the tape in ozs. per 100 ft. and F is the pull used in lbs.

TABLE OF SAG CORRECTIONS

(For tape weighing 10 oz. per 100 ft. under a pull of 10 lb.)

Span in feet.	Sag correction. Feet.	Span in feet.	Sag correction. Feet.	Span in feet.	Sag correction. Feet.
30	0.000	80	0.008	130	0.036
40	0.001	90	0.012	140	0.045
50	0.002	100	0.016	150	0.055
60	0.003	110	0.022	160	0.067
70	0.006	120	0.028	170	0.080

Thus, if a tape weighing 13.2 oz. per 100 ft. is used in a span of 150 ft. at a pull of 15 lb., the figure obtained from the above table is 0.055. Hence, the sag correction required is

$$0.055 \times \left(\frac{13.2}{15}\right)^2 = 0.043.$$

The above formula and table give the sag correction when the tape is used with both ends at the same level. If it is used on a heavy slope, the sag correction becomes :

$$C_1 = C \cdot \cos^2 \theta \left(1 + \frac{wl}{F} \sin \theta\right).$$

where the tension is that applied at the upper end, θ is the angle of slope and C is the ordinary sag correction for tension F and zero slope.

If the tension is applied at the lower end, the correction becomes :

$$C_1 = C \cdot \cos^2 \theta \left(1 - \frac{wl}{F} \sin \theta\right).$$

For a steel tape the formula may be taken :

$$C_1 = C \cdot \cos^2 \theta.$$

When a tape is used with intermediate supports lined in between the terminal points, so that there are two or more spans, such as are commonly used in catenary taping, the sag correction for the whole tape is taken as the sum of the sag corrections for the individual spans. This assumption would not be true if a very heavy tape were used, as all tapes have a certain amount of rigidity and the formula for the ordinary sag correction is based on the assumption

that the tape is perfectly flexible. With ordinary $\frac{1}{4}$ -in. and $\frac{1}{2}$ -in. tapes, however, the effect of rigidity is negligible in all cases that occur in practice, even when two or more spans are involved.

In all cases, wherever it is necessary to apply it, sag correction should be applied to the measured distance before the latter is corrected for slope.

(5) **Correction for Tension.**—If the pull applied to the tape is not the same as that used during standardisation, a correction for elastic stretch of the tape becomes necessary. Let F be the pull actually used in the field and F_m that used during standardisation. Then, if l is the length of the tape under pull F and l_m its length under pull F_m :

$$l - l_m = \frac{l_m \times (F - F_m)}{AE}$$

where A is the area of the cross-section of the tape in square inches and E is Young's Modulus of Elasticity in lb. per square inch. For ordinary steel tapes, E may be taken equal to 28,500,000 lb per square inch. If F is greater than F_m , l will be greater than l_m and, if F is less than F_m , l will be less than l_m . Also, of course, if a tape has been standardised in catenary and a pull different to that used in standardisation is used in the field, the standardisation correction will be affected. To correct for this, the sag correction for F_m should be worked out and applied so as to give the true length of the tape on the flat. The sag correction for F is then computed and applied to this length to give the true length in catenary.

These corrections for pull should very seldom be necessary, as, unless there are very good reasons for not doing so, a tape should always be used under the same pull as that at which it was standardised.

(6) **Correction for Height above Sea Level.**—Unless the height of the ground above sea level is large, it is neither usual nor necessary to apply this correction to ordinary work, although this has to be done on precise work, such as base-line measurement or the survey of precise framework traverses. For an explanation of the correction and for the formula for its computation see Vol. II, Chap III. For anything but work of the highest precision, its value may be taken as $0.00048 \times H$ per 1,000-ft. length, where H is the height of the line, in feet, above sea level. It thus amounts to about $1/50,000$ when the line is 400 ft. above sea level and to about $1/20,000$ when the line is 1,000 ft. above sea level. For heights above sea level it has to be subtracted from the measured length and for heights below sea level it has to be added. In many cases it can be allowed for by including it with the standardisation correction.



FIELD DETERMINATION OF WEIGHT OF TAPE

Owing to the handles and metal loops at the ends, it is not always easy to determine the correct weight of a tape even when a proper scale or balance is available. The weight can, however, easily be determined in the field by measuring the depth of the sag at the middle of the span. For this purpose one end of the tape is anchored, at a height of about 4 ft. above ground level, to a post or wall and the other end attached to a spring balance. This end is then raised until, when the desired pull is applied, the zero and end marks are exactly at the same height, a theodolite or level being used to level them up. This height is then marked on a post or staff held against the tape opposite its lowest point and the depth of the tape below this point very carefully measured. If y represents the measured sag, then :

$$wl^2 = 8Fy$$

$$\text{or} \quad w = \frac{8Fy}{l^2}$$

If an error δy is made in the measurement of y , the resulting error in the sag correction will be given by :

$$\frac{2}{3} \frac{wl}{F} \cdot \delta y.$$

Thus, with a tape weighing 1 lb. per 100 ft., used under a tension or pull of 15 lb., if an error of 0.005 ft. is made in the measurement of y , the error in the determination of the sag correction for a span of 100 ft. will be 0.0002, which is inappreciable in work with a steel tape.

If a tape has studs at foot intervals in the first hundred feet and there are no intermediate studs in the 100–200 and 200–300-ft. lengths, the sag correction for the first span will be different to that for the other two and should be separately determined.

STANDARDISING STEEL TAPES, SPRING BALANCES AND THERMOMETERS

If accuracy in taping work is to be secured, it is absolutely necessary to use tapes that are properly standardised, because errors due to faulty standardisation are cumulative in their effects. For this reason, it is desirable to keep at least one special tape as a field standard and to use it for nothing else but the standardisation of the tapes ordinarily used in the field. If possible, the standard tape should be standardised at the National Physical Laboratory. This laboratory issues two classes of certificate—Class A, in which an accuracy of 1 in 1,000,000 is guaranteed, and Class B, in which

an accuracy of 1 in 100,000 is guaranteed. For all ordinary work with a steel tape a Class B certificate will be all that is required. When sending a tape to the N.P.L. for standardisation, it is well to send its own spring balance with it and to specify very clearly the conditions under which it will be used—whether on the flat or in catenary, the pull and, if possible, the average temperature.

When comparing the standard and field tapes, it is necessary to put down some sort of base line, to find the true length of it from the standard tape, and then to use this value when comparing the length of the base and of the field tape. For work on the flat, the base should be on very level ground and it will be convenient if its ends are marked by fine lines ruled on sheets of zinc, nailed down very firmly to the pegs so that there is not the slightest chance of movement. The positions where the end marks of the tape come can then be scratched on the zinc and the small distances between marks measured to hundredths of an inch by means of a diagonal scale, such as that often given on boxwood protractors, and a pair of dividers.

For work in catenary, the ends should be marked by very stout wooden pegs, about 3 ft. 6 in. high above the ground, in each of which a 3-in. nail, with a fine scratch on the head, has been driven so that the head projects about 1 in. above the top of the peg. Alternatively, and this is the better plan, a strip of zinc can be nailed to the top of each peg, but the zinc and the top of the peg should slope slightly towards the centre of the base so as to allow for the slope at the ends of the tape when it is suspended freely. Marks against the end graduations can then be scratched or drawn for different settings of the tape and distances between these marks and the index marks on the zinc measured, as before, with dividers and diagonal scale. In every case, it will be well to rule a fine line down the centre of the zinc, and in the direction of the line of the tape, to give alignment and to mark a definite point on the mark on the peg from which measurements are to be taken.

As it is absolutely essential that there should be no movement of the pegs during measurement it is well to put three heavy struts at the side of each peg, arranged like the legs of a tripod, to steady it. Similarly, when zinc strips are used, care should be taken to see that it is quite impossible for them to move in the slightest degree.

In every case, a number of readings should be taken, the tape being moved slightly forwards or backwards between each pair of readings. Also, some means of anchoring and holding the tape, so that it is not necessary to hold it by hand, should be devised, and, at the spring balance end, a tourniquet can easily be improvised to enable the pull to be adjusted and maintained with reasonable accuracy.

The spring balance and thermometers should also be carefully tested and standardised. In the case of the spring balance the best way is to fix up some sort of pulley arrangement and to test

the balance by suspending it horizontally, so that a string attaching the hook to a known weight can be passed over the pulley. If this is not possible, the index error should be tested and found by the *method described on page 183 of Vol. II (fourth edition) of this book.*

For testing the thermometers, a standard thermometer, whose error, if any, is known, should be kept and used for testing purposes only. The thermometers used in the field can then be standardised over a suitable range of temperature by hanging them in hot water, close to the standard thermometer, and taking simultaneous readings on all of them, and on the standard, as the temperature of the water falls. If temperatures near freezing-point are likely to be met with in the field, it may be also necessary to carry out another test using iced water.

All thermometers, including the standard, should be constantly examined for broken threads of mercury, and, when not in use, are best kept standing in a semi-vertical position, with the bulb down, rather than left lying with their lengths horizontal.

ERRORS IN MEASUREMENTS WITH STEEL TAPES

The principal errors to be guarded against in work with steel tapes are, as in the case of every other type of measurement, those of a constant and cumulative kind. The main ones are errors of alignment, standardisation, temperature, and pull. Consequently, the chief efforts of the surveyor should be directed to removing these as far as possible and to minimising their effects. Needless to say, he should also take every precaution to avoid actual mistakes, such as those arising from false readings, wrong bookings or from movement of pegs or marks during measurement. The last is a likely source of error that has specially to be watched and guarded against when taping in catenary.

Certain kinds of error always have the same sign for all tapes. These are errors in alignment and errors due to the tape not being stretched straight, either horizontally or vertically, or both, all of which tend to make a measured length too long. Another class of error tends always to have the same numerical value, and the same sign, with a particular tape and the same surveyor, but to vary, both in magnitude and sign, with different tapes and different surveyors. Standardisation errors and errors in pull are of this type. The third kind is the purely accidental error of measurement, variable both in magnitude and in sign for every tape and every surveyor. According to the law of the propagation of error, the cumulative effect of constant errors will be proportional to the length of the line, while that of the variable errors will be proportional to the square root of the length of the line.

The different sources of error are :

(a) *Errors in Alignment.*—Always of one sign, tending to make the measured length too long. If d is the error in displacement of the end of a tape of length l , the resulting error in length $= \frac{d^2}{2l}$.*

(b) *Error due to Horizontal Curvature or Deformation.*—This is due to the tape not being stretched straight in the horizontal plane between its end points. Always of the same sign, tending to make the measured length too long. If error is due to a kink of depth D at middle of tape, error in measured distance $= \frac{2D^2}{l}$.

(c) *Error due to Vertical Deformation.*—This error arises from the tape being prevented from lying in a straight line in a vertical plane owing to small bumps and irregularities in the ground. Always of the same sign, tending to make the measured length too long. If error is due to a rise of h feet at the centre of every 100-ft. length, the error in a single tape length measured by a tape l feet long $= 2 \times \left(\frac{l}{100} \right) \times \frac{h^2}{100}$.

(d) *Error due to Error in Measurement of Slope.*†—There may be a constant error of slope, always of the same magnitude and the same sign for the same instrument and same observer, but varying, both in sign and magnitude, with different instruments and different observers. Such an error should be very small in catenary taping if reasonable care is taken, angles of slope are observed with theodolite, and these slopes do not exceed 1 in 10; but it may be fairly appreciable in the case of surface taping, especially if the slopes are measured with an Abney level badly out of adjustment or the target is not of the correct height. There will also be small errors

* This formula, and others similar to it, are derived from the well-known expansions for the trigonometrical functions, viz. :—

$$\sin \theta = \theta - \frac{\theta^3}{3!} + \frac{\theta^5}{5!} - \frac{\theta^7}{7!} + \dots$$

$$\cos \theta = 1 - \frac{\theta^2}{2!} + \frac{\theta^4}{4!} - \frac{\theta^6}{6!} + \dots$$

and

$$\theta = \sin \theta + \frac{1}{2 \cdot 3} \cdot \sin^3 \theta + \frac{1 \cdot 3}{2 \cdot 4 \cdot 5} \sin^5 \theta + \frac{1 \cdot 3 \cdot 5}{2 \cdot 4 \cdot 6 \cdot 7} \sin^7 \theta + \dots$$

When θ is very small, we have $\sin \theta = \theta$ and $\cos \theta = 1 - \frac{\theta^2}{2}$.

† An error in the measurement of the slope will also produce a small error in the amount of the computed sag correction for steep slopes. This error is given by $C \cdot \sin 2\theta \cdot \delta\theta \cdot \sin 1''$, where C is the sag correction on the flat. This is negligible for all ordinary steel tapes. Neglect to correct sag correction for slope for all reasonably moderate slopes produces errors that are negligible when a steel tape of ordinary section and weight is used.

in slope, varying for every tape length both in magnitude and sign. Let angle of slope be θ , constant error $\delta\theta_1$, and variable error $\delta\theta_2$, both measured in seconds of arc.

Then :

$$\text{Error due to } \delta\theta_1 = l \cdot \sin \theta \cdot \delta\theta_1 \cdot \sin 1''.$$

$$\text{Error due to } \delta\theta_2 = l \cdot \sin \theta \cdot \delta\theta_2 \cdot \sin 1''.$$

(e) *Error due to Error in Temperature.*—There will probably be a constant error in temperature, which may be constant for the same thermometer and for all bays in a line, but varying in sign and magnitude for different thermometers and different lines, together with an error varying in sign and magnitude for the same thermometer but different tape lengths.* Both types of error will almost certainly be greater in magnitude for ground than for catenary taping owing to the tape being in contact on the ground with objects of different capacities for absorbing, reflecting, and radiating heat. Let δT_1 be the constant error in temperature, δT_2 the variable one. Then

$$\text{Error due to } \delta T_1 = l \times c \times \delta T_1$$

$$\text{Error due to } \delta T_2 = l \times c \times \delta T_2$$

where c = coefficient of thermal expansion of the tape.

(f) *Error due to Different Elastic Stretches of Tape due to Variations in Tension.*—There may be an error in pull, which is constant for one particular spring balance, but which may vary in magnitude and sign with different balances. Let this error be δF_1 . Also, there may be a slight variation of pull from bay to bay with the same balance. Let δF_2 be this variable error. Both errors will cause variations in the amount of the stretch of the tape. Then :

$$\text{Error due to } \delta F_1 = \frac{l \times \delta F_1}{A \times E}$$

$$\text{Error due to } \delta F_2 = \frac{l \times \delta F_2}{A \times E}$$

where A = cross-sectional area of the tape and E = Young's modulus of elasticity.

(g) *Errors due to Variations in Sag Correction caused by Errors in Pull.*—Errors in pull will also affect the sag correction. One error may be constant for one particular spring balance, but vary in magnitude and sign for different balances. The other will be variable in magnitude and sign from bay to bay of the same line.

* In all probability, the thermometer in most cases registers a temperature lower than the actual temperature of the tape, at any rate when the sun is shining. See *Report of Proceedings. Conference of Empire Survey Officers, 1935.* pages 295 to 325.

Let length of tape be l and in this let there be n equal sags. Then for one tape length,

$$\text{Error due to } \delta F_1 = \frac{w^2 l^3}{12 n^2 F^3} \cdot \delta F_1.$$

$$\text{Error due to } \delta F_2 = \frac{w^2 l^3}{12 n^2 F^3} \cdot \delta F_2.$$

or, if C = the ordinary sag correction, for the whole tape,

$$\text{Error due to } \delta F_1 = 2C \times \frac{\delta F_1}{F}.$$

$$\text{Error due to } \delta F_2 = 2C \times \frac{\delta F_2}{F}.$$

(h) *Errors due to Faulty Standardisation.*—These will be constant and of the same sign for the one tape, but different in sign and magnitude for different tapes.

(i) *Errors due to Faulty End Readings and Settings.*—With proper care, there should be no appreciable constant errors, but there may be others varying in magnitude and sign according to the care and method used in reading and setting.

(j) *Errors due to Errors in Assumed Height above Sea Level.*—There will be a slight error if no correction is made for height above sea level or if the true height is not known exactly. The magnitude of this error is given by $0.000048 \times \delta H$ per 1,000-ft. length, where δH is the error of the assumed height, in feet, of the line above sea level. For the class of work here considered, the effect of this error will be assumed to be negligible or to be included in the errors of standardisation.

PROPAGATION OF ERROR IN MEASUREMENTS WITH THE STEEL TAPE

It is very important, not only to be able to estimate the magnitude of the different errors that are likely to occur in using the steel tape, but also to understand the way in which they are propagated and the effect which each of them has on the final result. Unfortunately, we do not know their exact values or signs. If we did, we could take their algebraic sum and apply this in the form of a correction to get the correct length of the line. However, we can get some idea of the likely total effect by estimating, or obtaining in some other way, the probable value of each single error and then proceeding to combine these probable values by the law of error.* To assist

* For definition of "probable error," see Vol. II, Chap. IV. If the reader is not familiar with the theory of least squares and wishes to follow the argument given here he would do well to read the first twelve pages of that chapter.

in doing this, the different possible errors are classified and grouped in the following table :

Errors due to	Errors always of the same sign for all tapes.	Errors of same sign for any one tape or line but varying in sign for different tapes and lines.	Errors of variable sign for different days.
Errors of Alignment	$u_1 = \frac{d^2}{2l}$	—	—
Errors due to Horizontal Deformation	$u_2 = \frac{2D^2}{l}$	—	—
Errors due to Vertical Deformation	$u_3 = 2 \times \left(\frac{l}{100} \right) \times \frac{h^2}{100}$	—	—
Errors in Measurement of Slope	—	$v_1 = l \sin \theta \cdot \delta \theta_1 \cdot \sin 1''$	$w_1 = l \sin \theta \cdot \delta \theta_2 \cdot \sin 1''$
Errors in Temperature	—	$v_2 = l \times c \times \delta T_1$	$w_2 = l \times c \times \delta T_2$
Errors in Stretch due to Errors in Pull	—	$v_3 = \frac{l \times \delta F_1}{A \times E}$	$w_3 = \frac{l \times \delta F_2}{A \times E}$
Errors in Sag Correction due to Errors in Pull	—	$v_4 = \frac{w^2 l^3}{12 n^2 F^3} \cdot \delta F_1$	$w_4 = \frac{w^2 l^3}{12 n^2 F^3} \cdot \delta F_2$
Errors in Standardisation	—	$v_5 = \Delta_1 l$	—
Errors due to Faulty End Readings or Settings	—	—	$w_5 = \Delta_2 l$

Here the u 's are to be taken as mean errors. If their values were known, they could, and should, be applied as corrections and not treated as errors. The v 's and w 's are to be taken as probable errors, and the error arising from any one of the v 's in the measurement of a line, in which there are N settings or tape lengths, is $v \times N$, while the probable effect of any one of the w 's is $w \times \sqrt{N}$. Now, regarding the matter from the general point of view and combining the above results by the ordinary rules for the combination of observations, we have :

Probable Error of Measurement of Line =

$$N(u_1 + u_2 + u_3) \pm [N^2(v_1^2 + v_2^2 + \dots + v_5^2) + N(w_1^2 + w_2^2 + \dots + w_5^2)]^{\frac{1}{2}}$$

This is simply one example of what may be called a purely statistical type of formula and only applies to the case of taping in general. It is useful in estimating what sort of error is to be expected, or considered reasonable, for any particular kind of taping and in judging the effect of individual sources of error; but, for any particular tape or particular observer, if the signs of v_1, v_2, v_3 , etc., were known, which they ordinarily are not, the expression would take the form:

Probable Error=

$$N(u_1 + u_2 + u_3) + N(v_1 + v_2 + v_3 + v_4 + v_5) \\ \pm [N(w_1^2 + w_2^2 + w_3^2 + w_4^2 + w_5^2)]^{\frac{1}{2}}$$

As an example of the use of the general formula, take the case of a 300-ft. tape, $\frac{1}{8}$ in. wide and 0.015 in. thick, weighing 12 oz. per 100 ft. length and used in catenary, in three equal spans of 100 ft. each, under a pull of 15 lb.

$$\begin{aligned} \text{Let } d &= 3 \text{ in.} & D &= 0 & h &= 0 \\ \theta &= 1\frac{1}{2}^\circ & \delta\theta_1 &= \pm 30'' & \delta\theta_2 &= \pm 30'' \\ \delta T_1 &= \pm 2^\circ \text{F.} & \delta T_2 &= \pm 2^\circ \text{F.} & c &= 0.000 \ 0062 \\ & & & & & \text{per } 1^\circ \text{F.} \\ \delta F_1 &= \pm \frac{1}{2} \text{ lb.} & \delta F_2 &= \pm 1 \text{ lb.} & E &= 29,000,000 \text{ lb.} \\ & & & & & \text{per sq. in.} \\ w &= \frac{12}{16 \times 100} = \frac{3}{400} = 0.0075 \text{ lb. per ft. run} \\ \Delta l_1 &= \pm 0.006 & \Delta l_2 &= \pm 0.007 & n &= 3 \end{aligned}$$

The different errors and their squares may now be written down as follows:

	u	v	v ²	w	w ²
1	0.000104	0.00114	12996 × 10 ⁻¹⁰	0.00114	12996 × 10 ⁻¹⁰
2		0.00372	138384 × 10 ⁻¹⁰	0.00372	138384 × 10 ⁻¹⁰
3		0.00276	76176 × 10 ⁻¹⁰	0.00552	304784 × 10 ⁻¹⁰
4		0.00208	43264 × 10 ⁻¹⁰	0.00417	173889 × 10 ⁻¹⁰
5		0.00600	360000 × 10 ⁻¹⁰	0.00700	490000 × 10 ⁻¹⁰
$\Sigma u = 0.000104^*$ $\Sigma v^2 = 630820 \times 10^{-10}$ $\Sigma w^2 = 1120053 \times 10^{-10}$					

The u terms are negligible and we may write for catenary taping under the conditions given above:

$$\text{Probable Error} = \pm [0.000063N^2 + 0.000112N]^{\frac{1}{2}}$$

* The Σ sign denotes summation, e.g. $\Sigma v^2 = v_1^2 + v_2^2 + v_3^2 + v_4^2 + v_5^2$.

For different lengths of line from a little over a quarter of a mile to about twenty miles this gives :

N	Length. Feet.	Probable error. Feet.	Probable error. Fractional.	N	Length. Feet.	Probable error. Feet.	Probable error. Fractional.
5	1,500	0.046	$\frac{1}{32,610}$	50	15,000	0.404	$\frac{1}{37,130}$
10	3,000	0.086	$\frac{1}{34,880}$	100	30,000	0.801	$\frac{1}{37,450}$
20	6,000	0.166	$\frac{1}{36,140}$	200	60,000	1.594	$\frac{1}{37,640}$
30	9,000	0.245	$\frac{1}{36,730}$	300	90,000	2.388	$\frac{1}{37,690}$
40	12,000	0.324	$\frac{1}{37,040}$	400	120,000	3.182	$\frac{1}{37,710}$

Hence, for very short lines, the fractional error is of the order of about $1/33,000$, but, as the length increases, it becomes more or less constant and of the order of about $1/38,000$. It thus becomes quite clear that the errors chiefly to be guarded against are those that tend to be constant in value and sign and whose total effect is directly proportional to N and hence to the length of the line. Incidentally, too, the figures show what a small part small accidental errors due to faulty end readings and settings have on the final result, and hence it follows that extreme accuracy in taking these end readings is not necessary and that multiplication of readings is more valuable as a check than as a means of reducing error.

If the length of the tape used is 100 ft. instead of 300 ft., the various external factors causing error remain much as before. Errors due to faulty end readings and settings will be practically the same for a 100-ft. tape as for a 300-ft. one, although they will be trebled in number for the same length of line, and therefore, with the same data as before but with $v_s = 0.002$, we find the total probable error is :

$$0.0003 N \pm [0.000 0070 N^2 + 0.000 0560 N]^{\dagger}$$

a formula which, for the same length of line, gives practically the same results as those for the 300-ft. tape. Hence, as regards accuracy, the results to be expected with the two tapes are very similar.

Now take the case of the 300-ft. tape, alike in all respects to that already considered, but used for surface taping. Here there are likely to be appreciable values for u_1, u_2, u_3 . Errors of stan-

dardisation will be practically the same as before, and there will be no errors at all due to sag correction that are not allowed for under u_3 . Errors due to temperature and pull are, however, likely to be greater than when the tape is hung in catenary, since control over them is not so good. In addition, there will be a slight increase in the errors due to reading and setting at the ends, while, as an Abney level is used for observing slopes, the errors arising from errors in the measurement of slopes will be greater.

On the assumption that the tape was used with spring balance and that temperatures were recorded, let

$$\begin{array}{lll} d = 0.25 \text{ ft.} & D = 0.25 \text{ ft.} & h = 0.5 \text{ ft.} \\ \theta = 1^\circ 30' & \delta\theta_1 = \pm 180'' & \delta\theta_2 = \pm 120'' \\ \delta T_1 = \pm 4^\circ \text{ F.} & \delta T_2 = \pm 2^\circ \text{ F.} & \\ \delta F_1 = \pm 1 \text{ lb.} & \delta F_2 = \pm 2 \text{ lb.} & \\ \Delta_1 l = \pm 0.003 \text{ ft.} & \Delta_2 l = \pm 0.012 \text{ ft.} & \end{array}$$

The resulting errors are tabulated as follows :

	u	v	v ²	w	w ²
1	0.000104	0.006853	469636×10^{-10}	0.004569	208759×10^{-10}
2	0.000417	0.007440	553536×10^{-10}	0.003720	138384×10^{-10}
3	0.015000	0.005517	304373×10^{-10}	0.011034	1217492×10^{-10}
5	—	0.003000	90000×10^{-10}	0.012000	144000×10^{-10}
$\Sigma u = 0.015521$		$\Sigma v^2 = 1417545 \times 10^{-10}$		$\Sigma w^2 = 3004635 \times 10^{-10}$	

For this case, therefore, we have :

$$\text{Probable Error} = 0.0155 N \pm [0.000142 N^2 + 0.000301 N]^{\frac{1}{2}}$$

Tabulating, as before, the quantity under the radical sign for different values of N, we have :

N	Length. Feet.	Probable error. Feet.	Probable error. Fractional.	N	Length. Feet.	Probable error. Feet.	Probable error. Fractional.
5	1,500	0.071	$\frac{1}{21,130}$	50	15,000	0.609	$\frac{1}{24,630}$
10	3,000	0.131	$\frac{1}{22,900}$	100	30,000	1.204	$\frac{1}{24,920}$
20	6,000	0.251	$\frac{1}{23,900}$	200	60,000	2.396	$\frac{1}{25,040}$
30	9,000	0.370	$\frac{1}{24,320}$	300	90,000	3.588	$\frac{1}{25,080}$
40	12,000	0.489	$\frac{1}{24,540}$	400	120,000	4.779	$\frac{1}{25,110}$

Thus, the fractional error decreases from about $1/21,000$ for short lines to a more or less stable value of about $1/25,000$ for long lines, so that we may say that, with a 300-ft. tape, of section 0.125 in. by 0.015 in., used on the flat under a mean pull of 15 lb. and with temperatures observed and allowed for, the order of error to be expected is somewhere about $1/23,000$; but, in addition, there may easily be an error of somewhere about $1/20,000$, due to various errors in horizontal and vertical alignment, which always tends to make the measured length of the line too long.

It must be noted that the formulae given above are derived from estimates of the contributory errors, and, of course, different results will be obtained if these errors differ from the values assigned to them. Apart from this, the definition of probable error in itself means that the errors obtained in practice will exceed the probable error as often as they will be less than it. Accordingly, estimates such as these are simply to be taken as a general indication of what can normally be expected, and are valuable because they are the only means available of judging how best to balance and control the various observations in order to obtain a desired result. In no case, though, should the actual error exceed about four or five times that given by the formulae; if it does, it is likely that there is a gross error or mistake somewhere.

In all the examples worked out, the fractional error tends to become reasonably constant for lines other than very short ones. This indicates that, for work with steel tapes, the error is more inclined to be proportional to the length of the line than to the square root of the length, and this is contrary to an assumption which is very often made. Thus, the commonly used Bowditch rule for the adjustment of traverse co-ordinates is based on the assumption that the linear errors are proportional to the square roots of the lengths of the legs of the traverse, but, considering the great difficulty in taping in controlling and eliminating constant errors of the same sign, it is very difficult to justify this assumption, and it is far more likely that, in the long run, errors in measurement of distance tend to be roughly proportional to the length of the line.

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(NOTE.—For references relating to geodetic base line measurement and first order work, see References at the end of Chapter V in Volume II.)

EXAMPLES

1. A 100-ft. chain which was 1 foot too short was used to measure a line and the result was 1346.1 ft. What was the true length of the line ?

2. A line whose true length was known to be 1201.44 ft. was measured by means of a 100-ft. tape and the measured length was 1205.75 ft. What was the correct length of the tape ?

3. A line was measured by means of a 300-ft. tape. This tape was standardised and its true length at 62° F. was found to be 300.033 ft. During the measurement the mean temperature was 78° F. and the following slopes were observed :

1° 40'	for 300	ft.
1 00	„ 300	„
2 20	„ 125	„
4 00	„ 175	„
3 40	„ 300	„
3 00	„ 300	„
5 20	„ 156	„
1 40	„ 144	„
1 00	„ 300	„
0 40	„ 264.13	„

The observed length of the line was thus 2364.13 ft. What was the true length and at what temperature was the tape of correct nominal length ? Assume c , the co-efficient of thermal expansion, = 0.000 0062 per 1° F.

4. A tape, 100 ft. long, of standard length at 84.3° F. was used to measure a line, the mean temperature during measurement being 58° F. The measured length was 636.94 ft., the following being the slopes :

1° 00'	for 100	ft.
1 40	„ 100	„
2 20	„ 100	„
3 40	„ 100	„
5 00	„ 60	„
7 20	„ 40	„
1 20	„ 100	„
1 40	„ 36.94	„

What was the true length of the line? Assume, as before, that the co-efficient of expansion of the tape was 0.000 0062 per 1° F. and that this line, as well as the one referred to in the last example, was measured on the flat.

5. Calculate, to three decimal places, the sag correction for a 300-ft. tape weighing 14 oz. per 100-ft. length and used, under a pull of 20 lb., in three equal spans of 100 ft. each. Use the formula for sag correction and check your results by means of the table given on page 162.

6. A base of 300 ft. nominal length was laid down to test a field tape. The tape used as a standard measured 300.007 ft. at a temperature of 65° F. The base was found to be 0.016 ft. shorter than the standard tape when the temperature was 76.8° F. The field tape, at a temperature of 78.3° F., measured 0.033 ft. shorter than the base. Assuming that the co-efficient of thermal expansion of both tapes was 0.000 0065 per 1° F., what was the length of the field tape and at what temperature was it exactly 300 ft. in length?

7. A tape of 300 ft. nominal length was standardised on the flat and its true length found to be 300.013 ft. at 70° F. It was then used in catenary, in three equal spans of 100 ft. each, to measure a level line, the apparent length of which was found to be 2698.67 ft. The weight of the tape was 12 oz. per 100 ft. length and the pull used, both during standardisation and during the field measurements, was 15 lb. Assume that the mean temperature during the field measurements was 60° F. and that $c = 0.000\ 0062$ per 1° F. What was the length of the line?

8. A tape, 100 ft. nominal length, was standardised on the flat and its length at 65° F., under a pull of 20 lb., was found to be 99.973 ft. It was used in catenary at the same pull and at a temperature of 57° F. to measure a short span, the measured length of which was 76.41 ft. What was the true length of the span between supports if the tape weighed 2 lb. per 100 ft.?

9. Assume that, instead of having been standardised on the flat, the tape in the last example was standardised in catenary. What was the length of the measured span between supports?

10. A tape 300 ft. long, $\frac{1}{4}$ in. wide, and 0.02 in. thick, was used to measure a line, the apparent length of which was found to be 6472.18 ft. The tape was standardised under a pull of 15 lb. but, after the line was measured, it was found that the pull actually used during the measurement was 17 lb. What was the true length of the line if the tape was standardised and used on the flat? Take $E = 28,000,000$ lb. per square inch.

11. If, instead of having been standardised and used on the flat, the tape in the last example had been standardised and used in catenary in three equal spans of 100 ft. each, what was the error in the measurement of the line? The weight of the tape was 24 oz. per 100-ft. length and the odd length at the end consisted of one span of 100 ft. and one of 72.18 ft.

12. The measured sag at the middle of a 100-ft. tape suspended in catenary with both ends at the same level was found to be 8 in. when the pull was 15 lb. What was the weight of the tape in ounces per 100-ft. length? Calculate, to four decimal places, the sag correction per 100-ft. length.

13. If there had been an error of $\frac{1}{4}$ in. in the measurement of the sag in question 12 what would be the error in the computed sag correction?

CHAPTER III

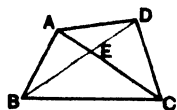
CHAIN SURVEYING

CHAIN or linear surveying is that method of surveying in which only linear measurements are made in the field. Although unsuitable for large areas or difficult country, it is quite well adapted for small surveys on open ground with simple detail, and is largely employed in such circumstances. To the beginner it forms a fitting introduction to the study of other methods on account of its simplicity and the general applicability of many of its operations.

The organisation of chain survey methods has been brought to a high state of perfection by the Ordnance Survey and a short general account of this organisation is given later in this chapter.

CHAIN SURVEYING—PRINCIPLES

Surveys with Straight Boundaries.—The simplest possible survey is that of a triangular plot with straight boundaries. If the horizontal lengths and sequence of the three sides are recorded,



sufficient information is available to enable a plan of the area to be drawn by the method of Fig. 1a.

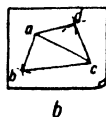
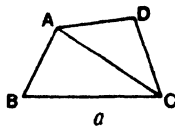


FIG. 119.

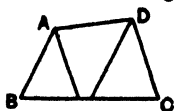
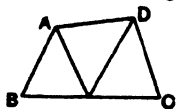
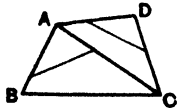
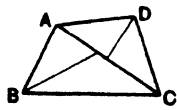


FIG. 120.

If, however, the area has more than three straight boundaries, it is no longer sufficient to measure the lengths of the sides only, as an infinite number of figures could be drawn satisfying the data. The field measurements must be so arranged that the area can be plotted by laying down triangles. Thus, if either diagonal of the four-sided field ABCD (Fig. 119a) is chained, as well as the sides, the plotting can be performed, preferably by first laying down the diagonal in a convenient position on the paper and erecting the triangles upon it (Fig. 119b).

Check Lines.—The surveyor should be able to guarantee that, within permissible errors of

measurement and plotting, the plan produced does actually represent the area surveyed. Consideration of Fig. 119 will show that a mistake made in the measurement or plotting of any of the lines will cause a distortion of the figure which may pass unnoticed. Such mistakes could be eliminated by performing the whole work twice, but may be discovered with less trouble by making cross measurements, called check or proof lines, which, although they may not be essential to the plotting of the chain lines, serve to indicate the correctness of the work by their falling into place on the drawing. Fig. 120 shows various ways in which the survey of Fig. 119 could be checked: the method to be used should depend upon local circumstances.

Notes.—(1) In the first method, the intersection E of the diagonals should be marked by a pole, and its chainage along each diagonal noted. Triangles ABC and CDA can then be checked independently, and a mistake localised.

(2) None of the methods shown affords an absolute check, as it is possible for two errors to combine and permit a proof line to fit in. They are, however, to be regarded as sufficiently good, as their object is to ensure that gross mistakes will not pass undiscovered, and these should be of sufficiently rare occurrence as to preclude the possibility of balance.

(3) If, on plotting, it is found that a mistake has been made in the field, careful examination of the notes may reveal how and where it has occurred. It is usually necessary, however, to rechain the doubtful portion, hence the importance of confining the effects of possible mistakes by distributing check lines to all parts of a survey. When surveying far from headquarters, the accuracy of the chaining may be verified by plotting the lines to a small scale before leaving the ground.

Offsets.—In the more general case in which the lines to be reproduced on paper are not all straight, the above methods are applied to the measurement of a framework of survey lines which can be plotted and checked, and from which the various details are located. The most rapid and commonly used system of location is that of Fig. 1b, the perpendiculars being called *offsets*, but points may also be tied in from the chain lines by the method of Fig. 1a.

Fig. 121 represents a field with four irregular sides. ABCD is a framework, the lines of which lie alongside the features to be

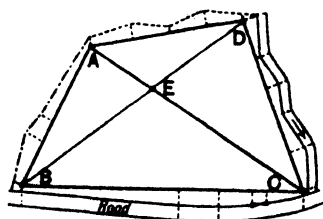


FIG. 121.

surveyed. From these lines as many points can be located as are necessary to define the irregularities of the boundaries. In plotting, the triangles are laid down as before, and from their sides short perpendiculars are erected in the proper positions, and the lengths of the offsets are scaled. The boundaries, etc., are then drawn through the points obtained.

Points to which Offsets are Taken.—To survey a straight line from an adjacent chain line, it is sufficient to determine correctly the positions of both ends by offsets. These points being plotted,

the straight line joining them represents the feature surveyed, the measurement and plotting of additional offsets imparting no further information.

In surveying an irregular line, it is treated as divisible into a series of lengths, each sensibly straight, and a sufficient number of offsets is taken to locate those lengths. Offsets must therefore be measured to every point at which the line has a marked change of direction. The degree to which minor irregularities may be neglected depends upon the character of the line being surveyed and also upon the scale to be used in plotting. Fewer offsets are required for surveying indefinite lines, such as the margins of woods or marshes, or those subject to variation, such as shore lines, than in the case of, say, a well-marked boundary equally irregular. The scale of plotting should be kept in view, as on it depends the refinement of detail which can be reproduced on the plan.

In the survey of lines of regular curvature, *e.g.* railway lines, it is sufficient to take offsets at regular intervals.

CHAIN SURVEYING—FIELD WORK

Reconnaissance.—On arriving on the field, the surveyor should first of all walk over and thoroughly examine the ground with a view to determining how he may best arrange the work. The importance of this step is sometimes overlooked by the beginner, but the utility of a thorough reconnaissance cannot be over-emphasised, the time spent being amply repaid in the greater ease with which the survey can be executed. The positions of stations are selected and marked, the poles being used to test intervisibility. During the reconnaissance the surveyor should prepare a sketch of the ground, showing the arrangement of lines and the numbering or lettering of the stations.

Selection of Stations.—In examining the ground for a good arrangement of survey lines, the surveyor should endeavour to meet the following requirements.

1. Survey lines should be as few as practicable, and such that the framework can be plotted.

2. If possible, a long line should be run roughly through the middle of the area to form a "backbone" on which to hang the triangles.

3. Triangles should be well-conditioned.

4. Each portion of the survey should be provided with check lines.

5. As few lines as possible should have to be run without offsets.

6. Offsets should be short, particularly for locating important features.

7. Obstacles to ranging and chaining should be avoided as far as possible.

8. Lines should lie over the more level ground.

9. In lines lying along a road, the possibility of interruption of the chaining by passing traffic is to be avoided by running the line at one side of the road.

Well-conditioned Triangles.—The third of the above requirements refers to the shape of the triangles, which should be such that the distortion produced by errors in measurement and plotting will be a minimum. It can be shown that a point located by the intersection of two arcs is least displaced by errors in the radii when the arcs intersect at 90° . If the three sides of a triangle are equally subject to error, the three angles should approach 90° as nearly as possible, *i.e.* the triangle should be equilateral, and triangles approximating to this form are termed "well-conditioned." An endeavour should always be made to avoid using triangles having one or two angles less than 30° : if they must be used, such badly-conditioned triangles require additional care in chaining and plotting.

Marking Stations.—Survey stations should be marked to enable them to be readily discovered during the progress of the survey, and preferably in a manner which will render them available after the lapse of some time, in case it may be necessary to repeat a portion of the work or to extend the survey. In soft ground wooden pegs about 18 in. long by $1\frac{1}{2}$ in. square are suitable. They should be driven firmly with only a small projection above the ground. In roads and streets it is necessary to substitute nails or spikes driven flush. The difficulty of recovering the position of a nail is avoided by noting the tie measurements to it from two, or preferably three, well-defined and permanent points near it. In important surveys it is highly expedient to locate the principal stations in this manner as a precaution against displacement of the peg. In many surveys pegs are dispensed with, and the stations are marked by inserting a twig into the hole made by a ranging pole. The marking is made conspicuous if the twig is cleft to receive a piece of paper. Plasterers' laths are sometimes used, as they are sufficiently straight to be used for ranging as well as marking. On turf a good marking is obtained by cutting out a sod in the form of an arrow or an acute isosceles triangle with the apex at the station.

In tropical countries wooden pegs, unless treated with creosote or solignum or some other form of wood preservative, cannot be depended on to last more than a few days as they are liable to be destroyed by white ants. In such a case, an inexpensive mark can be made, which will last for some little time, by pressing the pointed shoe of a ranging rod into the ground and filling the hole so made

with cement mortar in which a tack or nail can be driven when it is necessary to mark the exact point.

Running Survey Lines.—The routine of running a survey line comprises the chaining of the line and the location from it of the adjacent detail. Having stretched the chain and inserted the forward arrow, the leader leaves the chain lying on the ground, and returns to assist the follower with the offsetting. He takes the ring of the tape, and holds it at the various points to right and left of the chain as directed by the surveyor, while the follower remains at the chain with the tape-box. The latter holds the tape at right angles to the chain, estimating the perpendicularity by eye in the general case, and calls out to the surveyor the lengths of the offsets and the chainages from which they are projected. The distances along the survey line at which fences, streams, etc., are intersected by it must also be observed and noted.

Notes.—(1) The surveyor should be on his guard to detect gross mistakes in reading. He should remain beside the follower to check the readings and see that the tape is held perpendicular to the chain line and horizontal on slopes.

(2) Offsets should be taken in order of their chainages, and, before allowing the chain to be moved forward, the surveyor should make sure that no offsets have been overlooked.

(3) A common tendency is to take too many offsets. No attempt should be made to record irregularities too small to be shown on the scale to which it is proposed to plot.

(4) On flat ground offset measurements should be made on the surface of the ground; on slopes the chainmen should use poles for plumbing.

(5) The accuracy required in the perpendicularity of offsets increases with the length of the offset, the scale of plotting, the importance of the feature offsetted, and the angle between the chain line and the line being located. The cross staff, optical square, or box sextant may be used in setting out long offsets, but in general the perpendicularity is estimated. The correct position of the tape can be determined by swinging it through a small angle about the ring as centre. The smallest tape reading against the chain is the required perpendicular distance.

(6) The accuracy required in measuring offsets depends on the first three factors above. In particular, by keeping in view the scale to which the plotting is to be performed unnecessary refinement will be avoided. Assuming about .01 inch as the smallest distance on the paper which can be distinguished in plotting, the following figures indicate the refinement of measurement required.

If the scale is 1 in. = 100 ft. or more, measurements should be recorded to the nearest foot or link.

If the scale is 1 in. = 50 or 66 ft., measurements should be recorded to the nearest $\frac{1}{2}$ ft. or link.

If the scale is 1 in. = 20 to 40 ft., measurements should be recorded to the nearest $\frac{1}{4}$ ft. or $\frac{1}{2}$ link.

If the scale is 1 in. = 8 or 10 ft., measurements should be recorded to the nearest inch.

It is, however, better to be over- than under-accurate in view of the possibility that the survey, or a part of it, may be required on a larger scale than was originally intended.

(7) The allowable length of offset depends upon the accuracy desired, the method of setting out the perpendicular, and the scale of plotting. Survey lines should be so arranged that offsets to important objects are as short as

possible, but for the survey of indefinite features long offsets, up to, say, 100 ft., are permissible without instrumental setting out. Progress is considerably delayed, however, when offsets exceed the length of the tape, and it is false economy to reduce the number of survey lines at the expense of convenient offsetting.

(8) Special care is necessary in offsetting to buildings, and the measurements should be made self-checking by the use of ties and by noting the chainages at which the directions of the walls cut the survey line. Buildings should be taped round; those of irregular outline may have to be enclosed by subsidiary chain lines from which to offset.

(9) As a large part of the time occupied on a chain survey is spent in offsetting, the necessity for systematic routine is evident. The chainmen should not be allowed to exchange their duties, and the one who observes the measurements should be instructed to call them out in a uniform manner, thus—"Fence, 9½ off 285."

Note-keeping.—The field book, about 9 in. by 5 in., and opening lengthwise, may be of unruled paper, but more commonly each page has a single red line, or two red lines about $\frac{1}{2}$ in. apart, ruled down the middle. The single line represents the survey line, and against it are entered the total length of the line and the distances along the chain at which offsets are taken, intermediate stations are established, and fences, streams, etc., are crossed. The space on either side is available for sketches of the various features located from the survey line and for notes of the offset distances. In the double line system, the space between the lines is reserved for distances along the chain, which are thus kept entirely separate from other dimensions. In using a plain page book, the surveyor draws down the page either one or two pencil lines as preferred. This system has the great advantage that if more detail has to be surveyed on one side of a line than on the other, overcrowding of the notes is avoided by ruling the lines near one side of the page.

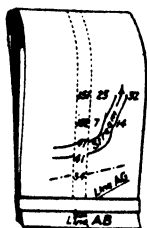


FIG. 122.

FIELD BOOK.

On commencing the chaining of a line, a fresh page should be started, and the designation of the line is noted at the foot. Booking proceeds from the bottom of the page upwards (Fig. 122), as it then progresses naturally with the chaining, and the right and left sides in the book correspond with those in the field when looking in the direction of chaining. As the various features within offsetting distance are reached, the surveyor sketches them and enters the chainage and length of each offset as shown in Figs. 123 and 124. For uniformity and ease in plotting, offsets should be entered as distances from the chain line only.

Notes.—(1) The complete record includes: (a) a general sketch of the lay-out of the lines; (b) the details of the lines; (c) the date of the survey; (d) a page index of the lines; (e) the names of the members of the party. Some surveyors ink in their notes after plotting. This is desirable on pages which have suffered from rain, but otherwise is a matter of choice.

(2) The two essentials in booking are accuracy and clearness. The surveyor, although he may intend to plot the work himself, should always aim at producing notes which could be plotted without difficulty by a draughtsman

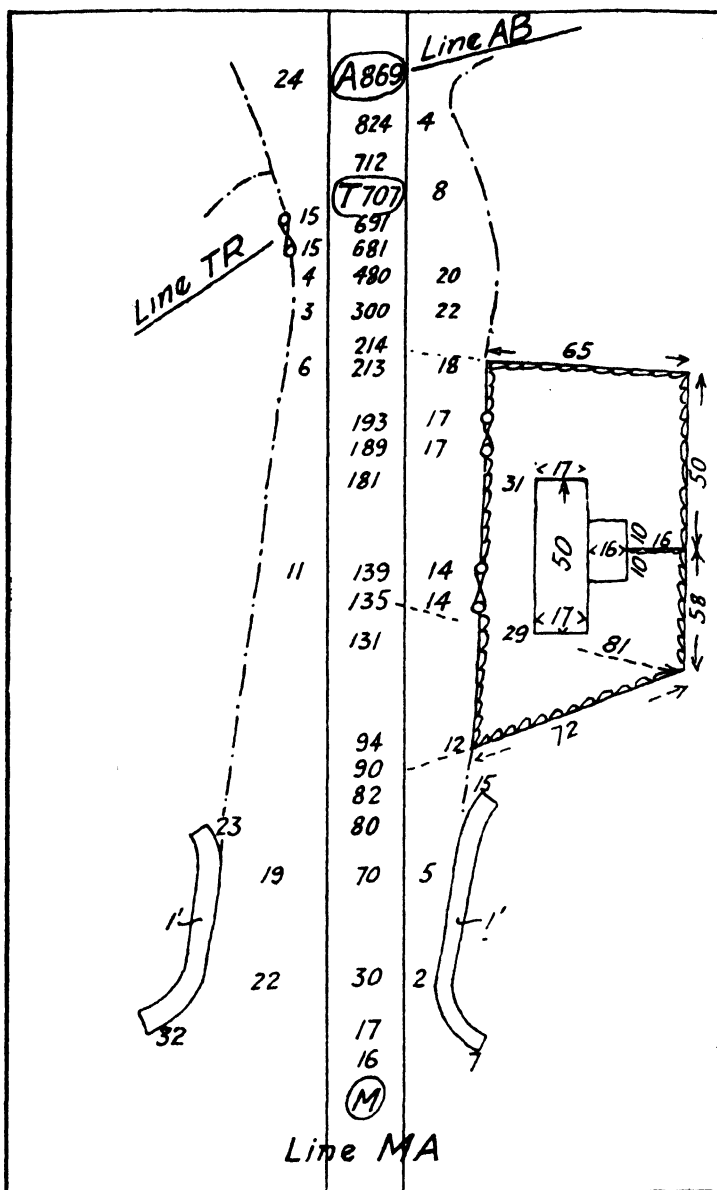


FIG. 123.—DOUBLE LINE BOOKING.

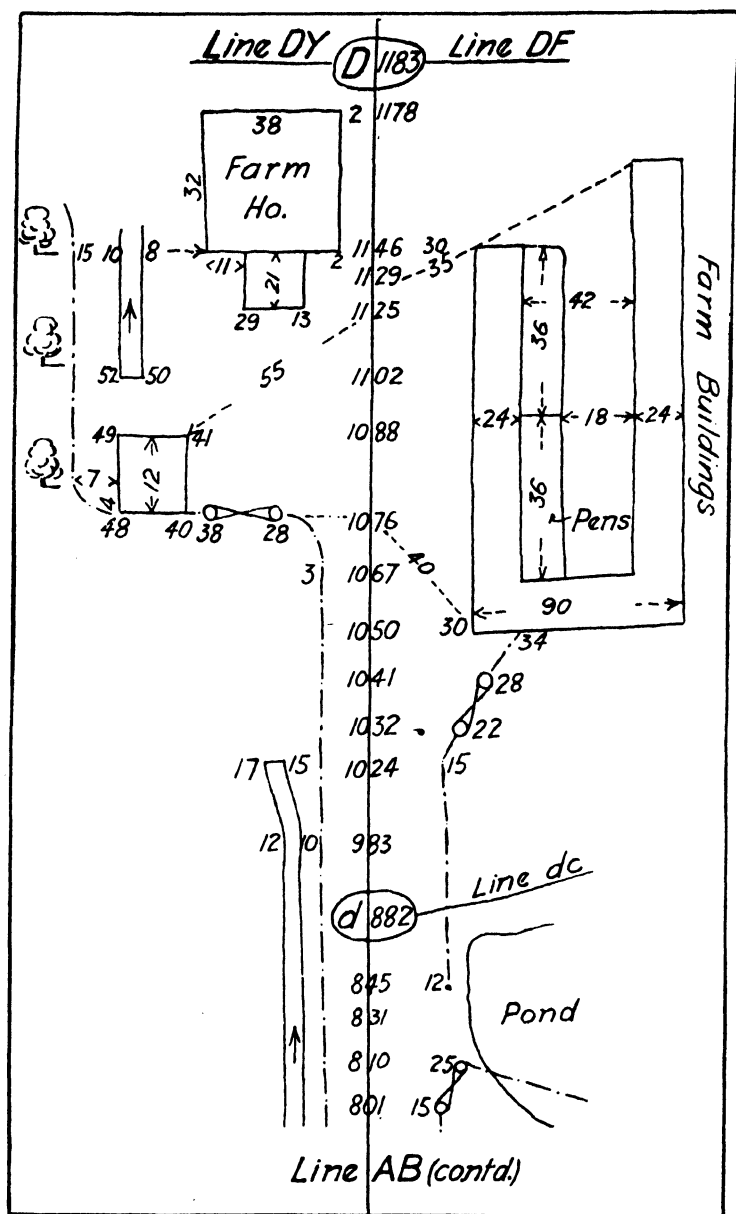


FIG. 124.—SINGLE LINE BOOKING.

quite unfamiliar with the ground, as the field book may be referred to, and extracts made, at a future date. This desideratum is promoted by : (a) neat figuring and legible writing ; (b) lucid sketches ; (c) clearness in representing the points to which offsets have been taken ; (d) the insertion of explanatory notes and references to other pages of the field book where misunderstanding might occur or difficulty in following the notes is likely to arise in plotting ; (e) leaving nothing to the memory ; (f) keeping the book clean.

(3) The making of satisfactory sketches proves troublesome to the inexperienced. The tendency is to allow insufficient room for sketches of intricate detail, so that dimensions cannot be entered clearly. No attempt should be made to sketch strictly to scale. The size of complicated parts should be exaggerated, the curvature of flat curves should be increased, and angles which are nearly 90° or 180° should have their divergence from those values emphasised. On the other hand, long straight lines should be shown shortened. A comparison of the sketches in Figs. 123 and 124 with the corresponding features on the scale diagram, Fig. 126, will exhibit the allowable distortion. It should be observed that, in the double line system, a line crossed by the chain is apparently broken at the intersection, because the survey line is represented in the book by a *space*, which has no physical existence on the ground.

(4) The pencil should be of good quality, to avoid smearing, and rather harder than is usually preferred for writing, to prevent the notes being washed out by rain. In wind, it is a convenience to have a strong rubber band to hold the leaves of the field book (Fig. 122).

Examples of Chain Surveys.—Fig. 125 shows a small survey of an irregularly shaped parcel of ground. The noticeable features are : (a) the provision of a long backbone line AB ; (b) the running of lines CH and EK merely as proof lines, to check triangles AEF and DJB respectively ; (c) the manner of plotting and checking triangle FGH by locating G on CH produced and checking its position by FG ; (d) the survey of the straight hedge to the right of CH simply from its intersections with lines AB and EF.

A farm survey is illustrated in Fig. 126. AB has been adopted as a base line in preference to a diagonal extending right across the

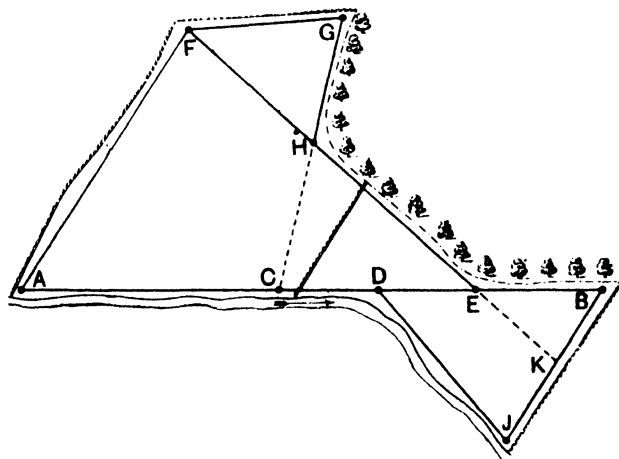


FIG. 125.—SMALL CHAIN SURVEY.

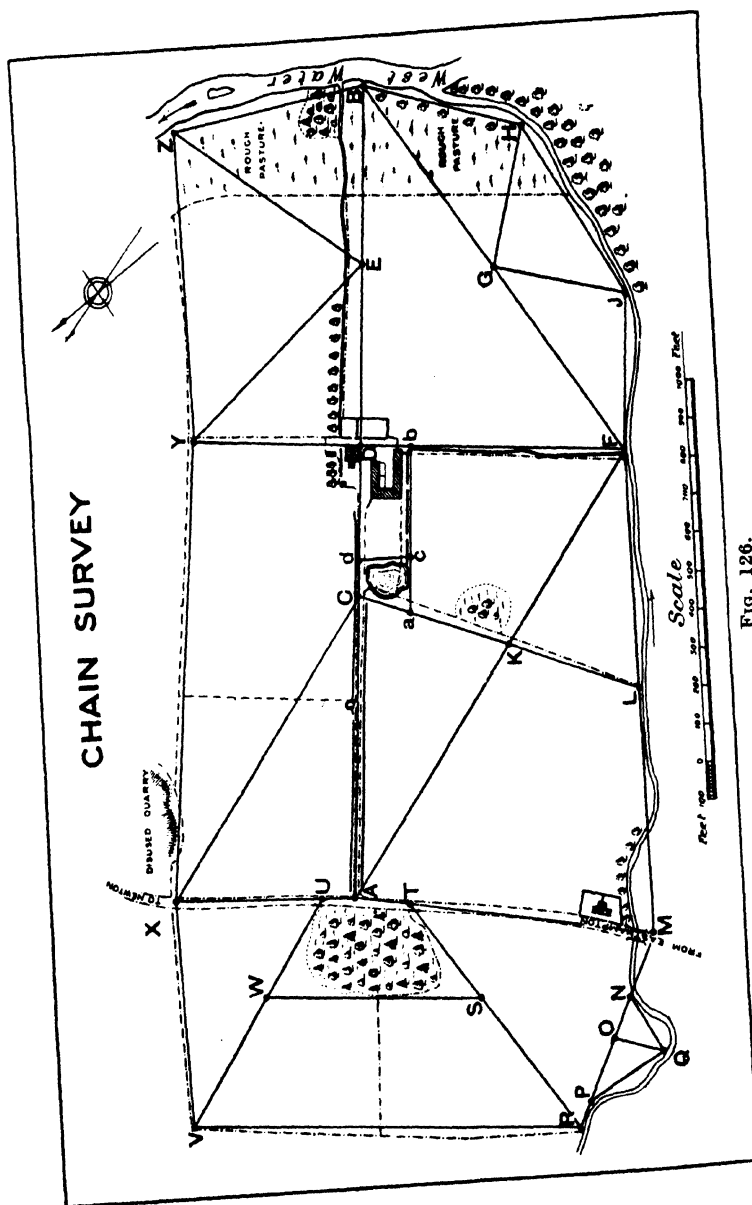


FIG. 126.

survey, as the outline would not permit of well-shaped triangles by the latter arrangement. Having fixed AB, there is little choice as regards the positions of D, F, M, G, H, and J. The line CK is inserted for offsetting to the pond and to the piece of rough ground: its extension KL is merely a proof line to check triangle AMF. No proof line is required in triangle DFB, as it forms part of triangle AFB, which is checked by DF and CK. The narrow area ABZX is best surveyed and checked as shown. The lines are self-checking, while the fact that Y is in range with FD affords a further check, so that the chaining of a line between C and Y is unnecessary. The arrangement of lines for the survey of area XVRM is preferred to running diagonals XR and VM, as being more convenient for the location of the outline of the wood. Triangle PQN, with the proof line OQ, is introduced to avoid long offsets to the stream.

Survey of Woods, etc.—Chain surveying methods may prove tedious when applied to the survey of a dense wood, a pond, or other area across which lines cannot be run. In such cases a system of lines must be laid out to surround the area. Wherever possible, the main lines should form a single triangle, checked by proof lines at one or more corners, and from the main lines subsidiary triangles may be projected inwards where required to shorten the offsets. Otherwise, as in Fig. 127, the sides of the enclosing polygon must be fixed relatively to each other by forming triangles at the corners. This example could be plotted by first drawing AB and constructing the triangles AEC and DFB. On producing AE and BF to G and H respectively, the measured length GH should fit in between G and H. A better check is, however, available by chaining triangle GKL. On plotting this triangle on base GK, and producing GL to H, the station H so obtained should coincide with its position as given by the line BH. It is desirable for the rapid location of a mistake that each of the triangles should be checked by a proof line.

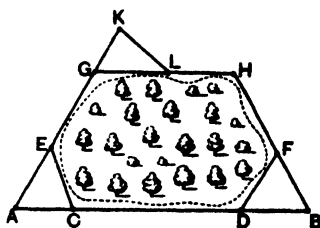


FIG. 127.—CHAIN SURVEY OF WOOD.

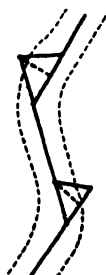


FIG. 128.

Survey of Narrow Belts.—Fig. 128 illustrated how the above principle is applied to the chain survey of a long narrow strip of country, the width of which can be controlled by offsets from a central chain line. The introduction of triangles between the survey lines in this, as in the previous example, is simply the linear survey method of enabling the angles between the lines to be plotted. Such surveys are more accurately and

rapidly executed by measuring the directions of the lines by the *theodolite*.

MISCELLANEOUS FIELD PROBLEMS

It is frequently impossible to arrange a survey so that all the lines can be run in the straightforward manner previously described. While the difficulties which may be encountered are most rapidly and accurately surmounted by the aid of an angular instrument, they may also be solved by a routine involving only the essential equipment used in chain surveying, and these methods will be considered here. Many of them involve the setting out of perpendicular and parallels, and such operations will be treated first.

To Erect a Perpendicular to a Chain Line from a Point on It.—

(a) Select points B and C on the line on either side of, and equidistant from the given point A (Fig. 129). Pin down the ring of the tape at B, and, having the end held at C, hold the mid-point D so that both halves are stretched tight. D is then on the required perpendicular.

Alternatively, sweep arcs of equal radii with the tape from B and C, scratching the arc on the ground with an arrow, and mark the intersection D.

By similarly locating D' on the other side of A, a longer base is available for production.

(b) With A as centre, sweep an arc of 30 ft. radius, and from B, 40 ft. along the chain from A, sweep an arc of 50 ft. radius. C, the

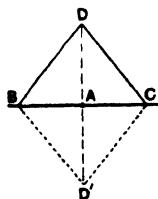


FIG. 129.

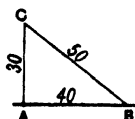


FIG. 130.

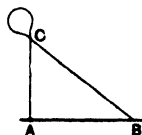


FIG. 131.

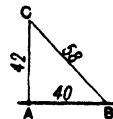


FIG. 132.

cut of these arcs, is on the perpendicular AC (Fig. 130), since $50^2 = 30^2 + 40^2$. The ring of a 100 ft. linen tape may be pinned at A and the 80 mark held at B, while the 30 mark is placed so that the two parts are taut. A steel tape cannot be bent sharply at C, but, by holding the 100 mark at B, C is obtained as the position of the 30 and 50 marks held together with a 20-ft. loop between (Fig. 131). Any multiples of 3, 4, and 5 can, of course, be similarly used, and the sides of the triangle are proportionately reduced when using a shorter tape. Integral values for the sides of right-angled triangles are proportional to $(2n+1)$, $2n(n+1)$, and $(2n^2+2n+1)$, where n is integral or fractional, but, of the various series of integers available, 3, 4, 5, and 20, 21, 29, corresponding to

$n = 1$ and $n = \frac{1}{2}$ respectively, yield the best intersections. With the latter, the whole of a 50 or 100 ft. tape is stretched (Fig. 132).

To Drop a Perpendicular to a Chain Line from a Point Outside It.—i. *When the Point is Accessible.*—(a) With the given point A as centre, swing an arc to cut the chain line at B and C (Fig. 133). D, the mid-point of BC, is the foot of the required perpendicular.

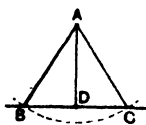


FIG. 133.

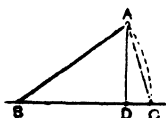


FIG. 134.

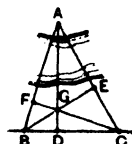


FIG. 135.

(b) Measure the distance from A to any point B on the line (Fig. 134). Set off BC on the chain equal to BA. Measure AC.

$$\text{Then } CD = \frac{AC^2}{2BC}.$$

ii. *When the Point is Inaccessible.*—(a) By repeated trial of erecting perpendiculars from BC, find that which passes through A.

(b) Select points B and C on the line (Fig. 135). Set out BE and CF perpendicular to AC and AB respectively. Mark their intersection G. AG produced to D is the required perpendicular.

To Run a Parallel to a Chain Line through a Given Point.—

i. *When the Point is Accessible.*—(a) From the given point A drop a perpendicular AB to the given line BC (Fig. 136). From C, as

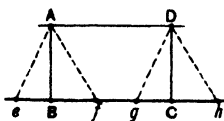


FIG. 136.

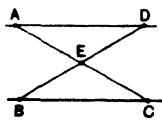


FIG. 137.

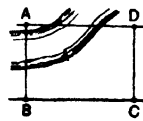


FIG. 138.

far from BC as convenient, erect a perpendicular CD equal to AB. AD is the required parallel. Alternatively, measure the sides of a triangle Aef, with apex at A, and reproduce the triangle at Dgh.

(b) From A measure AC to any point C on the line (Fig. 137). Mark E, the mid-point. From any point B on the chain measure BE, and produce BE its own length to D.

ii. *When the Point is Inaccessible.*—Proceed as in (a) above, obtaining the inaccessible perpendicular distance AB (Fig. 138) by the methods of page 191.

Obstacles.—The more important obstacles may be classed as :

1. Those which obstruct ranging but not chaining.
2. Those which obstruct chaining but not ranging.
3. Those which obstruct both.

1. Obstacles which Obstruct Ranging but not Chaining.—This type of obstacle, in which the ends of a line are not intervisible, is

difficult to avoid except in flat country, as it commonly occurs in the form of an intervening hill. Two cases may occur :

- (1) Both ends may be visible from intermediate points on the line.
- (2) Both ends may not be visible from any intermediate point.

Case (1). Having plumbed the terminal poles A and B (Fig. 139), the surveyor C and his assistant D, each with a ranging pole, proceed up the hill, and take up positions such as C_1, D_1 , in plan, as nearly in the line as they can judge, and such that C can see both B and D, while D can see A and C. C directs D to D_2 , in line with B, and then D guides C into position C_2 , in line with A. C now brings D to D_3 , and so on alternately. When C and D are simultaneously satisfied that the other is in line with the remote pole, they are both in the line AB, and fix their poles. This method is also serviceable in ranging a line across a hollow.

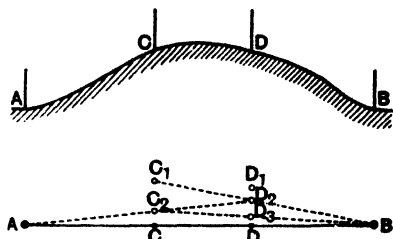


FIG. 139.

Case (2). The same principle is applied in this case by gradually bringing a series of intermediate poles into range with each other and with the stations. This involves either additional assistance or considerable delay in walking from pole to pole, but is often the most suitable method.

When only the length of the line is required, or if offsets have to be taken from only a part of it, the method of the *Random Line* is best. Proceeding from station A, a line such as AB' (Fig. 140) is ranged out at random, but as nearly towards B as can be judged. It is chained out to B' , where $B'B$ is perpendicular to AB' , and $B'B$ is measured. Then $AB = \sqrt{(AB')^2 + (B'B)^2}$, or, logarithmically, $AB = AB' \sec \theta = BB' \operatorname{cosec} \theta$, where $\tan \theta = \frac{BB'}{AB'}$. If C' is a point of known chainage on AB' , a point C can be located on AB by the perpendicular offset $C'C = \frac{AC'}{AB'} \times B'B$. In this manner a sufficient number of points can be obtained on AB to complete its ranging, and, if necessary, AB is then chained for offsetting.

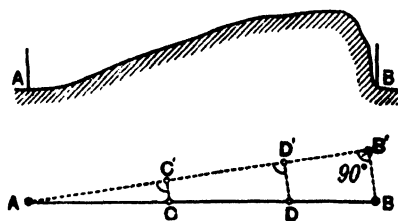


FIG. 140.

If, however, the detail to be offsetted is confined to a part CD of the line, it will suffice to chain between those points, since their chainages can be deduced from those of C' and D' . If the detail can be located satisfactorily from AB' , time is

saved by offsetting from that line, the plotting being performed by first obtaining B on the paper from the computed length AB.

2. Obstacles which Obstruct Chaining but not Ranging.—The typical obstacle of this class is a sheet of water the width of which in the direction of measurement exceeds the length of the chain or tape. The problem consists in finding the distance between convenient points on the survey line on either side of the obstruction. Two cases may be distinguished :

- (1) In which the obstacle can be chained round.
- (2) In which the extent of the obstacle prevents this.

Case (1). Figs. 141 to 146 illustrate six methods of finding the distance between poles A and B on either side of a pond.

(a) Set out equal perpendiculars AC and BD (Fig. 141); then $CD = AB$.

(b) Erect perpendicular AC (Fig. 142), and measure AC and CB; then $AB = \sqrt{BC^2 - AC^2} = \sqrt{(BC - AC)(BC + AC)}$.

(c) Find, by optical square or cross staff, a point C which subtends 90° with A and B (Fig. 143). Measure AC and CB; then $AB = \sqrt{AC^2 + CB^2} = AC \sec \theta = CB \operatorname{cosec} \theta$, where $\tan \theta = \frac{CB}{AC}$.

(d) Set out a straight line CAD (Fig. 144). Measure AC, AD, BC, and BD; then $AB = \frac{\sqrt{BC^2 \times AD + BD^2 \times AC}}{CD} \cdot AC \times AD$.

This formula may be computed by logarithms by finding the auxiliary angle θ from

$$\tan \theta = \frac{AD \times (BC - AC) (BC + AC)}{AC \times (BD - AD) (BD + AD)}$$

Then

$$\begin{aligned} AB^2 &= \frac{2 AC \times (BD - AD) (BD + AD) \times \sin 45^\circ \times \sin (45^\circ + \theta)}{AD \times \cos \theta} \\ &= \frac{2 AD \times (BC - CA) (BC + CA) \times \sin 45^\circ \times \sin (45^\circ + \theta)}{AD \times \sin \theta} \end{aligned}$$

a solution proposed by Capt. G. T. McCaw, C.M.G.

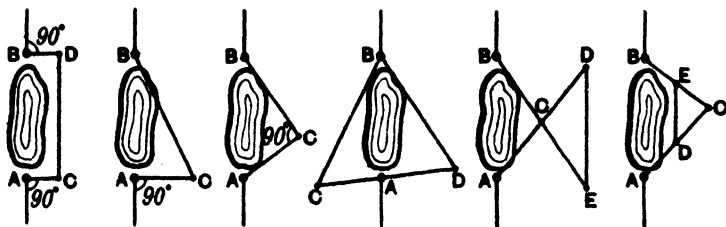


FIG. 141. FIG. 142. FIG. 143. FIG. 144. FIG. 145. FIG. 146.

(e) Mark a point C (Fig. 145). Range D in line with AC so that $CD=AC$. Range E with BC, making $CE=BC$; then $ED=AB$.

(f) Select a point C (Fig. 146), and measure AC and BC. Mark D and E such that $CD=\frac{CA}{n}$ and $CE=\frac{CB}{n}$; then $AB=nDE$.

In each case the obstacle is surveyed from the auxiliary lines. Method (d) is suitable for a wide obstacle, since offsets can be taken on each side, but long offsets are avoided in the other methods by repeating the work on the other side.

Case (2). A river is typical of this class of obstacle.

(a) From A and another point C on the line (Fig. 147) erect perpendiculars, or any parallel lines, AD and CE, such that E, D, and B are in line. Measure AC, AD, and CE; then $AB=\frac{AC \times AD}{CE - AD}$.

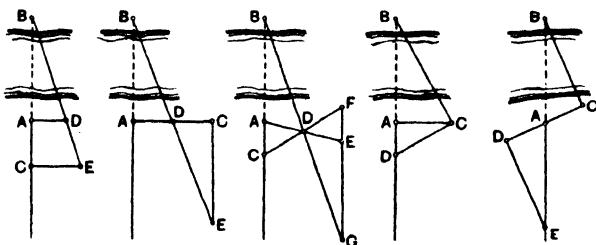


FIG. 147. FIG. 148. FIG. 149. FIG. 150. FIG. 151.

(b) Set out a perpendicular AC (Fig. 148), and mark its mid-point D. From C erect CE perpendicular to AC, and mark E in range with BD; then $CE=AB$. More generally, if D lies anywhere on AC, $AB=\frac{AD \times CE}{CD}$.

(c) Select a point D (Fig. 149). Measure AD and CD, and continue them their own lengths to E and F. Find G in range with BD and EF; then $EG=AB$.

(d) Erect a perpendicular AC (Fig. 150), and, using an optical square or similar instrument at C, find D on the chain line so that BCD is a right angle. Measure AC and AD; then $AB=\frac{AC^2}{AD}$.

(e) By instrument locate C which subtends 90° with AB (Fig. 151). Range D with AC, so that $AD=AC$. Find E at which the line is cut by the perpendicular DE; then $AE=AB$.

3. Obstacles which Obstruct Ranging and Chaining.—In this case the obstruction cannot be seen through, but the methods of Figs. 147 to 151 are applicable if, owing to the contour of the ground, the line can be ranged beyond the obstacle. In the general case, however, the problem consists both in prolonging the line beyond the obstacle and finding the distance across it.

(a) From A and B on the line erect equal perpendiculars AC and BD (Fig. 152). Range E and F with CD, and set out EG and FH perpendicular to DF and equal to AC. G and H are in the line, and $CE=AG$.

(b) Select a point C (Fig. 153). Measure AC and BC, and mark D and E so that $CD=\frac{AC}{n}$ and $CE=\frac{BC}{n}$. Range F and G with DE. Measure CF and CG, and produce them to H and K, making $CH=nCF$ and $CK=nCG$. H and K are in the line, and $AH=nDF$.

(c) Set out a triangle BCD (Fig. 154). Continue BC and BD to E and F, making $BE=nBC$ and $BF=nBD$. Mark G on EF so

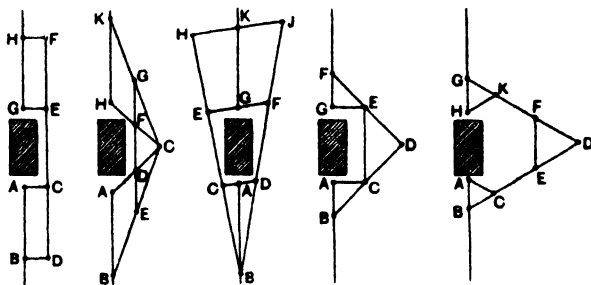


FIG. 152. FIG. 153. FIG. 154. FIG. 155. FIG. 156.

that $EG=nCA$. Similarly locate K. G and K are on the line, and $AG=(n-1)BA$.

(d) Erect a perpendicular AC, and mark B so that $AB=AC$ (Fig. 155). Produce BC to D. Set out DF perpendicular to DB, making $DF=DB$ and $DE=DC$. From F and E swing arcs of radii $=AC$, obtaining intersection G. G and F are in the line, and $CE=AG$.

(e) By swinging the tape, construct an equilateral triangle ABC (Fig. 156). Produce BC to D. Form an equilateral triangle DEF, and produce DF to G, making $DG=BD$. Obtain H on the line by an equilateral triangle GKH. $AH=BD-BA-HG$.

None of these methods ensures much accuracy in the prolongation of the chain line, and this type of obstacle should therefore be studiously avoided unless an angular instrument is available.

ORDNANCE SURVEY METHODS OF CHAIN SURVEY

The Ordnance Survey methods of chain survey are devised mainly for filling in details in fairly congested areas in towns and assume that some sort of control, of higher order than any that can be established by the use of the chain alone, is available. They are a very good example of that fundamental rule in surveying of working

from the whole to the part. There is nothing essentially different in them from the methods of chain survey already described, but a general description may be of interest from the point of view of seeing how the work is organised and subdivided and for the rules laid down to prevent excessive error creeping in.

The detail to be surveyed is first of all divided up into large areas or blocks, at the edges of which definite framework or control points exist, and these blocks are broken down by lines called "main lines," which commence and end on framework points. Each of the smaller blocks is then subdivided by lines called "split lines," or "splits," which begin and end on definite points on the main lines, and, if necessary, further subdivision is carried out by means of "detail lines." Hence, the order of survey is (1) framework, (2) main lines, (3) split lines, (4) detail lines. In this way, the detail falls more or less automatically into its proper place.

Main lines, being rather important ones, are usually measured by a standardised steel tape and are run as straight lines from one control point to another. In the case of short lines, of a quarter of a mile or so in length, alignment may be done by eye when the ground is level, but, for longer lines, the theodolite is generally used.

Split lines break down the blocks bounded by traverse lines or main lines into smaller blocks, and they are seldom allowed to exceed half a mile in length, alignment being done by theodolite, except for very short lengths on level ground. Distances are measured by ordinary wire chain. The maximum length of offsets depends on the scale on which the survey is to be plotted. For town survey work on the 1 : 2,500 scale, 25 ft. is the limit, but 50 ft. may be allowed in open country, except to definite points such as hedge junctions, house corners, etc., where offsets of more than 20 ft. are avoided as much as possible.

Before main or split lines are surveyed, the points at which other lines are likely to meet them are decided on, and during the survey these points are marked on the ground in some convenient way and their chainage noted.

Detail lines are used to split up still further the areas bounded by other lines and to pick up detail that is too far from these lines to be surveyed from them. They are surveyed by the ordinary wire chain and are run as close as possible to the detail to be surveyed.

Right angled offsets are not allowed to exceed 25 ft. in length except in the case of ill-defined detail, such as the edge of marsh or forest. Also, single offsets up to 20 feet may be used to fix important points, such as the corner of an extensive block of buildings, but, where possible, the surveyor runs his lines to touch corners of this kind. In the case of modern housing estates, garden cities, etc., where there is a good deal of re-entrant detail, offset triangles have sometimes to be used to pick up detail. These consist of small triangles, in which the base is a definite part of one of the chainage

lines and the other two sides are measured with an ordinary linen tape (see Fig. 1a).

For booking, the Ordnance Survey uses a thin book, about the size of an ordinary twopenny exercise book, with blank, unruled pages. In all probability, the ordinary surveyor, if he is not very good at drawing, would find it an advantage to use a book in which the pages were ruled into squares with faint blue lines about an eighth to a quarter of an inch apart. The opening page of the first book used in the survey contains a general diagram of the main blocks and shows all the main and split and other similar lines. The lengths of these lines, and the numbers of the pages in which their survey is recorded, are entered against each of them. Detail diagrams of the various subdivisions of the work, showing the different chain lines, are made where required, generally at the beginning of the book, so that the draughtsman can follow the work easily and plot the different survey lines in their proper positions from the data contained in the block and detail diagrams. In every case, the figures on any line are written in such a way that the direction in which they run shows the direction in which the line was chained. The various chain and detail lines are then booked on successive pages in order of survey, and all details, such as offsets, etc., are shown with reference to these lines, particular care being taken at all times to insert references from one line, or from one page, to another, so that all the information required for drawing can be found very easily.

An interesting account of the method of chain survey taught at the School of Military Engineering, Chatham, which is based on the Ordnance Survey system, will be found in the S.M.E. publication, *Notes on the Making of Plans and Maps* (H.M. Stationery Office. Price 17s. 6d. net). This publication gives very complete examples of the method of booking such work.

PLOTTING

Instruments.—The draughtsman will be provided with a set of the usual drawing instruments, which should include drawing pens, compasses with pen and pencil points and lengthening bar, dividers, spring bows, pricker, protractor, scales, set-squares, French curves, and colour brushes. In place of a T-square, which is of minor utility in plotting surveys, a 24-in. parallel ruler should be used. For drawing long straight lines a steel straight-edge is required, 6 ft. being a serviceable length. Beam compasses are necessary for describing arcs of large radius, and are very useful in plotting various kinds of surveys. For drawing railway lines, a set of railway curves is essential. These are flat strips of pearwood, celluloid, or vulcanite, each cut to a distinct curvature. A useful set consists of 100 curves of radii from $1\frac{1}{2}$ in. to 240 in.

Scales.—Drawing scales should be of well-seasoned boxwood or ivory and of superior quality. The most useful length is about 12 in., and a flat section with two bevelled edges accommodating two different fully divided scales proves more accurate and convenient than oval and triangular forms, having respectively four and six different scales. A useful pattern has one edge divided to chains and links with the same scale converted to feet on the other, so that dimensions laid down in links can be readily scaled in feet.

The more commonly used scales are 10, 20, 30, 40, 50, 60, 80, 100, 200, etc., ft. or links to 1 in. It is to be noticed that one set of graduations is employed for plotting to several scales decimally connected, *e.g.* the 20 scale, having 20 divisions to 1 in., can be used as a scale of 2, 20, 200, or 2,000 ft. or links, according as the smallest division is treated as $\frac{1}{2}$, 1, 10, or 100 ft. or links.

Since Ordnance Survey maps are extensively used, alterations, new works, etc., have frequently to be plotted to the Ordnance scales. The larger scales are :

60	in. to one mile or	1/1,056,
50·688	„ „	1/1,250,
25·344	„ „	1/2,500,
6	„ „	1/10,560,
1	„ „	1/63,360.

The 1/1,056 and 1/1,250 scales apply to town maps only, and the first is now available only for London. Town maps were formerly published on a scale of 1/500, but this practice has been discontinued.

Materials.—*Paper.*—Only the best hand-made paper should be used for survey plans other than those intended merely for temporary use. Plans which will be subjected to much handling are rendered more permanent by having the paper mounted on holland or cotton. Mounting is also necessary for very large plans and long rolls, and should always be done before plotting, otherwise the plan will suffer distortion.

Notes.—(1) Paper expands under the influence of a damp atmosphere, and contracts when dried. The variations are less marked in seasoned than in fresh paper, and are also reduced in mounted papers. Tight rolling of a drawing tends to stretch it, so that it is desirable that drawings should be kept flat.

(2) Mounted sheets and rolls are sold in various sizes and qualities, but mounting may at times have to be performed by the draughtsman. A sheet of the fabric, a few inches longer and wider than the paper, is tacked on to a table at intervals of 6 in. or less round the margin, and is then thoroughly damped. A quantity of flour paste is freshly prepared, and this is well rubbed into the back of the paper with a stiff brush or by the fingers. Superfluous paste is brushed off to leave a uniform film over the whole sheet. The paper is stretched on the cloth, and, by means of a squeegee roller or the edge of a large clean set-square, worked from the middle of the sheet towards the margin, the paste is pressed into the interstices of the fabric, and all surplus paste is squeezed out at the edges until the paper is perfectly flat. The whole is left to dry before the tacks are removed, and the sheet is finally trimmed, or a narrow margin of cloth is left for pasting on the front of the sheet to protect the edges.

If a large mounted sheet is to be prepared by joining individual sheets, unsightly overlaps are avoided if the sides to be joined are each given a feather edge. This is effected by cutting each sheet half-way through along a straight line near the margin. On tearing off the strip by pulling on the side of the paper away from the knife cut, the paper is brought to an edge, and these edges are to be overlapped in mounting.

Pencils.—It is uneconomical to use other than superior pencils. 4H is the most suitable hardness for plotting, but 2H or 3H may be employed for sketching, lettering, etc. A chisel point may be used for drawing straight lines, but for all other work a conical point is required. Points should be kept very sharp by the use of sandpaper, pencil points on compasses requiring similar attention.

Ink.—The ink should be good quality Indian or Chinese ink, preferably prepared fresh each day from the stick. Most bottled waterproof inks, however, prove quite satisfactory, and prepared inks of various colours are often required.

Preliminary Considerations.—*Scale.*—The scale is usually definitely, or at least approximately, fixed from considerations of the extent and purpose of the survey before the field work is undertaken. If a choice remains, the final selection may be governed by the size of sheet required to contain the survey.

Position of Survey on Sheet.—Having drawn a border line giving a margin of 1 in. to 1½ in. round the sheet, the position of the survey, title, scale, north point, etc., within the rectangle must be arranged so that the drawing will present a balanced appearance. Whenever possible, the north side of the survey should lie towards the top of the sheet. The title may be placed either centrally at the top or in the lower right-hand corner. The scale is usually drawn at the bottom.

In the absence of an existing plan of the area, a suitable position of the survey on the sheet is best determined by first plotting the outline roughly on tracing paper to the scale of the drawing. This plot is placed over the drawing, and, when in a suitable position, the direction of the principal survey line and the position of one of its stations are pricked through. If the preliminary plot is prepared on a smaller scale, a suitable position must be estimated within a border rectangle similarly reduced.

The Pencil Plot.—*Plotting Survey Lines.*—The longest line is first drawn, and its total length, as well as the positions of intermediate stations, is carefully scaled. Stations are marked with a prick or a fine conical pencil point, and the marks are rendered conspicuous by having a small circle sketched round them. The triangles attached to the base line are then erected by describing short intersecting arcs with the lengths of the sides as radii, beam compasses being used if necessary. When these triangles are checked by fitting in the proof lines, the remaining stations are similarly laid down. The whole framework must be plotted and checked.

before the filling in of detail is begun. As the accuracy of the finished drawing is largely dependent upon the precision with which the framework is plotted, care should be exercised in scaling dimensions, and pencil lines should be very fine.

If the main points are to be plotted from rectangular co-ordinates, and the plan is to cover a large sheet of paper, it is well to commence by drawing a square "grid," so as to divide the sheet up into a number of squares of 3 to 6 in. side. Each point should then be plotted with reference to the bottom left-hand corner of the square in which it falls, so that this corner virtually becomes a local origin of co-ordinates. The reason for doing this is that the paper is liable to expand and contract slightly while work is in progress, and plotting from a grid square in this way helps to minimise errors due to this cause.

Plotting Detail.—There are two ways of plotting offsets. In the first, the chainages from which offsets were taken are marked out along the survey line, and the lengths are scaled off at right angles. In plotting long offsets and those which were set out by instrument, a pencil line should be drawn perpendicular to the survey line by set-square, but otherwise it is sufficient to estimate the perpendicularity by eye.

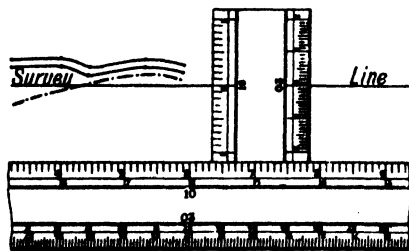


FIG. 157.—USE OF OFFSET SCALE.

In the second method, a short scale, called an offset scale, is used as shown in Fig. 157. The ordinary scale is laid parallel to the survey line and so that (a) the zero of the offset scale coincides with the line, (b) the chainage of the working edge of

the offset scale can be read on the long scale. The long scale being held by weights, the offset scale is slid along to the various chainages, and the offset lengths can be pricked off rapidly.

Where much detail has to be shown, offset points should be joined up as they are plotted, and in doing so it should be remembered that changes of direction occur only at the offsets. The more common mistakes in plotting detail are: (a) plotting offsets from wrong points; (b) plotting offsets on wrong side of survey line; (c) omitting offsets; (d) joining up wrong points; (e) scaling chainages from wrong end of line.

Inking In.—When inking in the drawing, the pencil lines must be followed exactly. Lines should be rather finer than are suitable for structural or mechanical drawing on account of the smaller scales, but very fine lines should be avoided as they often present a ragged appearance and become indistinct when the plan gets dirty. Shade lining may be adopted in representing buildings. A writing pen

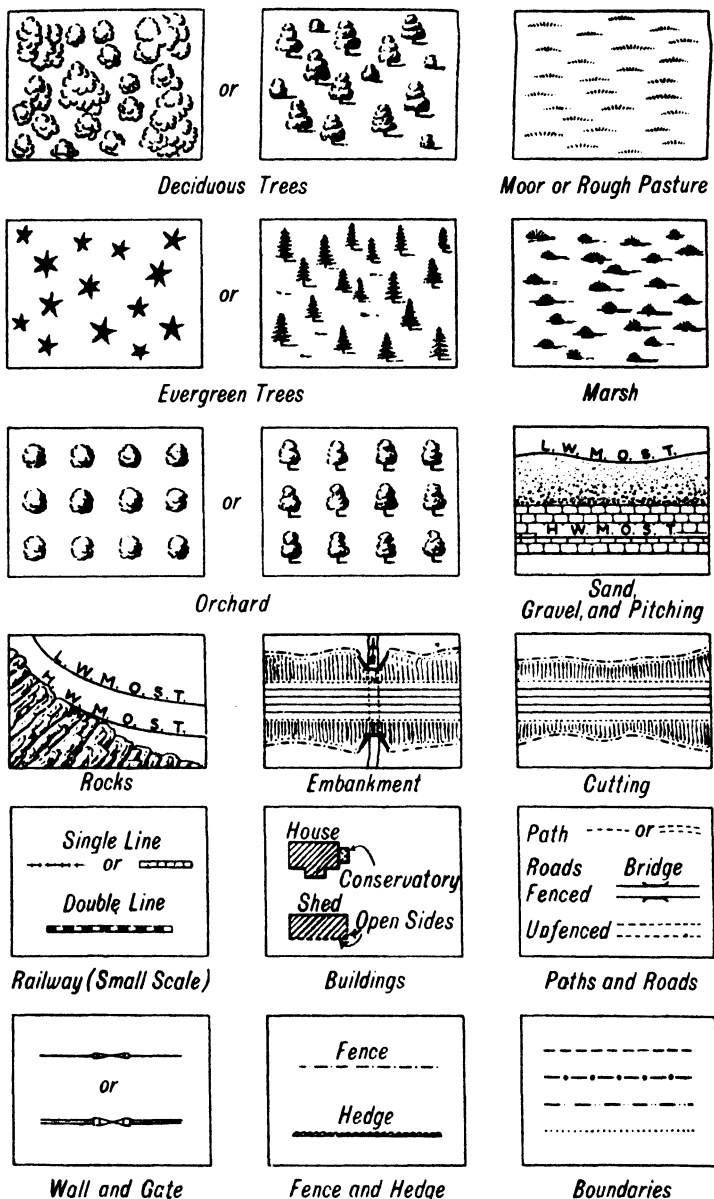


FIG. 158.—CONVENTIONAL SYMBOLS.

should never be used in place of the drawing pen, as the lines produced have a varying thickness.

Black ink being used for existing features, new works are distinguished by being shown in red. On large scale plans railway lines are often drawn in blue. Telegraph poles may be joined by a thin blue or green line to indicate the line of wires. Survey lines are not inked in, but it is sometimes a convenience to have the positions of stations recorded, say, by a small red circle.

Conventional Symbols.—Symbols, more or less standardised by custom, are employed to suggest various features. The form taken by some of these depends upon the scale: thus, on a small scale it is usual to represent fences, hedges, and walls by a solid line, but on a large scale these are differentiated. Fig. 158 illustrates the more common symbols used in ordinary work. The convention of showing trees in elevation instead of in plan is generally favoured, as it is easier to make the result look effective: the trunks are sketched parallel to the side borders of the drawing. In large scale plans of railways, the running edge, not the centre line, of each rail is represented. A list of symbols relating to signals and other details of railway track has been drawn up by the British Engineering Standards Association, and forms the subject of their Specification No. 376, Part I (1930).

Colouring.—The application of large washes of colour to survey plans is unnecessary, and is open to the objection that it distorts the paper. Roads should be shown up in pale burnt sienna, water is indicated by an edging of Prussian blue, preferably shaded off, and buildings should be given a wash of grey, formed of very much diluted black ink. Boundaries have commonly to be made conspicuous by edgings of colour. Tints should be prepared from the best cake colours only, and should be applied with a large sable brush.

Lettering, etc.—Where appearance is important, neat hand printing is preferable to stencilling. To ensure proper spacing, letters should first be sketched in with pencil between top and bottom guide lines. Only severely simple styles of lettering are required, and the beginner should practise these either from a lettering copy-book or from well-finished drawings. Block characters, in the form of upright or sloping capitals and sloping smalls, are most commonly used, and, on account of the uniform thickness of the lines forming them, are much easier to draw than Roman characters. The title should be formed of upright capitals, while the use of upright and sloping capitals and italics of various sizes imparts sufficient variety to the appearance of the notes throughout the drawing. The size of letters should accord with the size of the sheet and with the scale, and should indicate roughly the relative importance of the descriptions. As far as possible, lettering should be parallel to the top and bottom

borders ; otherwise, descriptions should be capable of being read from the lower right-hand corner of the sheet.

The scale must always be drawn, and its value should be written above it. A plain north point should be placed in a convenient position, its direction being obtained from a field observation of the magnetic bearing of any of the lines.

Reducing and Enlarging Drawings.—To reproduce a drawing on a different scale from the original, the most accurate method is to replot the work from the field notes. It is especially desirable to adopt this method for enlargements, in preference to utilising the original drawing, in order to avoid multiplying the errors of plotting of the latter.

For reductions, or enlargements only slightly greater than the original, a simpler method is to rule a network of squares on the original drawing or on a sheet of tracing paper covering it. On the new sheet a similar series of squares is drawn, the linear dimensions of the latter being greater or smaller than those of the first, according to the enlargements or reduction required. By reference to the sides of the squares, the detail may be transferred from the one sheet to the other by scaling, by means of proportional compasses, or simply by estimation.

The reduction or enlargement of drawings may also be performed entirely by mechanical means by the use of either the pantograph or the eidograph. These instruments are, however, not extensively employed, and modern practice tends more and more to use photographic methods for both reductions and enlargements. Several different cameras and outfits, specially designed for this class of work, are now on the market and there are also numerous firms who undertake copying, reducing and enlarging drawings at reasonable rates. When preparing drawings which are likely to be photographed, it is well to take special precautions with inking in, as sometimes lines in which the ink is not very dense do not photograph well. Also, when drawings are to be enlarged by photography, particular care should be taken with the lettering, because any imperfections become magnified and so may be unsightly or look untidy in the enlarged copy.

Use of Metal Plates as a Substitute for Drawing Paper.—In order to avoid the uneven distortion to which drawing paper is liable, the Ordnance Survey now plot all their new plans and maps on flat metal plates with a specially prepared enamelled surface which takes ink and on which small erasures and corrections can easily be made. Details of these plates and of the special plotting boards used to take them in the field will be found in articles on "Improvements in Reproduction Methods of the Ordnance Survey" and "The 1/1,250 Re-Survey of Great Britain" in the *Empire Survey Review*, Vol. X, No. 75 and Vol. IX, No. 68.

CHAPTER IV

THEODOLITE AND COMPASS TRAVERSING

A **TRAVERSE** survey is one in which the framework consists of a series of connected lines, the lengths and directions of which are measured. The system of fix involved is therefore that of Fig. 1c. When the lines form a circuit which ends at the starting-point, the survey is termed a closed traverse : otherwise it is unclosed. The latter type is applied in the survey of long strips of country, but for wide areas the survey usually takes the form of a network of closed traverses, with or without unclosed figures in parts.

Scope.—There is a considerable range in the character of traverse surveys according to the instruments used and the degree of accuracy necessary. In countries unsuited for triangulation, extensive theodolite traverses are required for the establishment of control points from which subsidiary surveys for the mapping of detail may proceed. In such primary traverse work great refinement is called for in both angular and linear measurements, and in certain countries the standard of accuracy now expected, and regularly attained, is of the order of about 1 in 70,000 to 1 in 100,000, a standard comparable with that of modern primary triangulation. In small surveys, particularly if they form closed figures, or if they are run between points whose relative positions have been otherwise determined, the precision aimed at may be considerably less, and at the bottom of the scale we may have, as in certain exploratory surveys, distances estimated from the rate of march, and directions taken by compass towards sound signals instead of to visible points.

In the present chapter the methods ordinarily used in general cadastral and large scale engineering surveys are considered, the applications of traversing in small scale mapping being dealt with in Vol. II, Chap. VII.

Comparison with Chain Surveying.—Angular surveying is of much wider applicability than linear surveying, and is susceptible of greater accuracy, while the field work can generally be more rapidly executed. In many cases, little or no detail has to be surveyed during the measurement of a theodolite traverse, as the latter may be intended merely to supply fixed points to be used either for setting out purposes or to form a framework on which the detail survey may be based. Otherwise the routine of chaining and offsetting has to be performed just as in chain surveying, but

the arrangement of the survey lines is quite different. The running of lines from which no offsets can be taken, as is frequently necessary for the plotting and checking of a chain survey, is obviated, and the lines can be arranged to follow the detail. The employment of angular instruments is essential in the many cases where it is impossible, on account of the shape of the survey, the character of the ground as to obstacles, etc., to lay out a satisfactory triangular system of chain lines.

BEARINGS

Direction.—The directions of survey lines may be defined (a) relatively to each other, (b) relatively to some reference direction, or meridian. In the first case they are expressed in terms of the angles between consecutive lines, and in the second by bearings.

The use of a meridian of reference from which the directions of survey lines may be measured has so many advantages, particularly regarding the facilities afforded for checking and plotting, that this system is adopted in preference to the other. Only in small closed surveys involving few instrument stations should it be regarded as permissible to dispense with the establishment of a meridian.

The reference direction employed may be one of the following :

- (a) True meridian.
- (b) Magnetic meridian.
- (c) Grid meridian.
- (d) Any arbitrary direction.

True Meridian.—The true or geographical meridian passing through a point is the line in which the earth's surface is intersected by a plane through the north and south poles and the given point. It therefore lies truly north and south. The determination of its direction through a station involves astronomical observations, and is described in Vol. II, Chap. II.

The meridians converge from the equator to the poles, and consequently the true meridians through the various stations of a survey are not parallel to each other. All the survey lines, however, are to be referred to one meridian, *viz.* that through the initial station, or station at which the meridian has been established. The bearings of line situated east or west of the initial station therefore differ from their azimuths, or directions from their respective local meridians. In consequence, a line common to two adjoining surveys is usually designated by different bearings in the two surveys. For ordinary small surveys the discrepancy is slight, and, when necessary, the correction for convergence (Vol. II, Chap. V) can be applied.

The direction of true meridian at a station is invariable, and a record of true bearing therefore assumes a permanence not

otherwise possible. This is a matter of considerable importance for large surveys in unmapped or imperfectly mapped country. In engineering location surveys the adoption of true meridian may save much time in retracement of the lines during final location and construction, more particularly if the ground is rough or densely wooded. For small surveys, on the other hand, true bearings need be used only if they can be measured from a meridian already established.

Magnetic Meridian.—The magnetic meridian of a place is the direction indicated there by a freely floating and properly balanced magnetic needle, uninfluenced by local attractive forces. Magnetic meridian does not coincide with true meridian except in certain localities, and the horizontal angle between the two directions is termed the *Magnetic Declination*, or declination of the needle. The amount and direction of the declination is different at different parts of the earth's surface: in some places the needle points west, and in others, east of true north. Its value at any place may be determined by making observations for true meridian, or may be interpolated approximately by reference to published isogonic charts.

Isogonic lines, or isogons, are imaginary lines passing through points at which the magnetic declinations are equal at a given time. Those through places at which the declination is zero are termed agonic lines. Across Great Britain and Ireland the value of the magnetic declination in 1957 ranges from about $7\frac{1}{2}^{\circ}$ W. at Dover to about 14° W. in the west of Ireland, and the isogons have a bearing of roughly 15° to the east of true meridian.

Variation of Declination.—The declination at any place is not constant, but is subject to fluctuations, which may be divided into: (1) regular or periodic variations; (2) irregular variations.

(1) This class of variation may itself be analysed into several components of different periods and amplitudes, but only two of them, secular and diurnal variation, are sufficiently pronounced to merit attention by the surveyor.

(a) *Secular Variation* is a slow continuous swing having a period of several centuries. Thus, at London, previous to about the year 1657, the declination was easterly and decreased annually. In 1657 the needle pointed towards true north. Thereafter the declination gradually increased westwards until 1819, when a maximum westerly declination of about $24\frac{1}{2}^{\circ}$ was attained. Since then the declination west has been decreasing until at present, June, 1957, its value at Greenwich is $8\frac{1}{4}^{\circ}$ W., with an annual decrease of $7\frac{1}{2}'$, which itself has a tendency to diminish slightly. At Dover and at Valencia, in south-west Ireland, the present values of the declination are 7.3° W. and 13.6° W. respectively.

While in this country the magnetic meridian has, within the recorded cycle, moved from one side of true meridian to the other;

similar records in other places show that a complete swing may be performed on one side of the meridian and that the range and period of the oscillation vary in different localities. The change produced annually by secular variation amounts in different places to from 0 to about ± 12 min., but does not remain constant at any place. The annual change is greatest near the middle point of the complete cycle, and is least as the extreme limits are reached.

(b) *Diurnal Variation* is an oscillation of the needle from its mean position during the day. The amount of this variation ranges from a fraction of a minute to over 12 min. at different places, being greater in high latitudes than near the equator, and more in summer than in winter at the same place. In the northern hemisphere, the needle is east of its mean position during the night, and attains its maximum easterly position at about 8 a.m. It then moves westwards, and is farthest west from its average position at about 1 p.m. The mean position of the magnetic meridian occurs in this country at about 10 a.m., and again between 6 and 7 p.m. The direction of the daily swing is reversed in the southern hemisphere.

(2) *Irregular Variations* are caused by magnetic storms. A variation within a quarter of an hour of as much as 5° has been recorded, but this is very exceptional, and variations exceeding 1° are rare.

On account of the secular variation it is always well, when magnetic bearings are used in a survey, to note on the plan the date of the survey, and, if possible, the magnetic declination and its annual variation on that date. Thus: "Magnetic Declination (February 1958) $11^\circ 35'$ W. Decreasing $7\frac{1}{2}'$ per annum."

Grid Line Meridian.—In some countries the government survey and official plans and maps are based on one or more basic geographical meridians, so chosen that they are either centrally situated with respect to the whole country, or else are the central meridians of definite belts, bounded on either side by other geographical meridians. Thus, some of the older maps of Great Britain are based on a number of different meridians, practically one for each county, but steps are now being taken to replace these local meridians of reference by a single one—the meridian 2° W.—so that the whole country is contained within strips, each 3° of longitude in width on either side of this meridian. Similarly, Nigeria, for instance, which extends over 12° in longitude, is divided into three belts, each 4° wide and each with its own basic meridian in the centre. In cases such as these, the maps generally have a rectangular "grid" plotted on them, and the north and south lines of this grid are parallel to the line representing the central meridian. Bearings referred to, and reckoned from, this central meridian or grid line are called "grid bearings," so that, at any particular place, the true meridian at that place is inclined at a small angle to the

grid line. The positions of definite points are defined in terms of rectangular co-ordinates in which the central axis is the central meridian. Consequently, if bearings are computed directly from the rectangular co-ordinates, the result is a "grid bearing" and not the true bearing through either of the terminal points of the line.

In general, it is an easy matter to compute the "convergence" between the grid bearing and the true bearing, or azimuth, at any point, although the formulæ differ slightly for the different systems of co-ordinates in use in different countries.

Arbitrary Meridian.—For small surveys, especially in mapped country, any convenient direction may be assumed as a meridian. This artificial meridian is usually the direction from a survey station either to some well-defined and permanent point or to an adjoining station. It is desirable that its magnetic bearing should be known. An arbitrary meridian has the merit of being invariable, and its direction can be recovered when required if the station or stations defining it are permanently marked or fixed by ties from permanent objects. If it is subsequently found necessary, the bearings can be converted to true bearings by the establishment of a true meridian.

Designation of Bearings.—Bearings are specified on either of two systems of notation: (a) the whole circle system; (b) the quadrantal system. In the whole circle or azimuth method, bearings originate from north, which is marked 0° or 360° , and are measured clockwise from the meridian, through E., or 90° , S., 180° , and W., 270° . In the quadrantal system, they are numbered in four quadrants, increasing from 0° to 90° from N. to E., S. to E., S. to W., and N. to W. Thus, if O (Fig. 159) is a survey station, and ON the meridian through it, the bearings of the lines from O are:

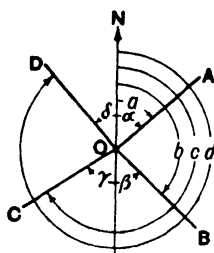


FIG. 159.

Line.	Whole Circle bearing.	Quadrantal bearing.
OA	a	N. α E.
OB	b	S. β E.
OC	c	S. γ W.
OD	d	N. δ W.

Note.—Quadrantal bearings are never reckoned from the E. and W. line, so that the letter which precedes the figure must be either N. or S.

Comparative Merits of Whole Circle and Quadrantal Reckoning.—In the whole circle method a bearing is completely specified by an angle, and the convention of reckoning clockwise from N. is so simple that the noting of the cardinal points as required in the quadrantal method must strike one as unnecessary trouble. The former system lends itself to the measurement of bearings on a

continuously graduated circle as fitted to the theodolite and some compasses. The circles of many compasses are, however, divided in quadrants, and in using such instruments quadrantal reckoning is naturally adopted. The fact that quadrantal bearings never exceed 90° is an advantage in extracting the values of their trigonometrical functions from ordinary tables, but the alternate clockwise and anti-clockwise direction of increase of angle in the different quadrants is sometimes inconvenient and may very easily lead to mistakes being made. Thus, if the true bearings of Fig. 159 have to be converted to magnetic bearings, given that the declination of the needle is 16° W., each of the whole circle reckonings has to be increased by 16° (subtracting 360° if the result exceeds 360°), but in the quadrantal method the correction is positive in the 1st and 3rd, and negative in the 2nd and 4th quadrants. Care must be exercised that the appropriate cardinal points are applied to the resulting figures.

Back Bearings.—The bearing of a line designated by the stations between which it lies is to be taken as that from the first station mentioned. Thus, in referring to the bearing of OA (Fig. 159), the sign of the direction is from O towards A. The bearing from A to O is termed the back, or reverse, bearing of OA, which latter may be distinguished as the forward bearing. By referring to a parallel meridian through A, it will be evident that the back and the forward bearings of the line differ by 180° .

In whole circle reckoning, the back bearing of a line is obtained from the forward bearing by applying 180° . To apply 180° , add when the given bearing is less than 180° , and subtract when the given bearing exceeds 180° . In the quadrantal system, it is only necessary to change the cardinal points by substituting N. for S. and E. for W., and *vice versa*. Thus, if the bearing of a line AB is observed as 300° , or N. 60° W., the back bearing of AB, or the forward bearing of BA, is 120° , or S. 60° E.

Reduced Bearings.—In finding the values of the trigonometrical functions of a whole circle bearing exceeding 90° , one must refer in the tables to the corresponding angle, less than 90° , which possesses the same numerical values of the functions. This angle is called the reduced bearing, and is that between the line and the part of the meridian, whether the N. or S. end, lying adjacent to it. It is therefore the angle used in the quadrantal reckoning of bearings.

To Reduce Whole Circle Bearings.—The following rule is applied :
If the bearing lies between 0° and 90° , no reduction is required.

"	"	90° and 180°, subtract it from 180°.
"	"	180° and 270°, subtract 180° from it.
"	"	270° and 360°, subtract it from 360°.

It follows that the reduced bearing of a line lying due N. or S. is 0° , while that of a line due E. or W. is 90° .

FIELD WORK OF THEODOLITE TRAVERSING

Field Party.—The minimum party consists of the surveyor and two chainmen, and this is sufficient for small surveys, if no clearing is necessary. In large theodolite surveys, particularly where rapid progress is important, the party may consist of a chief, an assistant surveyor or instrument man, a note-keeper, two or more chainmen and a number of labourers, the number depending upon the object of the survey, the nature of the ground, and, in forest or bush country, the amount of clearing required.

With a large party, the chief of party directs the survey and, in particular, reconnoitres the forward ground, fixes the position of stations and of permanent marks, and sees to their being properly pegged and flagged. The assistant surveyor or instrument man, assisted by the note keeper, is responsible for the angular observations and notes and may also control the main taping. The labourers clear the lines of all bush and obstacles, fetch and carry poles, drive and make pegs, erect signals and permanent marks, support and carry the tapes, if long ones are used, and do any other unskilled or semi-skilled work required. In forest country, an ordinary traverse party may easily number between twenty and thirty men, the whole party being split up into a number of small gangs or sub-parties, each with its own particular work to do.

Equipment.—For the linear measurements, the equipment is much the same as for chain surveying (page 140). In theodolite traversing, the steel tape is to be preferred to the wire chain because the errors arising from the use of the latter are much larger than those arising from angular measurements made with even the smallest theodolite. When the steel tape is used, the equipment may also include spring balance, chaining forks, tape clips, and thermometer.

The theodolite should be a transit of 4-in. to 6-in. size, with either vernier or microscope reading, and it is sometimes useful to carry a prismatic compass for use on minor subsidiary traverses, where the same degree of accuracy as that needed on the main work is not necessary.

By including traverse or trigonometrical tables in the miscellaneous equipment, the accuracy of closed circuits, when these only consist of a very few sides, can be tested before leaving the field. As a general rule, however, computations will be done at night on the return of the party from the field.

Balancing the Accuracy of the Linear and Angular Measurements.—In deciding on the equipment necessary, and the methods to be used for both linear and angular measurements, it is essential to bear in mind the standard of accuracy desired, and this is controlled by the object for which the survey is needed. When this point has been settled, the equipment and methods chosen should be such as

to make the relative accuracy of the linear and angular measures about the same, and, in doing this, it is useful to remember the ratio of the linear displacement at the end of a line, subtended by a second, a minute, and a degree of arc, to the length of that line.

Thus :

1 Second of arc corresponds to a displacement ratio of 1 : 206,300.

1 Minute of arc corresponds to a displacement ratio of 1 : 3,440.

1 Degree of arc corresponds to a displacement ratio of 1 : 57.

While remembering these figures, it must not be forgotten that angular errors tend to propagate themselves along a traverse, not directly as the number of stations, but as the square root of the number of stations, a point that will be referred to later, while, as we have already seen, errors arising from the linear measures tend to be roughly proportional to the length of the line.

Selection of Stations.—If the traverse is being surveyed with the primary object of picking up detail, considerations of easy chaining and short offsets should, as in linear surveying, be given due weight. If, on the other hand, it is needed as a means of establishing control points, or for other work in which a relatively high degree of accuracy is required, the primary consideration should be to avoid short legs and to obtain as long sights as possible. In addition, it is well to endeavour to keep the lines of sight as high above ground level as possible and to avoid "grazing rays," or rays which come very close to the surface of the ground, as, otherwise, shimmer or horizontal refraction, or both, will cause minor inaccuracies in observing the angles. The ground at stations should be firm to afford an unyielding support for the instrument, and should be moderately level rather than on a steep slope. Unless in exceptional circumstances, stations should not be established in situations, such as on roads, streets, or railways, where the observations will be delayed, and the instrument possibly endangered, by passing traffic.

It sometimes happens that difficulties on the ground prevent long legs being chosen for the linear measurements, but it is possible to sight a forward station some distance ahead on the line that the traverse has to follow. Thus, in Fig. 160, it is impossible, or at best

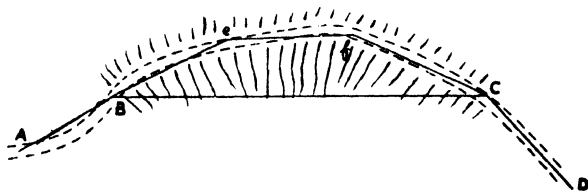


FIG. 160.

difficult or inadvisable, to carry the taping direct along the line BC, but B and C are intervisible. In this case, the taping line is

made to follow the line $BefC$. The angles at e and f are measured, as well as those at B to both e and C and at C to both B and f . The portion $BefC$ is then treated as a subsidiary traverse to determine the length, and the length only, of the line BC , so that main bearings are brought forward along $ABCD$, which is now the main traverse, with a computed length for the line BC . In this way, errors in bringing forward bearings through the short legs Be , ef , and fC are avoided. Such a lay-out of a traverse line, where the taping line departs or deviates from the main angular line, may be called a "deviation."

In other cases, schemes of combined traverse and triangulation may sometimes be used to cover the ground quickly; if necessary, the bases for the triangles being short traverses. A "traverse base" is one in which the distance between the terminal points is computed from a traverse run between them and it is sometimes used when the nature of the ground makes it impossible or inconvenient to measure a straight base. Much ingenuity may often be exercised by the surveyor in avoiding short traverse legs or in working round, or over, obstacles. Some of the methods available are dealt with in detail in the miscellaneous problems at the end of this chapter.

Marking of Stations.—Stations may be marked by pegs as described on page 180, but more permanent marks are often required for some or all of the stations of a traverse. These may take the form of a bolt or spike set in a block of concrete or a large stone. Reference marks to aid in the recovery of the station point should be established in its vicinity, their distance and bearing from the station being measured. The notes should include a detailed description of the site of each station with particulars of the reference marks.

Signals at Stations during Observing.—When observing angles with the theodolite, the signals to which observations are taken can be of various kinds which vary according as to whether legs are long or short. If legs are long, the signal may consist of a ranging pole, with a red and white flag at the top, carefully plumbd over the station mark and held in position by a labourer or supported by light wire stays or wooden struts. In the case of short legs, it may be possible to see, from the theodolite, the tack or mark in the centre of the peg or pillar, and, if this is so, the sight can be taken direct to the mark. If this is not possible, an arrow, held with its point on the mark, may be visible. For legs that are not too long, a very good mark is a plumb bob with a piece of white paper threaded on the string, the sights being taken to the string and the paper serving merely to enable the position of the string to be found easily in the field of the telescope. The plumb bob can be kept fairly steady if it is suspended from a long pole, held firmly, at a slight inclination to the vertical, and with its lower end on the ground. Otherwise, it may be hung from a "bush tripod," made by lashing three poles

together and fixing them so that they stand over the mark in the form of a tripod.

In all cases in traverse work, particular care should be taken to see that the signal is properly centered and plumbed over the station mark, and it should be sighted as low down as possible. Lack of proper care in plumbing signals is a frequent cause of minor error.

Special Equipment for Observing Short Legs.—Very short traverse legs are often unavoidable in surveys in mines and tunnels, and in such work it is essential to avoid large centering errors. Some makers therefore provide a special equipment for this kind of work which consists of a theodolite, three or more tripods, and at least two special targets which fit on the tripods and are interchangeable on them with the theodolite. When observations at a station are complete, the theodolite is taken off its tripod, without disturbing the latter, and then moved and placed on the tripod marking the forward station, from which the target has been removed. Meantime, the target at the rear station is taken off its tripod and placed on the original theodolite tripod, the forward target being moved on and placed on a tripod at the next forward station. In this way, centering errors are reduced to a minimum since the vertical axes of the theodolite and targets always occupy the same positions on the tripods. The targets are all provided with some sort of artificial illumination because in mines and tunnels work has to be done in darkness or in very subdued light. (See Fig. 388.)

A system of this kind has been used in the survey of precise traverses in the Gold Coast and Nigeria, about six tripods and five or six targets being in use at the same time. Several tripods and targets are set up and centered ahead of the surveyor by labourers specially trained to do this. Consequently, not only are centering errors almost entirely eliminated, but also, with sufficient labour and equipment, work proceeds very rapidly, as the surveyor has no fine centering to do at each station and has only to set his theodolite on a tripod and give it a final levelling.

In the equipment provided by some makers the theodolite is easily detachable from the levelling head, and a levelling head is provided with each target as well as with the theodolite. All levelling heads are of exactly the same pattern and size so that either the theodolite or a target can be fitted into each. This form of equipment is generally to be preferred to that in which the theodolite and targets have to be screwed on the tripod and not merely placed on a levelling head waiting to receive them.

Order of Field Work.—If the party is small it is preferable in minor surveys to complete the observation of the angles or bearings before taping is begun or vice versa. A full party on long traverses, however, is divided into two main groups; the first selects stations, clears the lines, erects permanent marks, and does the check taping, while the other follows and observes angles and does the main taping.

ANGULAR MEASUREMENTS

Up to fairly recent years, it has usually been taken for granted that, in theodolite traversing, the angular measures were almost invariably more accurate than the linear ones. The long steel tape, however, when properly used, can be an instrument of precision, and, if lines are measured in catenary, as already described in Chap. II, the accuracy of the linear may exceed that of the angular work, especially if a vernier instrument is used. In modern precise first order work, intended for primary framework, and in which invar tapes are used instead of steel ones, the tendency is for the linear work to be more accurate than the angular, and hence special precautions have to be taken with the angular observations. Even in ordinary engineering or cadastral surveying, in which the steel tape is used, it cannot be assumed that angular measurements can be made "just anyhow," and reasonable care with them is essential if a sound balance is to be struck with the linear work.

The methods of measuring the angles and bearings of a traverse may be divided into two classes :

(a) Those in which the angles at the different stations are measured directly, and the bearings subsequently calculated from the measured angles and the given bearing of an initial line.

(b) Those in which the setting of the theodolite is so arranged as to give direct readings of bearings.

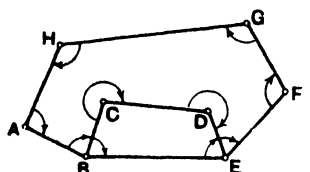
As a general rule, the first method is to be preferred and is the one most generally used for long traverses, or where precision is required, while the second may be used for short traverses where great precision is not necessary and the traverse is either a closed one or ends on a line of known bearing.

Theodolite Observations by Direct Observation of Angles.—The horizontal angles measured at the several stations may be either (a) included angles, (b) deflection angles. An included angle is either of the two angles formed at a station by two survey lines meeting there. A deflection angle is that which a survey line makes with the preceding line produced beyond the station occupied, and its magnitude is the difference between the included angle and 180° .

The minimum routine to be adopted for the observation of included angles is given on page 86, and it is often sufficient in small surveys when the linear measurements are made by chain. When, however, it is decided that considerations of accuracy demand the use of the steel tape, each angle should be read "face right" and "face left" (page 92), and both verniers or micrometers should also be read, not only to minimise the effects of instrumental error but also to provide a check against mistakes in reading. In the better classes of work, it is usual not to endeavour to set the instrument to read 0° or 360° exactly when the instrument is sighted on the back station, but to set it somewhere near that value and then,

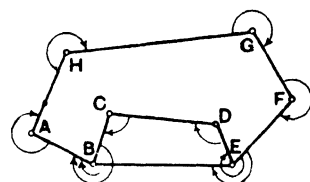
after the mark has been intersected with the cross-hair, to read both verniers. The mark at the forward station is then sighted and intersected and both verniers again read. This gives one value for the angle. For the next observation, on the other face, it is as well to "change zero," before the back station is resighted, by setting the verniers to read somewhere near 90° and 270° . Face having meantime been changed, the instrument is directed to the back station and the angle measured again. Changing zero in this way not only tends to avoid errors in measurement, since the two values of the angle are derived from two entirely different sets of figures, but it also tends to eliminate errors due to small periodic errors of graduation in the horizontal circle.

Included angles can be measured clockwise or counter-clockwise from either the back or the forward station, but it is well to adhere to the regular routine of measuring from the previously occupied station and in a clockwise direction, since the graduations of the theodolite circle increase in this direction. This does not necessitate that the telescope should always be turned clockwise, although it is better to wheel or swing it in a constant direction for observations on one face and in the reverse direction for observations on the other. Figs. 161 and 162 show that, in a closed polygon, angles measured



Stations occupied in order ABC--

FIG. 161.



Stations occupied in order AHG--

FIG. 162.

clockwise from the back station are either interior or exterior according to the direction of progress round the survey. Interior angles are obtained by proceeding counter-clockwise round the figure, but these will be exterior to subsidiary circuits as at C and D.

In measuring deflection angles, having bisected the mark at the back station by using the lower clamp and tangent screw and read one or both verniers, the theodolite is transited and is then pointed to the forward station, the upper clamp and tangent screw being used for this and to intersect the station mark. The verniers are again read, and the difference between the first set of readings and the second gives the angle of deflection. The measurement is either right- or left-handed from the production of the back line, and this direction must be most carefully noted in the field book. It is usual, when deflection angles are being measured, to set the horizontal circle to read zero when the back station is sighted, so that the reading when the forward station is sighted gives the angle of deflection direct.

Included angles are to be preferred to deflection angles. The latter are often used in surveys for railways, roads, pipe lines, etc., in which a series of traverse lines may make small deflection angles with each other, but they are open to the objections that right- and left-handed angles may be confused in booking or plotting or in working out co-ordinates and that the transiting of the telescope introduces possible errors of non-adjustment if observations are not made on both faces. Moreover, whatever other advantages the method of deflection angles may have, these largely disappear if zero is changed between face right and face left readings. Consequently, for all but entirely specialised work in which deflection angles are usually employed, it is preferable in every way to read and book the angles of a traverse as the included angle, read clockwise from the back station.

Booking the Angles.—The angles can be booked in an ordinary field book which may also include the linear measurements. Fig. 116, page 154, will explain the method in the case of observations by included angles, the angular observations being written at the right of the page. With instrument set at face right over station 64 the reading on the circle when the telescope was sighted on station 63, the back station, was $0^{\circ} 15' 20''$ on "A" vernier and $180^{\circ} 15' 40''$ on "B." These readings are set out in two columns as shown, minutes and seconds only being recorded in the case of the "B" vernier readings. The mean angle is $0^{\circ} 15' 30''$ and this is written out in full in a third column. When the instrument, still face right, was sighted at station 65, the forward station, the reading on "A" vernier was $196^{\circ} 32' 40''$, and on "B" the minutes and seconds read $33' 00''$. The mean angle was therefore $196^{\circ} 32' 50''$, and all three sets of figures are written down as shown above the corresponding readings to station 63. Hence, subtracting the figures in the third column, the included angle at station 64, as read with the instrument set face right, was $196^{\circ} 17' 20''$. Face was now changed and the horizontal circle set to a new zero near 270° on "A" vernier. When the back station was sighted, the readings were $272^{\circ} 12' 20''$ on "A" vernier and (again omitting the degrees) $12' 20''$ on "B," the mean being $272^{\circ} 12' 20''$. These are written down as shown. Similarly, when the forward station was sighted, the readings were $108^{\circ} 29' 49''$ and $30' 00''$, the mean being $108^{\circ} 29' 50''$. Consequently, the second value for the included angle was $108^{\circ} 29' 50''$ minus $272^{\circ} 12' 20''$, which, after adding 360° to the first angle, since it is numerically less than the second, gives $196^{\circ} 17' 30''$. The mean value of the included angle was therefore $196^{\circ} 17' 25''$, and this is shown underlined below the readings for the first observation.

In all cases, when observing included angles with the theodolite, all observations should be worked out before a station is vacated, because, if a mistake is made and noticed in time, it can be corrected at once and there is no necessity to have to revisit the station a second time to take new readings.

When using the method of included angles it is most important to avoid readings taken to the forward station being booked in the place usually reserved for booking readings taken to the back station, as otherwise the interior angle will be obtained instead of the exterior one, or vice versa. In the example, it will be noticed that, as it is usual to work up a page of a field book, the first observation taken—that to the back station—is booked at the bottom of the page, with the next observation—that to the forward station—booked immediately above.

Deflection angles can be booked in a somewhat similar manner, but, if the instrument is set to zero every time the back station is sighted and only one face used and one vernier read, the only entry will be the observed deflection angle. The figures *must* be followed in every such case by the letter "R" or "L," signifying a right-hand or left-hand deflection respectively. or else a small sketch made in the book to show clearly which direction the deflection takes.

Obtaining the Initial Bearing.—In order to calculate the bearings of the different lines the bearing of one line at the initial station must be known or assumed. Thus, in Fig. 163, AB is the first line of the traverse and AM a line which is already marked out in some way on the ground and whose bearing is known. If bearings are to be referred to a true meridian, that of AM may be obtained either direct from astronomical observations, or, if the points A and M have been established by the government survey department, on application to the head or local representative of that department. The bearing of AM being known, the angle MAB is measured and from this the bearing of the first line, AB, of the traverse can be calculated.

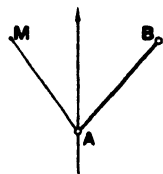


FIG. 163.

Calculation of Bearings from Angles.—Having obtained an initial bearing from which to start, the calculation of the other bearings of a traverse is very simple, and can be formulated as follows:

In Fig. 164 let B be the station at which the angle is measured and let the bearing of the line AB or BA be known. The point B may either be the first station or it may be any intermediate station on the traverse. If it is the first station, the point A will correspond to the initial reference mark M in Fig. 163. If B is an intermediate point in the traverse,

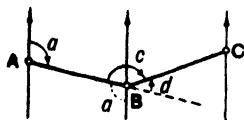


FIG. 164.

BA will be a traverse leg, the bearing of which has already been computed.

Let c = the measured angle ABC reckoned clockwise from A .

d = the deflection angle.

(1) Bearing BC = Bearing $BA + c$,
(subtracting 360° if the result exceeds 360°).

(2) Bearing BC = Bearing $AB \pm d$,
(using the $+$ sign if d is clockwise from AB produced, and the $-$ sign if d is counter-clockwise, as in Fig. 164; and adding 360° if the result is negative, and subtracting it if the result exceeds 360°).

Quadrantal bearings should be converted to whole circle reckoning before applying the formulæ.

In the reverse process the formulæ become:

(3) c = Bearing BC - Bearing BA ,
(adding 360° if the result is negative).

(4) d = Bearing BC - Bearing AB ,
(if d is positive, it is clockwise from AB produced).

When the bearings of a traverse have been calculated, and the bearing of each line obtained, the work should be checked by adding together the initial bearing and all the included angles or deflection angles. For included angles, the result should equal the bearing of the last line plus some multiple of 180° .

As an example, take the following:

Point.	Observed included angle.			Calculated bearing.			Line.
	°	'	"	°	'	"	
M				37	14	10	MA
A	224	15	25	81	29	35	AB
B	210	36	40	112	06	15	BC
C	135	14	10	67	20	25	CD
D	120	08	30	7	28	55	DE
E	167	42	35	355	11	30	EF
F							
	857	57	20				
	37	14	10				
	895	11	30				
3×180	= 540						
	355	11	30				

Here, the bearing of the line MA is $37^\circ 14' 10''$ so that the bearing of the line AM is $217^\circ 14' 10''$. Adding the included angle at A , the sum is $441^\circ 29' 35''$, and, subtracting 360° , the bearing of the line AB is $81^\circ 29' 35''$. Also, the sum of the included angles is $857^\circ 57' 20''$, and this, plus $37^\circ 14' 10''$, the bearing of MA , and less $3 \times 180^\circ = 540^\circ$, is equal to the bearing of the line EF .

The number by which 180° has to be multiplied in order to obtain the final bearing from the sum of the included angles and the initial bearing can be obtained by noting, for each station, the number of additions or subtractions of 180° or 360° . Thus, at A , 180° is added once, and 360° subtracted once, so that the net result is a subtrac-

tion of 180° . There are also similar subtractions of 180° at B, C, and D, and an addition of 180° at E. Hence, in all, 180° has been subtracted 3 times, so that 3 is the multiplier required. This, however, is not a very satisfactory check as far as an error of 180° is concerned, and, as the sum, provided too many angles are not included in it, can be relied on to check the minutes and seconds and the tens and units in the degrees, it is far better to make a rapid examination of the degrees column only. This should show at once whether or not a mistake of 180° has been made.

If deflection angles are used, it is best to tabulate right-hand deflections as positive and left-hand ones as negative. Thus, with the same angles as before but transformed into deflection angles :

Point.	Observed deflection angle.						Bearing.	Line.		
	+			—						
	°	'	"	°	'	"	°	'	"	
M		—			—		37	14	10	MA
A	44	15	25		—		81	29	35	AB
B	30	36	40		—		112	06	15	BC
C		—		44	45	50	67	20	25	CD
D		—		59	51	30	7	28	55	DE
E		—		12	17	25	355	11	30	EF
F										
	74	52	05	116	54	45				
	37	14	10							
	112	06	15							
	116	54	45							
	355	11	30							

Here the plus and minus columns are added, and the sum of the plus deflection angles added to the initial forward bearing from M to A, and the sum of the negative angles subtracted from the result. This gives the bearing of the last line, and this acts as a check.

This method of computing can also be used as a very useful check on bearings computed direct from the included angles by remembering that, if the included angle is greater than 180° , the deflection angle is right-hand or positive and equal to the included angle minus 180° , while, if the included angle is less than 180° , the deflection angle is left-hand or minus and equal to 180° minus the included angle. These are the rules for included angles measured clockwise from the back station, but equally simple ones can be devised for angles measured clockwise from the forward station.

THEODOLITE OBSERVATIONS BY DIRECT OBSERVATION OF BEARING

In this method the theodolite is oriented so that, when the telescope is sighted along an initial line, the reading on the vernier

corresponds with the bearing of the line. Hence, if the lower circle is kept fixed during observations, and the telescope is sighted along some other direction, the reading on the vernier gives the bearing of that direction also.

Three distinct systems of procedure are available :

- (a) Direct method involving transiting of the telescope.
- (b) Direct method without transiting.
- (c) Back bearing method.

In every case, the routine at the initial station is the same and involves setting the instrument to the correct orientation. To avoid repetition in the following descriptions of the observations, it is supposed that a single observation only of each bearing is made, but the observations may be duplicated by being taken with both faces of the instrument.

Orienting on the True Meridian or on a Grid Line.—Here, as in the case of finding bearings when using the method of observed angles, it is necessary that the first station occupied should be one end of a line whose bearing is already known, either from the results of astronomical observations, from the results of a previous survey, or from the data provided by the government survey department.

Referring to Fig. 163, let A be the first station of the traverse, AM the line of known bearing and AB the first leg of the traverse. To orient the theodolite by use of the bearing AM, one of the verniers is first set to read that bearing, and the vernier plate is clamped to the circle. With the lower clamp slack, the telescope is then turned towards M. When M appears near the intersection of the hairs, the lower clamp is tightened, and the line of collimation is brought exactly on the signal by the lower tangent screw. This completes the orientation, since the vernier still reads the true bearing of the line along which the line of collimation is directed. Under these conditions, if the upper clamp is released, and the circle is kept fixed, the telescope will point towards true north when the vernier is set to zero or 360° . To observe the bearing of the survey line AB, the telescope, carrying the vernier plate with it over the fixed circle, is directed to B by the upper clamp and tangent screw. The bearing is then read off on the vernier previously used.

Orienting on Magnetic Meridian.—In orienting from magnetic meridian, reference is made to the compass on the theodolite. The compass box, if of circular form, is mounted on the vernier plate, as in Fig. 53, while the trough pattern is either connected to the vernier plate by being screwed to a standard or is attached to the lower or graduated plate. In the case where the compass is connected to the vernier plate, the line joining the N. and S. graduations in the circular form, or that joining the zeros of the scales in the trough form, bears a fixed relationship to the line of collimation

of the telescope, and is intended to be parallel to it. When the compass is attached to the lower plate, the line of zeros is parallel to the line of collimation only when the horizontal verniers read 0° and 180° respectively.

To orient, with either arrangement, the vernier is first set and clamped to zero, the lower clamp being slack. The needle is then lowered upon its pivot, and the instrument is turned about the outer axis until the N. and S. graduations are brought opposite the ends of the floating needle. The lower clamp is then tightened, and the lower tangent screw is used to bring the zero graduation to exact coincidence with the point of the needle. The instrument is now oriented, since the line of collimation is directed towards magnetic north while the vernier reads zero. To observe the bearing AB, it is only necessary to set the line of collimation on B by means of the upper clamp and tangent screw, and note the reading of the vernier previously set to zero. The result of the observation may be checked by a glance at the reading of the north end of the needle in the circular box form.

Notes.—(1) It is to be noticed that when the needle rests on its pivot, the instrument can be rotated about it without disturbing its direction.

(2) The needle cannot be set properly if it is looked at from one side. The eye should be in the vertical plane of the needle as nearly as possible.

(3) When the plate has been oriented, the telescope may be pointing south, in which case it must be transited before observing bearings.

(4) While the needle of a trough compass is more sensitive than the shorter needle of a circular box, neither is capable of defining the meridian with the refinement with which angular measurements can be made by the theodolite. This is not always important, since in theodolite surveys, as distinct from compass surveys, the bearings of all lines after the first are measured from the bearing of the first line, and the needle is consulted at subsequent stations only as a check. Inaccurate orientation at the first station therefore merely turns the whole survey through a small angle, but does not distort it.

(5) If the value of the magnetic declination is known, the needle may be used for approximately orienting to true meridian by setting off the declination. It is, however, better, as regards facility for applying the compass check, to adhere to magnetic bearings in the field, subsequently converting them to true bearings if required.

(6) It must not be forgotten that, in practice, nearly every compass, whether of the trough or circular variety, has its own individual error, so that the magnetic axis does not coincide with the geometrical axis of the needle, and, in consequence, the magnetic meridian indicated by the instrument is not the true magnetic meridian. Individual compass errors of anything from half a degree to two or three degrees are quite common, and, in any particular instrument, the error may easily undergo a sudden change due to some outside influence upsetting the magnetisation of the needle and so altering the position of the magnetic relative to the geometrical axis. Hence, if absolutely correct orientation is desired, it is necessary to have the compass compared from time to time with a standard instrument or to test it on a line whose true magnetic bearing is known. Such a line can be laid out if a Connelly Standard Compass is available.

Most trough compasses are constructed so that the needle is accessible. Consequently, if a trough compass of this type is used, the error can be found, by using the method described on page 71, and adjusted or allowed for.

Orienting on Arbitrary Meridian.—Having set and clamped the vernier to zero, it is only necessary to sight the object or station defining the direction from A of the adopted meridian. When the signal is bisected by the use of the lower clamp and tangent screw, the circle is oriented. The upper clamp is then released, and bearing AB is observed as before.

Carrying Forward the Bearing.—While the measurements in these methods are virtually those of the angles between the lines meeting at the various stations, the instrument is so manipulated that the required whole circle bearings are read directly on the vernier. In following the methods, it is necessary to distinguish between the two horizontal verniers. They are designated here as 1 and 2, the former being supposed used at the initial station. If they are not given distinctive marks on the instrument, they can easily be distinguished by their positions with respect to the vertical circle or a plate level.

In each of the three methods, after observing the bearing of AB from A, the lower clamp is released, and the instrument is carried to B with vernier 1 kept clamped at the reading obtained. A signal is left at A.

Direct Method with Transiting.—(a) Set up and level the instrument at B. See that vernier 1 still records the bearing AB. If the plate has slipped during transfer of the instrument, correct the reading by the upper tangent screw.

(b) By using the lower clamp and tangent screw, sight back on A.

(c) Transit the telescope. The line of sight has now the same direction as it had at A, and vernier 1 still records the bearing AB. The instrument is therefore oriented.

(d) Bearing BC can now be observed by releasing the upper clamp and sighting C, the upper tangent screw being used in bisecting C. Vernier 1 now records the bearing BC.

(e) This reading being maintained on the vernier, the instrument is transferred to C, and the routine is repeated. On transiting at C, the instrument is returned to the same face as

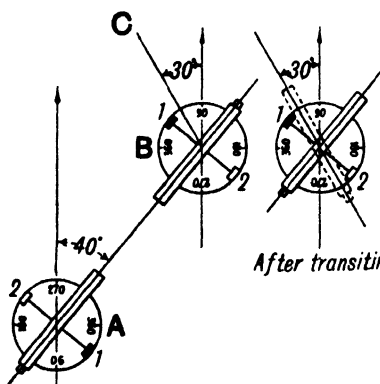


FIG. 165.

Example.—Fig. 165 illustrates the case where the bearings of AB and BC are respectively 40° and 330° . It will be seen that, on taking the backsight BA, the

verniers occupy the same positions relative to the telescope as at A, but, since the telescope is at 180° to its previous direction, the orientation of the circle is 180° different from what it was at A. This is neutralised on transiting the telescope. To bring the telescope into the position shown dotted, in order to sight C, it must be turned counter-clockwise through 70° , or clockwise through 290° , either movement resulting in vernier 1 (shown in black) being brought opposite the reading 330° as required.

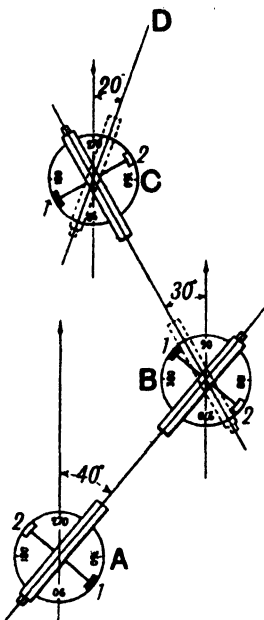


FIG. 166.

Direct Method without Transiting.—The manipulation of the instrument at B is similar to that in the previous method, except that the telescope, instead of being transited after the backsight is taken, is turned directly on to C. The difference of 180° in the orientation of the circle at B from its orientation at A therefore remains uncompensated, and the reading of bearing BC on vernier 1 is 180° out. A correction of 180° has therefore to be applied to the reading or readings taken at B, adding the correction if the observed value is less than 180° , and subtracting if the reading exceeds 180° . At C the orientation, being 360° out, is correct, and the results need no adjustment. The application of 180° is therefore necessary only at the 2nd, 4th, 6th, etc., stations occupied.

Example.—Fig. 166 shows this system applied to the previous case. To sight C from B, the telescope must now be turned through 110° clockwise, or 250° counter-clockwise, and vernier 1 then reads 150° , which falls to be increased by 180° . Following the process to station C, let the bearing CD be 20° . After backsighting on B, the rotation necessary to bring the telescope into the position shown dotted, in order to sight D, is 130° counter-clockwise, or 230° clockwise, and the vernier then reads 20° .

Notes.—(1) Some surveyors adopt the routine of reading opposite verniers alternately to eliminate the 180° correction, but it is simpler to read one vernier throughout and note the corrected values.

(2) Notwithstanding that a number of successive instrument stations may lie in the same straight line, the correction must be applied at alternate stations.

Back Bearing Method.—(a) Set up and level the instrument at B as before.

(b) Before sighting back on A, set vernier 1 to read the back bearing of AB, and fix the upper clamp.

(c) By using the lower clamp and tangent screw, sight back on A. The instrument is now oriented, since vernier 1 records the bearing BA, along which the line of sight lies.

(d) Release the upper clamp, and, without transiting, direct the telescope towards C. Clamp, and adjust by the upper tangent screw. Vernier 1 records the bearing BC.

(e) Apply the same method at all the subsequent stations.

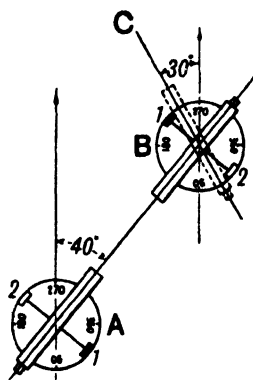


FIG. 167.

Example.—Fig. 167 shows that, on backsighting with vernier 1 set to the back bearing, the circle has exactly the same orientation at B as at A. To turn the telescope into the dotted position for sighting C, the same rotation is required as in the last case, but vernier 1 gives the required bearing directly.

Precautions in Carrying Bearings Forward.—(1) It is necessary to guard against using the wrong clamp and tangent screw. The routine at all stations consists of the two steps: (a) orientation by backsighting; (b) measurement of a bearing or bearings. In backsighting, only

the lower clamp and tangent screw must be used to bring the intersection of the hairs on to the signal, and these must not be touched again until the observations at the station are completed.

(2) If the telescope is set upright in carrying the instrument, it should be restored to its previous position before backsighting. Confusion may arise by inadvertently transiting.

(3) When several bearings are measured from one station, the round of bearings should be completed by observing the backsight a second time in order to detect possible movement of the circle. The observation to the station to be next occupied may then be repeated, so that the vernier may be left at the required reading.

Relative Merits of Methods of Carrying Bearings Forward.—In point of speed there is little difference between the three methods, as the time occupied in the first two in checking the vernier reading before backsighting is not much less than that required for setting the back bearing. The first method is probably the most mechanical, but, if only single observations are made, the transiting of the telescope introduces possible errors of non-adjustment, and in respect of accuracy the others are preferable. On the whole, the second method is the most satisfactory. The necessity for applying 180° at every second station is not likely to lead to error, as an omission to apply the correction is easily traced.

Booking the Bearings.—Bearings may be booked on the right-hand side of the field book in a manner somewhat similar to that already described for booking deflection angles. If only one vernier is read, there will only be a single entry—the observed bearing of the forward line. If both verniers are read, the readings to the back station should be booked as well as the means of these readings. On top of these, the readings to the forward station and their mean should be entered, and, from these results, a correction can be worked out to give the corrected forward bearing.

As the method of direct observation of bearings is usually used only for short unimportant traverses, mainly run for the survey of detail, it generally happens that the party is a small one and that detail has to be surveyed at the same time as the linear and angular measurements are made. In that case, the entries in the field book relating to the angular measures should be kept as low down on the page as possible, so as to leave plenty of room for entries relating to the survey of detail. Alternatively, a special book can be kept for the angular measurements.

The field notes should always include a sketch of the framework of survey lines, roughly to scale, so that the relative directions of the lines may be shown approximately correctly. The stations should be lettered and numbered on the sketch, and references given to the pages of the field books in which the measurements for each line or station are to be found.

LINEAR MEASUREMENTS

Various methods of measuring lengths have been described in Chap. II, and the particular one to be adopted should be chosen to suit the degree of accuracy required and the type of instrument available for the angular work. In general, however, a steel tape, and not a wire chain, is used in theodolite work, except, possibly, in the case where bearings are measured on one face of the instrument only.

Survey of Detail.—Owing to the ease with which mistakes are made in traversing and the resulting need for concentration on the essential observations, it is advisable, on long or important traverses, not to throw any more subsidiary work on the main observing party than is absolutely necessary. Consequently, the survey of detail, the greater part of which can be done by offsets, should generally be done by the party which does the setting out and check taping. On the other hand, when the traverse is only a short minor one and the party is a small one, the detail survey can be done by it at the same time as the angles or bearings and distances are measured. For this, the radiation, or angle and distance, method is often useful for the survey of detail near instrument stations. In this, bearings to the various points are observed, and the lengths of the radial lines from the instrument are measured. Unimportant detail

and distant and inaccessible points may be fixed by the intersection of bearings from two instrument stations.

In city surveying, where there is a large amount of detail, the best procedure is to survey only the frontages of the buildings during the running of the traverse lines. The miscellaneous detail can be subsequently located from the buildings and from subsidiary traverse lines projected where possible towards the back of the main buildings. This system is particularly advantageous if the main survey has to be executed during the night, when traffic is suspended.

SOURCES OF ERROR IN THEODOLITE TRAVERSING

Errors of linear measurement are dealt with on page 166. Those to which the angular observations are liable may be treated as :

- (1) Instrumental errors ;
- (2) Errors and mistakes in setting up and manipulating the instrument ;
- (3) Observational errors ;
- (4) Errors due to natural causes.

1. Instrumental Errors.—The effects of residual errors of adjustment, as well as of non-adjustable errors, can be satisfactorily reduced only by the adoption of a system of multiple observations, either on the repetition or direction principle (Vol. II, Chap. III). In employing any of the methods which have been described above for carrying forward the bearing, it is advisable to measure each bearing twice, face right and face left, and to read the opposite verniers at each observation. It is unusual to take more than one face right and one face left observation in ordinary traversing ; but, if the work is required to be of a precise character, and suitable precautions are taken in the linear measurements, a greater number of angular observations are taken for averaging. In such a case, the included angles between the lines should be measured, the bearings being deduced from the average values. In thus endeavouring to obtain results of superior accuracy with a small theodolite, the necessity for rigidity and stability in the instrument and tripod must be recognised.

2. Errors of Manipulation.—(a) *Defective Centering.*—If the centre of the instrument is not vertically over the station point, the angle or bearing is not measured from the point to which it is presumed the foresight has previously been taken. The angular error produced depends upon the error of centering, the lengths of sights, the magnitude of the angle being measured, and the position of the instrument relatively to the station and the points sighted, no error being introduced if the four points are concyclic. Under the worst

circumstances, an error of centering of about $\frac{1}{8}$ in. may produce an error of 1 min. in the angle measured if the sights are only 100 ft. long, but centering would have to be at least 9 in. out to produce a similar error with sights a mile long. Considering the ease with which centering may be performed, the error is not likely to be appreciable in ordinary work, except in very short courses (cf. page 355).

(b) *Defective Levelling*.—Careless levelling, particularly of the plate level perpendicular to the telescope, may be productive of more serious error than the last, especially if the points sighted are at considerably different elevations. In the absence of a striding level, the trunnion axis of the telescope is made horizontal by reference to this plate level. Although the plate levels are in adjustment, and the levelling of the instrument has been carefully performed, unequal settlement may cause the bubbles to move out of centre, and, if the instrument is not relevelled, the dislevelment of the horizontal axis introduces error whenever the inclination of the telescope is changed.

Let θ be a small angular error of dislevelment of the horizontal axis, or the inclination to the vertical of the plane in which the line of sight travels when the telescope is elevated or depressed. If a sight is taken from A (Fig. 168) to an object B at an inclination of α to the horizontal, the direction AB or AC is erroneously recorded by the horizontal circle as AD, since a horizontal line of sight has to be placed along AD in order that B may be bisected on elevating the telescope. The error of direction, β , is given by

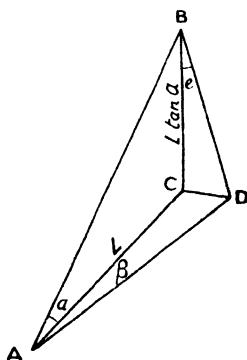


FIG. 168.

$\sin \beta = \tan \alpha \tan \theta$, or $\beta = \theta \tan \alpha$ as β and θ are both small angles*

The error in the value of an angle or bearing is the algebraic difference of the errors developed in the backsight and foresight, so that the greater the change of inclination of the telescope for the two observations the greater is the error. In the most precise angle measurements corrections are computed from the striding level readings (Vol. II, Chap. III).

(c) *Slip*.—Observations are vitiated if the orientation of the instrument is disturbed by the instrument turning: (a) on the tripod head by not being screwed firmly home, so that it slackens back; (b) on the lower parallel plate of a four-screw head by the levelling screws not being turned until they grip; (c) by neglecting to clamp a shifting head; (d) by insufficient tightening of the lower clamp.

* The derivation of this formula is given in Appendix II, page 627.

(d) *Using Wrong Tangent Screw.*—If the upper tangent screw has been used in backsighting, the mistake will be discovered by a glance at the vernier before sighting forward, but the mistake of turning the lower one in bisecting the forward signal is not evidenced until a check sight reveals a discrepancy.

3. Observational Errors.—These consist of errors and mistakes in sighting and reading.

(a) *Inaccurate Bisection of Signal.*—This may occur through defective vision or carelessness, particularly as regards the elimination of parallax. The bisection of a pole should always be made at the point of intersection of the hairs, as the vertical hair may not be truly vertical.

(b) *Non-verticality of Signal.*—Poles should always be sighted as far down as possible. When the foot cannot be seen, there is every likelihood of error being introduced through non-verticality, unless the pole has been tested by plumb line. The error is of very common occurrence, and since the error of direction of the line of sight, viz. $\tan^{-1} \frac{\text{error of verticality}}{\text{distance}}$, is inversely proportional to

the distance, particular care is necessary with short sights. A deviation of 1 in. in the position of the point sighted produces at a distance of 100 ft an error of bearing of nearly 3 min.

Whether the error is caused by non-verticality or by the signal not having been erected at the point intended to be used as an instrument station, its effect may be eliminated by centering the instrument at the forward station with respect to the point to which the foresight was taken.

(c) *Errors of Reading.*—The precautions to be observed in reading verniers and micrometers have been dealt with on pages 44 and 74.

(d) *Errors due to Displacement of Pegs or Signals.*—If pegs are not very firmly driven, one may get displaced while work is in progress. For instance, after observations at a station have been completed, the peg marking it may be moved inadvertently, without the movement being noticed, before the observations to that peg from the next station are completed. The only way to guard against errors arising from this cause is to choose instrument stations on firm ground, drive pegs firmly and warn everyone concerned to use the greatest care when walking near, or moving round, a peg.

Similar care must be taken to see that no movement of signals takes place while observations are in progress.

When work ends for the day, the pegs required to commence from for the next day's work should either be carefully referenced or guard pegs put around them to prevent people or animals walking into them.

(e) *Errors due to Wrong Booking.*—A common source of error in

traversing when included angles are observed is to book readings taken to a forward station as being taken to the back station or vice versa. This mistake, already referred to on page 215, is best avoided by adopting and keeping throughout to a standard system of booking.

4. Errors due to Natural Causes.—*Wind.*—It is impossible to perform accurate work in a high wind because of the vibration of the instrument. If careful centering is required, the plumb bob and line must be sheltered.

High Temperature.—On hot sunny days the heating of the ground during the midday hours causes warm air currents to ascend, and the irregular refraction produced gives rise to an apparent trembling of the signal near the ground. The effect is avoided if the line of sight is at all points above the disturbance, which may be taken as appreciable only within a height of 3 ft. from the ground.

The effect of the sun shining on one side of the instrument is to throw it out of exact adjustment, but in ordinary traversing uncompensated errors from this source are negligible.

Haze.—A hazy atmosphere increases the probability of inaccurate bisection of the signal, and may necessitate a suspension of the work.

FIELD CHECKS IN TRAVERSING

Various checks on a traverse are sometimes available in the field, and, when the opportunity offers itself, these should almost invariably be taken advantage of. They may be divided into two kinds:

- (a) Those available when the traverse is a closed surround.
- (b) Those available when the traverse is unclosed or does not form a closed surround.

In some cases, these checks are not only useful merely as checks, but the extra observations involved can also often be used in the office as additional data for adjustment purposes.

Field Checks in Closed Traverse Surround.—Let AB . . . KA (Fig. 169) be a closed circuit. If there is no intervisibility except between adjacent stations, then, on proceeding from the initial station A and carrying the bearing forward to B, C, etc., all the bearings will have been determined when the observations at K are completed. The instrument should, however, be again set up at A, and the bearing AB read from a backsight on K. Any difference between the new observed value of bearing AB and that first determined represents the angular error accumulated in the circuit. More usually the bearings of both AK and

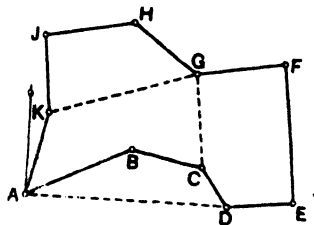


FIG. 169.

AB will be determined at the first occupation of A, and the error is discovered at K by comparison of the value of bearing KA with that previously obtained for AK.

The magnitude of the angular closing error will reveal whether any gross mistake has occurred. If the error is appreciably greater than that to be expected from accidental errors of angle measurement, the observations must be repeated. It is therefore desirable to be able to detect a mistake as soon as committed, or at least to localise it, so that it may be discovered and eliminated without undue additional labour. The following methods of checking are directed to this end.

Double Observations.—The method of obtaining each bearing from observations on both faces of the instrument is valuable because it reduces the effects not only of instrumental error but also of accidental errors of observation and it should almost invariably be used on extended traverses. At the same time, it must be recognised that, although it reduces the possibility of gross error very greatly, especially if care is taken to see that the degrees and minutes of the zero used for observations with the second face are different from those used in the case of the first face, it is not an absolute preventative of error. For this reason, other checks on the bearings should be used whenever they are available, not only to detect or prevent the possibility of gross errors, but also to provide a control on the growth of the purely unavoidable and accidental errors of observation.

Check taping helps to prevent the occurrence of gross error in the linear work, but, again, other checks should be taken advantage of when they can be obtained conveniently.

Cross Bearings.—Possible mistakes may sometimes be localised if the survey is divided up by observing cross bearings between such non-adjacent stations as are intervisible. Thus, in Fig. 169, the signal at D being visible from A, a measurement of bearing AD is included in the observations at A. On reaching D, after having occupied stations B and C, a sight is taken on A, and, if the bearings DA and AD differ by 180° , the circuit observations up to D are checked. Cross bearings CG and GK similarly serve to verify the angular work from C to G and from G to K respectively.

Check bearings are also useful for the location of a mistake in the linear measurement. When the magnitude of the closing error shows that a mistake in chaining has occurred, each of the compartments into which the survey is divided by the cross bearings can be tested for closure either by means of co-ordinates or by plotting. For example, in Fig. 169, the bearing of a line such as CG can be computed from the co-ordinates, or obtained from plotting, and the computed or plotted bearing compared with the observed bearing. It will then be seen which portion of the circuit is affected by the mistake.

Compass Check.—If the theodolite is fitted with a circular compass box, a reading of the compass affords a rough but useful check on each observation. If the bearings are being referred to magnetic meridian, the compass readings should be the same as the observed bearings; otherwise the difference should be the constant angle between the magnetic and reference meridians. The compass is insufficient to check the bearings nearer than to about half a degree, but it serves to show up serious mistakes, and the check should not be neglected if opportunities for taking cross bearings are few. The trough form of compass is less direct in this respect, but serves to verify the orientation of the instrument in terms of magnetic meridian, except at those stations where the orientation of the circle is 180° from that at the initial station. The modern tendency, however, especially in the better class of instrument, is not to have a compass attached to the theodolite.

Summation Test for Angles.—Adding together all the angles of a closed figure gives a useful check on the angular measurements. Assuming that included angles have been measured and that all are measured clockwise from the back station, the following two rules apply:

(a) If the closed figure always lies to the right of the observer as he walks around it in the direction followed by the survey, the measured angles are all the exterior angles of the figure, and their sum should be equal to $(2n+4) \times 90^\circ$, where n is the number of stations or corners in the figure.

(b) If the closed figure lies always to the left of the observer as he walks around it in the direction followed by the survey, the measured angles are all interior angles, and their sum should equal $(2n-4) \times 90^\circ$.

If deflection angles are measured, the difference between the sum of the right-hand and that of the left-hand angles should be equal to 360° .

It will happen very seldom that the observed angles add up exactly to their theoretical sum, and there will usually be a small excess or deficiency, which will be measured in minutes or seconds of arc. This represents the "closing error" due to the unavoidable and accidental errors of measurement, and this closing error should be distributed or adjusted among the various stations, by methods to be described in the next chapter, before co-ordinates are computed.

Linear Measurement.—The only reliable safeguard against error in taping or chaining is to have two independent measures of each line. As a council of perfection, the separate measurements should be of similar precision and done in opposite directions, on different dates, by totally different parties. However, in actual practice, it is usual to measure the check taping by less accurate methods than

the main taping, the object being to prevent the occurrence of gross errors, such as dropped tape lengths, or of errors of the order of a foot or over, rather than the detection of minor errors of a fraction of a foot. If time does not permit of a regular check taping or chainage, check measurements, to detect the grosser type of error only, may be made by pacing or pedometer, or, if legs are short, by tacheometer. If legs are long, tacheometrical methods are no faster than ordinary check taping.

Field Checks in Unclosed Traverse.—In the case of an unclosed traverse, opportunities may present themselves of observing check bearings as in a closed traverse. Thus in Fig. 170, illustrating part of an unclosed traverse, if E is visible from A, bearing AE may be observed from A, and, on reaching E, the orientation of the instrument and the angular work between A and E will be checked by a backsight on A. The forward and back bearings between E and H serve to check the carrying forward of the bearing from E to H. Mistakes in the linear work may also be discovered by means of these check bearings on computing the co-ordinates or on plotting.

In the case illustrated, the portions ABCDE and EFGH would best be treated as deviations (see page 210), the distances AE and EH computed, and the main bearings and co-ordinates brought forward along the lines AE and EH.

An alternative method consists in observing at intervals the bearing to a prominent object to one side of the traverse. Thus, in

Fig. 170, the bearing of a point P is observed from stations such as A, E, and H. The co-ordinates of P are obtainable from the observed parts of figure ABCDEP. A computed value of bearing HP is derived from the co-ordinates of H and P, and agreement between this value and that observed serves to test the traverse from A to H. If the co-ordinates are not computed, the check may be applied on the drawing by trying if the rays from A, E, and H intersect in one point.

Practically, it will rarely be possible to maintain either of the above systems of checking throughout the complete survey. Double observation of each bearing and distance is therefore specially desirable in an unclosed traverse to avoid the carrying forward of a gross mistake.

A thorough check is afforded in the case where the traverse is run between the stations of a refined triangulation. Constant errors as well as inconsistencies in the linear measurements then contribute towards the closing error, which may be regarded as a real index of the quality of the work.

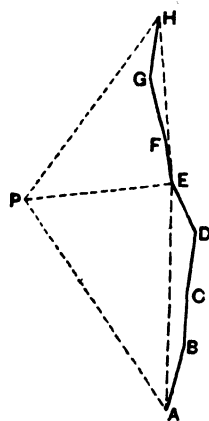


FIG. 170.

Astronomical Checks on Bearings of an Unclosed Traverse.—In long unclosed traverses, where any sort of precision is required, and if no state established trigonometrical or other control points, by means of which reliable bearings are marked on the ground and can be used as checks, are available, it is necessary to check the bearings by means of astronomical observations for azimuth taken at regular intervals. The methods of taking and computing these observations are described in Vol. II, Chap. I. The number of stations to be allowed between azimuth stations will depend on circumstances and the accuracy required, and may vary from ten to fifteen for the highest class of precise traverse to sixty to a hundred for ordinary cadastral or engineering traverses. In bush and forest country, the interval will depend, to some extent at any rate, on whether or not farm or other clearings exist at suitable intervals, as otherwise a good deal of clearing may be necessary in order to obtain a sufficient view of the sky to enable the observations to be taken.

Observations for azimuth may be taken either to the sun or to stars. Sun azimuths suffice for most ordinary traverse work, but, whenever possible, several observations should be taken and these should be properly "balanced." (See Vol. II, page 79, fourth edition, for definition of this term.) For precise work, the azimuth control is confined to observations of stars.

When azimuth observations are used to control bearings, it is necessary, when the survey does not run almost due north and south, to apply the correction for convergence of the meridians (Vol. II, Chap. V), as the bearings worked out through the traverse are referred to the meridian adopted as the standard meridian of the traverse, which may, or may not, be the meridian through the initial station of the survey, whereas, of course, the bearing obtained directly from the azimuth observations is referred to the meridian through the azimuth station itself.

In some cases, where trigonometrical points established by the government are available, it may not be possible for the ordinary surveyor to use these points to check his bearings, as the distant points to which the bearings are known may be too far away—perhaps anything up to forty or fifty miles. Here, astronomical observations will normally be the quickest and most convenient method of obtaining a check, unless clouded skies make them impossible. Even if the trigonometrical points are not of much use from the point of view of obtaining check bearings, the traverse should still be connected to them where possible, as the co-ordinate values, when compared with co-ordinates computed through the traverse, afford a most useful check on the accuracy of the survey as a whole, including the taping.

Unless the main triangulation is well "broken down" by "minor" triangles with comparatively short sides, it often happens that the triangulation points are on high hills which make a direct connection to them by ordinary traverse difficult, if not impossible. In such

cases a connection can often be made by means of a traverse base and sights to the trigonometrical point from both ends of the base.

Latitude and longitude observations are practically useless for checking co-ordinates computed from theodolite traverses, as, even when the work is done with large instruments and with the highest possible precision, the observations are liable to be effected by what is called "local attraction," which is due to irregularities in the earth's gravitational field, and errors arising from this cause may be greater, except for traverses of abnormal length, than the error to be expected in the traverse itself. Latitude and longitude observations may, however, be used to check and control long compass traverses.

Testing Closing Error as Check against Gross Error in Bearing.—

If r is the probable error of a single observation of an angle, the probable error in bearing at the end of n stations, assuming that there is no constant error in the angular measurements and that the probable error of an angle is independent of the length of traverse leg, is $\pm r\sqrt{n}$. If the closing error is greater than about five times this amount, that is if it exceeds $\pm 5r\sqrt{n}$, the probability is that there is a gross error somewhere in the angular measurements. This rule holds, of course, for closed surrounds as well as for "straight" traverses.

The probable error of a single angle can be determined very simply for any particular instrument by making a number—say about twenty, but the more the better—of independent measurements of the same angle on different zeros of the instrument. The formula required to work out the results is given and explained in Vol. II, third edition, page 247.* One objection to this method is that it involves measurements that are almost certainly made under more or less ideal conditions and not the conditions usually met with in the field. Consequently, it is much better to determine the probable error from observations actually taken in the field under varying field conditions. This can be done if the closing errors for a number of different surrounds or between fixed bearings are determined.

Let e_1 be the closing error when there are n_1 angular stations.

e_2 be the closing error when there are n_2 angular stations.

.....

e_n be the closing error when there are n_n angular stations.

N = Total number of closing errors available.

* Note that the first result obtained in the example given at the bottom of the page is the probable error of the arithmetical mean, whereas what we now require is the probable error of a single observation. This is given by the second result, viz. $\pm 1''.34$. Note, also, that the figures are for an instrument to be used on geodetic work.

Then, if r = the probable error of a single observation :*

$$r = \pm 0.674 \times \left[\frac{\frac{e_1^2}{n_1} + \frac{e_2^2}{n_2} + \frac{e_3^2}{n_3} + \dots + \frac{e_n^2}{n_n}}{N} \right]^{\frac{1}{2}}$$

OFFICE TESTS FOR LOCATING GROSS ERRORS IN TRAVERSES

When both ends of a traverse are known fixed points, and it is known that a gross error in bearing exists, the station at which it occurred can often be located by plotting or computing the traverse from each end, when the station having the same co-ordinates in each route will be the one where the error lies.

Another method, suggested by a writer in the *Empire Survey Review*, obviates the labour of plotting backwards, and is as follows.† In Fig. 171, the correct line of the traverse is *Oabcdefgh*, but a gross error has been made in the measurement of the angle

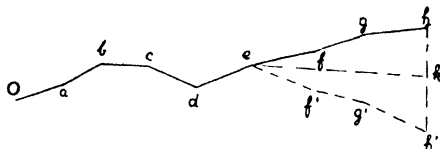


FIG. 171.

at *e*, with the result that, from *e* on, the traverse plots as *ef'g'h'*, so that the terminal point comes to *h'* instead of *h*. As the point *h* is a fixed one, its position is known and can be plotted. Join *hh'* and bisect it at *k*. At *k* erect a perpendicular, *ke*, to the line *hh'*. This perpendicular will then pass through *e*, the point at which the error was made. This follows from the fact that the triangle *ekh'* is isosceles.

If the closure of a traverse indicates the existence of a gross error in taping, the exact leg in which the error occurred can sometimes be found by examining the bearing of the closing error. Provided that there is only one gross error and that it is much larger in magnitude than the normal accidental errors, the bearing of the closing error will be approximately the same as that of the leg in which the gross error exists.

* This formula will usually give a somewhat higher value for the probable error than that obtained from a number of measurements of a single angle. This is because it may include a small constant error as well as the true probable error of the normal accidental errors of observation. The formula neglects the effect of the probable errors of the azimuth observations or of the fixed bearings used in the test, which are assumed to be small.

† "Two Simplified Checks," by B. Goussinsky, *Empire Survey Review*, Vol. III, No. 20, April, 1936.

If there are two or more gross errors, either in bearing or in length, it will be impossible to locate the exact spots at which they occurred by any of the methods just given, though they may be localized to some extent by one or other of the field checks given on pages 227 to 230.

PROPAGATION OF ERROR IN TRAVERSING

The propagation of error in linear measurement has been considered in Chapter II and we now have to consider the propagation of (a) angular error and (b) error of position. The latter question, however, is a more complicated one than can be dealt with fully here and all that can be attempted is a general discussion of the more important points.

Propagation of Angular Error.—In traverse work the measurement of angles is only a means to obtaining the bearings of the different legs, and an error in the measurement of an angle at any station affects all the bearings from that station on. If angles are measured on both faces of the theodolite and on different zeros, such angular errors as there are will tend to have different signs at different stations and thus to be of a compensating and accidental type. Constant errors may arise from errors in centering when legs are short, slip and from "phase" at the signals on a bright day. Usually, with reasonably long legs, the constant errors are very small and the main ones to be considered are the purely accidental ones, varying both in magnitude and sign at the different stations. Errors of this type are, for all practical purposes, independent of the length of the individual legs.

Let $\delta\theta_1, \delta\theta_2, \delta\theta_3 \dots \delta\theta_n$ be the true errors in the angles at the 1st, 2nd, 3rd, . . . n th stations occupied. The error $\delta\theta_1$ in the first angle will produce an error of the same amount in the bearing of the first and all subsequent legs. Similarly, the error $\delta\theta_2$ in the second angle will produce an error of $\delta\theta_2$ in the bearing of the second and following legs so that the total error in the bearing of this leg is $\delta\theta_1 + \delta\theta_2$. Proceeding in this way, we see that the error in the bearing of the n th leg is :

$$\delta\theta_1 + \delta\theta_2 + \delta\theta_3 + \dots + \delta\theta_n.$$

If, instead of actual errors, whose values and signs we do not know, we take $\delta\theta_1, \delta\theta_2, \delta\theta_3 \dots \delta\theta_n$ as the probable errors of measurement of each angle, we have, by the usual laws of propagation of error :

$$r_i^2 = \delta\theta_1^2 + \delta\theta_2^2 + \delta\theta_3^2 + \dots + \delta\theta_n^2$$

where r_i is the probable error of the n th bearing.

Hence, if $\delta\theta_1 = \delta\theta_2 = \delta\theta_3 = \dots = \delta\theta_n = r_a$, we have :

$$r_i^2 = nr_a^2$$

$$\text{or } r_i = \pm r_a \sqrt{n}.$$

Thus, the probable error of the final bearing at the end of the

traverse depends on the square root of the number of stations in the traverse.

The initial azimuth or fixed bearing will not itself be free from error, and, if we assume this probable error to be r_z , we will have :

$$r_b^2 = n r_a^2 + r_z^2$$

$$\text{or if } r_z = p \times r_a$$

$$r_b = r_a \sqrt{n + p^2}.$$

Similarly, if the end azimuth or fixed bearing also has a probable error of $p \times r_a$, the probable value of the closing error in bearing will be :

$$r_b = r_a \sqrt{n + 2p^2}.$$

In general, however, we may say that the error to be expected in the bearing of the last leg of a traverse depends on the square root of the number of stations and not directly on the number of stations. Compass bearings, of course, do not obey these laws, the probable error of the bearing at any station depending solely on the probable error in reading the compass and not on the probable errors of the bearings of preceding legs.

Propagation of Error of Position.* Consider the displacements, δx and δy , along axes parallel to those of X and Y, produced at one end of a line, of length l and bearing a , by an error of δl in l and one of δa in a . Let x be the latitude and y the departure of the line.

We have :

$$x = l \cos a$$

$$y = l \sin a$$

and, by differentiation :

$$\delta x = \delta l \cos a - l \sin a \cdot \delta a$$

$$\delta y = \delta l \sin a + l \cos a \cdot \delta a.$$

Geometrically, these formulæ may be derived as follows :

Let OA (Fig. 172) be the true position of the line of length l and bearing a . Let there be small errors in length and bearing of

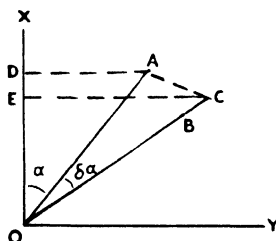


FIG. 172.

* If the reader is not familiar with the properties of rectangular co-ordinates, he should postpone reading this section until he has read the next chapter. Also, if so desired, he can postpone reading it until the second reading of the book, or until he has read Chapter IV of Volume II.

δl and δa respectively. Then the line OA becomes the line OBC, where $OB = OA = l$, and $BC = \delta l$ and angle $AOC = \delta a$.

$$OD = x = l \cos a$$

$$\begin{aligned} OE = x + \delta x &= (l + \delta l) \cos (a + \delta a) \\ &= (l + \delta l) (\cos a \cos \delta a - \sin a \sin \delta a). \end{aligned}$$

Since the angle δa is very small, we may write $\cos \delta a = 1$, $\sin \delta a = \delta a$. Hence,

$$\begin{aligned} x + \delta x &= (l + \delta l) (\cos a - \delta a \sin a) \\ &= l \cos a - l \cdot \delta a \sin a + \delta l \cdot \cos a - \delta l \cdot \delta a \sin a. \\ &= x - l \delta a \sin a + \delta l \cos a - \delta l \cdot \delta a \sin a. \end{aligned}$$

Also, since δl and δa are both small, the term containing the product $\delta l \times \delta a$ is very small in comparison with the other terms and may therefore be neglected. Hence, it follows that

$$\delta x = \delta l \cos a - l \cdot \delta a \sin a$$

and similarly,

$$\delta y = \delta l \sin a + l \cdot \delta a \cos a.$$

Let OA be the r th leg of the traverse and let the suffixes r denote quantities referring to that particular leg. Then

$$\delta x_r = \delta l_r \cos a_r - l_r \cdot \delta a_r \sin a_r.$$

Now the error in the bearing δa_r is compounded of the errors in all the angles up to r , so that

$$\delta a_r = \delta \theta_1 + \delta \theta_2 + \delta \theta_3 + \dots + \delta \theta_r$$

$$\delta x_r = \delta l_r \cos a_r - l_r (\delta \theta_1 + \delta \theta_2 + \delta \theta_3 + \dots + \delta \theta_r) \sin a_r.$$

If the different errors, $\delta l_1, \delta l_2, \delta l_3 \dots \delta l_n$; $\delta \theta_1, \delta \theta_2, \delta \theta_3 \dots \delta \theta_n$, were known, the total error in X up to the n th leg would be :

$$\begin{aligned} \delta X &= +\delta l_1 \cos a_1 + \delta l_2 \cos a_2 + \delta l_3 \cos a_3 + \dots + \delta l_n \cos a_n \\ &\quad - [l_1 \delta \theta_1 \sin a_1 + l_2 (\delta \theta_1 + \delta \theta_2) \sin a_2 + l_3 (\delta \theta_1 + \delta \theta_2 + \delta \theta_3) \sin a_3 \\ &\quad + \dots + l_n (\delta \theta_1 + \delta \theta_2 + \delta \theta_3 + \dots + \delta \theta_n) \sin a_n] \end{aligned}$$

with a somewhat similar expression for δY .

In this expression we can substitute $x_1 = l_1 \cos a_1, x_2 = l_2 \cos a_2, x_3 = l_3 \cos a_3 \dots x_n = l_n \cos a_n$; $y_1 = l_1 \sin a_1, y_2 = l_2 \sin a_2, \dots y_n = l_n \sin a_n$ where the x 's and y 's are the latitudes and departures of the different legs. If we do this, and collect all the coefficients of $\delta \theta_1, \delta \theta_2, \delta \theta_3 \dots \delta \theta_n$ together, we have

$$\begin{aligned} \delta X &= \left[\frac{\delta l_1}{l_1} x_1 + \frac{\delta l_2}{l_2} x_2 + \frac{\delta l_3}{l_3} x_3 + \dots + \frac{\delta l_n}{l_n} x_n \right] - \left[\delta \theta_1 (y_1 + y_2 + y_3 + \dots + y_n) \right. \\ &\quad \left. + \delta \theta_2 (y_2 + y_3 + \dots + y_n) + \delta \theta_3 (y_3 + y_4 + \dots + y_n) + \dots + \delta \theta_n y_n \right] \end{aligned}$$

In this we note that $\frac{\delta l_r}{l_r} x_r$ is the error produced in X by the error δl_r and that $\delta \theta_r (y_r + y_{r+1} + \dots + y_n)$ is the error produced by $\delta \theta_r$. Unfortunately, we do not know the values of $\delta l_1, \delta l_2, \delta l_3 \dots \delta l_n$; $\delta \theta_1, \delta \theta_2 \dots \delta \theta_n$, but we may have some idea of what their probable values are likely to be. Consequently, if we treat them as probable errors and call R_x the probable error in X , we have, by the ordinary rules for the combination of probable errors :

$$R_s^2 = \left[\frac{\delta l_1^2}{l_1^2} x_1^2 + \frac{\delta l_2^2}{l_2^2} x_2^2 + \frac{\delta l_3^2}{l_3^2} x_3^2 + \dots + \frac{\delta l_n^2}{l_n^2} x_n^2 \right] + \left[\delta \theta_1^2 (y_1 + y_2 + y_3 + \dots + y_n)^2 + \delta \theta_2^2 (y_2 + y_3 + \dots + y_n)^2 + \delta \theta_3^2 (y_3 + y_4 + \dots + y_n)^2 + \dots + \delta \theta_n^2 y_n^2 \right]$$

If the traverse is a straight one, with legs equal in length, we have $x_1 = x_2 = x_3 = \dots = x_n = x$; $y_1 = y_2 = y_3 = \dots = y_n = y$. Also, if we assume that the probable error in distance is proportional to the length, so that $\delta l_1 = kl_1$, $\delta l_2 = kl_2$, \dots , $\delta l_n = kl_n$ and that the probable errors of all the angles are equal, so that $\delta \theta_1 = \delta \theta_2 = \delta \theta_3 = \dots = \delta \theta_n = r_a$, we have :

$$\begin{aligned} R_x^2 &= nk^2x^2 + [n^2y^2r_a^2 + (n-1)^2y^2r_a^2 + (n-2)^2y^2r_a^2 + \dots + 2^2y^2r_a^2 + 1^2y^2r_a^2] \\ &= nk^2x^2 + y^2r_a^2[1^2 + 2^2 + 3^2 + \dots + n^2] \\ &= nk^2x^2 + \frac{n(n+1)(2n+1)}{6} y^2r_a^2. \end{aligned}$$

Similarly,

$$R_y^2 = nk^2y^2 + \frac{n(n+1)(2n+1)}{6} x^2r_a^2,$$

and the total probable error

$$\begin{aligned} &= R_x^2 + R_y^2 \\ &= nk^2(x^2 + y^2) + \frac{n(n+1)(2n+1)}{6} (x^2 + y^2)r_a^2 \\ &= nk^2l^2 + \frac{n(n+1)(2n+1)}{6} l^2r_a^2 \end{aligned}$$

where l is the length of one leg. Here r_a is in radians and, if it is expressed in seconds of arc, we must write $r_a \sin 1''$ for r_a . Also, $nl = L$, where L = total length of traverse. Making these substitutions we finally have :

Total probable displacement

$$= L \left[\frac{k^2}{n} + \frac{(n+1)(2n+1)}{6n} r_a^2 \sin^2 1'' \right]^{\frac{1}{2}}$$

This expression gives the probable linear displacement for a "straight" theodolite traverse in which no adjustment for closing error has been made to the bearings. In practice, when the traverse is a closed one or ends on a line of known bearing, the individual bearings are adjusted, usually by applying to each angle an equal proportion of the closing error, before co-ordinates are computed. Therefore what we really want is the probable displacement, taking into account the adjusted, and not the unadjusted, angles and bearings. This complicates the problem and adds very considerably to the labour in working out a solution but one has been worked out in detail by Mr. F. Yates in *Records of the Gold Coast Survey Department*, Volume III. The full formula for the general case is a long and cumbersome one, and so is not given here, but Mr. Yates

has obtained some fairly easy rules, giving the probable displacements due to the probable angular errors, for some simple common forms of traverse. These formulæ are all based on the assumption that all legs are equal in length and they give the probable displacements in two directions at right angles to one another, the direction of X being shown in Fig. 173. Let R_x = probable displacement in the X direction due to the angular errors only, and R_y = probable displacement in the Y direction due to the angular errors only. Then, for the cases illustrated, in which a thick line indicates an azimuth line or line of fixed bearing :

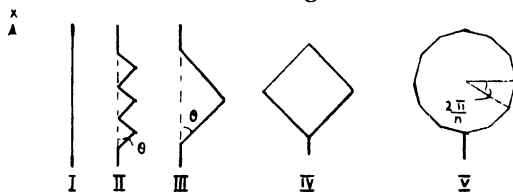


FIG. 173.

I. A straight traverse.

$$R_x = 0.$$

$$R_y = r_a L \sin 1'' \left[\frac{(n+1)(n+2)}{12n} \right]^{\frac{1}{2}}$$

II. Zigzag traverse, each leg inclined at an angle $\pm \theta^\circ$ alternately to the general direction (n even).

$$R_x = r_a L \sin \theta \sin 1'' \left[\frac{(n+2)}{4n(n+1)} \right]^{\frac{1}{2}}$$

$$R_y = r_a L \cos \theta \sin 1'' \left[\frac{(n+1)(n+2)}{12n} \right]^{\frac{1}{2}}$$

III. Traverse with bend at middle point, each half being inclined at angle $\pm \theta^\circ$ to the general direction (n even).

$$R_x = r_a L \sin \theta \sin 1'' \left[\frac{(n+2)(n^2+2n+4)}{48n(n+1)} \right]^{\frac{1}{2}}$$

$$R_y = r_a L \cos \theta \sin 1'' \left[\frac{(n+1)(n+2)}{12n} \right]^{\frac{1}{2}}$$

IV. Square surround with external azimuth at one corner (n a multiple of 4).

$$R_x = r_a L \sin 1'' \left[\frac{n^2+8}{96n} \right]^{\frac{1}{2}}$$

$$R_y = r_a L \sin 1'' \left[\frac{(n+2)(n^2+2n+4)}{96n(n+1)} \right]^{\frac{1}{2}}$$

and, very approximately, when n is large,

$$R = \sqrt{R_x^2 + R_y^2} = r_a \sin 1'' \left[\frac{1}{3} A (n+1) \right]^{\frac{1}{2}}$$

where A is the area enclosed.

V. Polygonal surround with an external azimuth (n a multiple of 4).

$$R_x = \frac{r_a L \sin 1''}{\sin \frac{\pi}{n}} \left[\frac{1}{8n} \right]^{\frac{1}{2}}$$

$$R_y = \frac{r_a L \sin 1''}{\sin \frac{\pi}{n}} \left[\frac{n+3}{8n(n+1)} \right]^{\frac{1}{2}}$$

and, approximately, when n is large,

$$R = \sqrt{R_x^2 + R_y^2} = r_a \sin 1'' \left[\frac{A}{\pi} (n+1) \right]^{\frac{1}{2}}$$

In the last two cases we can put as a very good approximation

$$R = r_a \sin 1'' \sqrt{\frac{1}{3} A n}.$$

If the probable error of the taping is assumed to be proportional to the length of the leg, the total probable displacement due to errors in both taping and angular measurements can be found from any of the above cases by substituting for R_x and R_y in the formula :

Probable error in displacement

$$= \left[\frac{k^2 L^2}{n} + R_x^2 + R_y^2 \right]^{\frac{1}{2}}$$

A comparison of the formula for the straight traverse with unadjusted angles and bearings with that for the same traverse with adjusted angles (Case I), shows that adjusting the angles reduces the probable displacement by about half.

Another interesting case is to compare a straight traverse (Case I) with a square surround (Case IV) of the same length.

For the straight traverse

$$\begin{aligned} R &= \sqrt{R_x^2 + R_y^2} = r_a L \sin 1'' \left[\frac{(n+1)(n+2)}{12n} \right]^{\frac{1}{2}} \\ &= r_a L \sin 1'' \left[\frac{n}{12} + \frac{1}{4} \right]^{\frac{1}{2}} \text{ approximately.} \end{aligned}$$

For the square traverse, $A = \left(\frac{L}{4} \right)^2$

$$\begin{aligned} R^1 &= r_a L \sin 1'' \left[\frac{(n+1\frac{1}{2})}{48} \right]^{\frac{1}{2}} \\ \therefore \frac{R^1}{R} &= \frac{\left[\frac{n}{48} + \frac{1}{32} \right]^{\frac{1}{2}}}{\left[\frac{n}{12} + \frac{1}{4} \right]^{\frac{1}{2}}} = \frac{1}{2} \text{ approximately.} \end{aligned}$$

Hence, for the square traverse, the probable linear displacement due to the angular errors alone is approximately one-half of the

corresponding probable linear displacement for the straight traverse. Thus, quite apart from the taping errors, one would expect a closed surround to give a much smaller closing error than a straight traverse between fixed points, and this fact should be borne in mind when comparing the results from both kinds of traverse.

As a numerical example, take a traverse 30,000 feet, or about 6 miles, long, with legs of an average (assumed equal) length of 1,000 feet and let $k = 1/30,000 = 0.000033$, $r_a = \pm 10''$, $n = 30$.

$$R = \left[\frac{(0.000033)^2 \times (30,000)^2}{30} + \frac{(10)^2 \times (30,000)^2 \times \sin^2 1'' \times 31 \times 32}{12 \times 30} \right]^{\frac{1}{2}} \\ = \pm 2.42 \text{ feet or } 1 : 12,400.$$

This example shows the necessity for keeping the angular errors down to a minimum when any degree of accuracy is required.

For the square surround, using the same data, we have $R = \pm 1.19$ feet, or 1 : 25,200.

In the above, we have assumed that the linear errors, though proportional to the length of the leg, are of the accidental type with variable signs. If, however, as is generally the case, there is a constant linear error throughout the whole traverse, the probable error of displacement may be taken as:—

$$P. E. = \left[C^2 L_1^2 + \frac{k^2 L^2}{n} + R_x^2 + R_y^2 \right]^{\frac{1}{2}}$$

where C is the probable constant error per unit length and L_1 is the direct distance between the terminal points of the traverse.

The formulæ given above only apply, of course, to one single traverse in which there are fixed bearings at the ends and no intermediate ones. Long traverses will usually consist of a number of sections, or minor traverses, each beginning and ending on fixed azimuths or bearings, and, for these, the probable final displacement will be the square root of the sum of the squares of the probable displacements of each section.

MISCELLANEOUS PROBLEMS IN THEODOLITE SURVEYING

By means of the theodolite, obstacles to ranging or chaining may be more rapidly and accurately surmounted than by the use of chain surveying equipment only. The examples given below also include some problems frequently encountered in the course of setting out works. While all possible cases cannot be included, the solutions given will suggest the geometrical methods to be applied in particular circumstances.

Obstacles.—*Obstacles which Obstruct Ranging.*—(a) When an intermediate point can be selected from which the ends are visible, the line may first be ranged by eye (page 137). On setting the theodolite in this line, it may be adjusted to the exact line by trial

and error, until, on sighting one end station and transiting, the line of sight is found to cut the other station.

(b) From A (Fig. 174) project a random line AC'B' by the theodolite in the estimated direction towards B, leaving a point C' visible from A. Measure AB' and the offset B'B. Compute $a = \tan^{-1} \frac{BB'}{AB'}$, and, with the theodolite at A, set off a from AC', and establish points on AB.

(c) Select a point C (Fig. 175) from which A and B are visible. Measure AC, CB, and c . Solve for a and b , and set off one or both of these angles at A and B respectively.

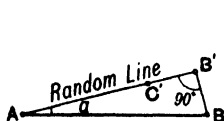


FIG. 174.

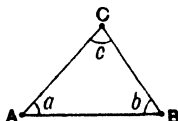


FIG. 175.

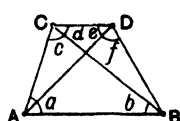


FIG. 176.

(d) If AB is very long, select two intervisible points C and D (Fig. 176) from both of which A and B are visible. Measure angles c , d , e , and f . Solve triangle ACD for AC and AD, taking CD as unity, and from triangle BCD obtain BC and BD. Now solve triangle ACB for a and b , and check by solving triangle ADB. By orienting on C, set off a and b from A and B respectively, and establish points on AB. This method will not be very satisfactory if CD is short, but is useful for clearing AB.

(e) If intervisible points C and D cannot be obtained, run a traverse from A terminating on B, and compute the bearing of AB. Set off this bearing from A.

As a general rule, when any of the methods (b), (c), (d), or (e) are used to run a straight line between two points A and B which are not intervisible, and the line has to be set out on a computed bearing or angle, the line laid out will not strike the end point exactly. This is due to small errors in observation which cannot be avoided. Thus, in Fig. 174, the line actually laid out on the ground follows the direction AB' instead of the desired direction AB. With care, however, B' should be very close to B. The line AB' is chained and the distance BB' measured, when the length of the offset at any intermediate C' can be calculated from the formula

$$\text{Offset at } C' = \frac{BB' \times AC'}{AB'}$$

Consequently, permanent marks should not be put in on a line of this description until the distance BB' has been obtained and any necessary offsets calculated.

Obstacles which Obstruct Chaining.—(a) The methods of page 191, involving the use of right angles, are available, the theodolite being employed to erect perpendiculars.

(b) When both points, A and B (Fig. 177), are accessible, set out an equilateral triangle by laying off a and b each $= 60^\circ$, obtaining the intersection C. Measure AC, and check by BC. Alternatively, set out any triangle ABC, and measure a sufficient number of the parts to enable the triangle to be solved for AB.

(c) When B is inaccessible, set out AC perpendicular to AB (Fig. 178). Measure AC and c , and solve for AB. Alternatively,

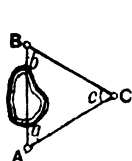


FIG. 177.

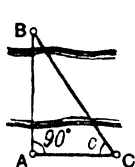


FIG. 178.

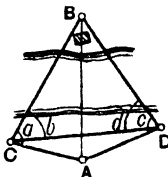


FIG. 179.

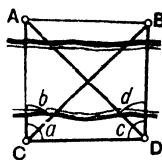


FIG. 180.

measure out AC in any convenient direction, observe the angles at A and C, and solve for AB.

(d) When B is inaccessible and invisible from A, select two intervisible points C and D (Fig. 179) from which A and B are visible. Measure CD, AC, and AD and angles a , b , c , and d . Solve triangle BCD for BC and BD, then solve triangle ABC for AB, and check from triangle ABD. Time is saved in the measurements if C, A, and D are collinear.

(e) When A and B are both inaccessible, select two points, C and D (Fig. 180) from which both A and B are visible. Measure CD and angles a , b , c , and d . Solve triangle ACD for AC and AD, and BCD for BC and BD. Obtain AB by solving triangle ABC, and check from triangle ABD.

Perpendiculars and Parallels.—*To Set Out a Perpendicular to a Given Line from a Given Point.*—(a) When both the point P and the

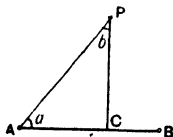


FIG. 181.

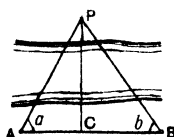


FIG. 182.

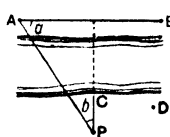


FIG. 183.

line AB are accessible, select a point A on AB, and measure a (Fig. 181). At P sight A, and set off $b = (90^\circ - a)$; then PC is the perpendicular. Alternatively, measure a and AP, and compute AC.

(b) When P is inaccessible, select two points A and B on AB (Fig. 182). Measure AB, a , and b . Then $AC = \frac{AB \tan b}{\tan a + \tan b}$ or, in a form suitable for logarithmic calculation, $AC = \frac{AB \cos a \cdot \sin b}{\sin(a+b)}$

(c) When P is accessible, but AB is not, deduce a (Fig. 183) by observations from P and a point D, as in Fig. 180. From P sight A, and set off $b = (90^\circ - a)$; then PC is the direction of the perpendicular.

(d) When both P and AB are inaccessible, set out any accessible line parallel to AB (see below), and proceed as in (b).

To Set Out a Parallel to a Given Line from a Given Point.—

(a) When both the point P and the line AB are accessible, select

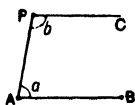


FIG. 184.



FIG. 185.

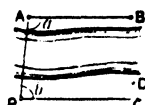


FIG. 186.

a point A on AB, and measure a (Fig. 184). From P sight A, and set off $b = (180^\circ - a)$. PC is the required parallel.

(b) When P is inaccessible, select points A and B on AB (Fig. 185), and measure AB, a , and b . The perpendiculars AC and BD to the

required parallel $= \frac{AB}{\cot b - \cot a} = \frac{AB \sin a \sin b}{\sin (a - b)}$.

(c) When P is accessible, but AB is not (Fig. 186), deduce a by observations from P and a point D, as in Fig. 180. From P sight A, and set off $b = (180^\circ - a)$; then PC is the required parallel.

The Traverse Base and its Use.—It is sometimes inconvenient or difficult to measure the direct distance between the two ends of a line which it is proposed to use as a base of a triangle for fixing a third point. In such a case a “traverse base” may be used. This is a base the length of which is obtained by computation from a traverse run between the terminal points.

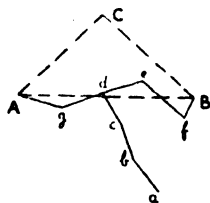


FIG. 187.

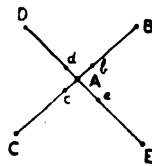


FIG. 188.

In Fig. 187, it is desired to connect the traverse $abcd$ to the point C, the ground between d and C being unsuitable for linear measurements. C can be seen from the points A and B, but the ground between these points is not suitable for the direct measurement of the line AB. In this case, the subsidiary traverse $AgdefB$ is observed,

and the length and bearing of AB computed. If A and B are inter-visible, the angles CAB and CBA and, if possible, ACB can be observed, and the triangle ABC computed. If A and B are not inter-visible, the bearings of the lines AC and BC can be obtained from the traverse by measuring the angles gAC and fBC , and these, together with the computed bearing and length of AB, give all the essential data for the solution of the triangle.

Referencing Marks and Beacons.—When there is any possibility of temporary or permanent marks being moved and it is desired to be able, at any future time, to replace them in the exact positions in which they were originally, they should be carefully “referenced.” There are several ways of doing this. In a town, several careful linear measurements to the corners of permanent buildings or to other clearly defined points of a permanent nature will serve to fix the position of the point. In other places the following is a suitable method.

Let A, Fig. 188, be the point which it is required to reference. Set up the theodolite at A and put in a mark, permanent or otherwise, at a point B in such a position that it is unlikely to be moved. Sight on B and then set out a mark at a point C, likewise in a position where it is not likely to be moved, and on the straight line BA or BA produced. Now put in two similar marks D and E on the straight line DAE and at a suitable angle with the line BAC. Measure and record the distances AB, AC, AD, and AE. These measurements will be sufficient to enable the point A to be re-established, but it is better to do this by theodolite. If two theodolites are available, set one at B and one at D and sight on C and E respectively. Then the intersection of the two lines of collimation will be at the point A.

If only one theodolite is available, set up at B and sight on C and then line out two marks, b and c , on the line BC, close to, and on either side of, the position where A will come. Now set up at D, sight on E, and set out two pegs, d and e , also close to A and on either side of it. The position of A can then be found by stretching a string from b to c and one from d to e , when the intersection of the two strings will give the position required.

This method is very commonly used on construction work to enable points that are liable to be moved while construction is in progress to be recovered at any time.

For very important points, it is advisable to put in three or four marks, instead of two, on each line or else to set out additional lines. Failing this, linear measurements from two points will suffice to recover the position fairly accurately, though not so accurately as by the use of the theodolite in the manner described.

COMPASS TRAVERSING

In theodolite surveying, direct reference is made to the meridian at the initial station only, and from the observed bearing of the first line those of the others are obtained mechanically or by calculation by the methods already described. In compass surveying, on the other hand, every bearing may be observed directly from magnetic meridian, established at each station by floating the needle. Alternatively, those forms of compass which are fitted with a vernier, connected with the line of sight and moving over a fixed graduated circle, can be used in the manner of a theodolite to measure angles and to carry forward the bearing, the needle only being consulted as a check at stations after the first. These two methods of compass traversing are distinguished as free needle and fixed needle surveying.

Fixed Needle Surveying.—In deciding upon which of the two systems to adopt, consideration may be given to the following.

Advantages of Fixed Needle Work.—(1) The precision attainable, although inferior to that of theodolite surveying, is greater than in free needle working.

(2) Readings are not influenced by local attraction.

Disadvantages.—(1) Most forms of vernier compass are heavier and less portable than those adapted for free needle observations only.

(2) The time taken to observe is greater than in free needle work.

The fixed needle system is used extensively in mine surveying, but the general surveyor has recourse to the compass only in cases where lightness of equipment and speed are of greater account than refinement of observation. For this reason, the free needle method is that more generally followed. As the methods of observation used in theodolite surveying (other than that involving transiting of the telescope) are equally applicable to fixed needle surveying, the latter need not be further considered.

Relative Merits of Theodolite and Free Needle Surveying.—*Advantages of the Compass.*—(1) Some forms, notably the prismatic compass, are very light and portable.

(2) Observations are more rapidly completed than with the theodolite.

(3) Each bearing is determined independently of the others, and errors do not accumulate, but tend to compensate.

(4) The bearing of a line can be observed from any point on it.

(5) Intermediate points can be established quickly on a long line of known bearing which cannot be ranged completely from one end.

(6) The bearings of all the courses of a traverse can be observed by occupying every second station only, but this procedure is not recommended because of the possibility of local attraction.

Disadvantages.—(1) Under the best conditions, readings cannot be relied upon to closer than about 10 min. which corresponds to a linear displacement of about 1 : 344. The uncertainty arises both in the estimation of the position of the needle and in the existence of diurnal variation (page 205). On the other hand, cases do arise when the compass actually gives more accurate results than the theodolite unless very special care is taken with the centering of the instrument and of the signals. This is in long traverses when legs are excessively short, and the reason is that, when the compass is used, errors in reading the bearing at one station do not affect subsequent bearings, as errors in reading the angles with the theodolite—which may be large when legs are very short—do.* Also, very close sights mean altering the focus of the telescope for almost every sight and this may lead to error. For all normal work, however, the theodolite is far the more accurate instrument.

(2) Precautions must be taken against the effects of local attraction.

Application of Compass Surveying.—The portability of the compass makes it specially suitable for reconnaissance and exploratory surveys, where rapidity of observation is important. Short compass traverses may, however, be introduced with advantage between stations of a theodolite survey for the rapid survey of detail which need not be recorded with a high degree of precision. The time saved by using the compass in place of the theodolite is greatest when survey lines have to be short on account of obstructions or irregularities of detail. The method is therefore suitable for tracing streams, particularly in wooded gorges, irregular shore lines, clearings in woods, etc.

Local Attraction.—The magnetic needle does not record the direction of magnetic meridian when under the influence of attracting bodies in its vicinity. Local attraction, as it is termed, may arise from mineral deposits in the ground, particularly magnetic iron ore, but is also caused by the proximity of steel structures, rails, electric cables conveying current, fences, lamp-posts, iron pipes, etc. Care should be exercised to see that the chain, clearing axes, and similar articles are at a safe distance from the needle, and that the observer has no sources of attraction, such as keys, knife, etc., about his person. The possibility of local attraction is much greater in cities than in the country, but in no case can compass bearings be regarded as reliable unless the routine of observation is designed to detect its presence and eliminate its effects.

* See Briggs' *The Effects of Errors in Surveying*, page 95.

Detection of Local Attraction.—To test for local attraction, it is only necessary to observe the bearing of each line from both its end stations. If the forward and back bearings differ by 180° , it may be taken that no local attraction exists at either station, provided the compass is free from instrumental error and that no mistake is made. On finding that the back bearing does not agree with the forward bearing within the limits of permissible error of reading, the latter should be checked if it is thought improbable that the disagreement proceeds from local attraction. The discrepancy may have been caused by avoidable attraction from articles on the person, chains, etc. If not, local attraction exists at one or both stations, and the two bearings should be noted for subsequent correction.

Elimination of Effects of Local Attraction.—The deflection of the needle at a station under the influence of local attraction causes the bearings read there to originate from a direction other than magnetic meridian. The error is the same for each bearing read at the station, and from the erroneous bearings the true included angles at the affected stations can be deduced in the usual manner. Starting from a bearing unaffected by local attraction, and applying these included angles, the correct bearings are obtained.

A more rapid method of correction is illustrated in the following example :

Line.	Observed Bearing.	Correction.	Corrected Bearing.
AE	319°	$-2\frac{1}{2}^\circ$	$316\frac{1}{2}^\circ$
AB	$72\frac{1}{2}^\circ$	$-2\frac{1}{2}^\circ$	$70\frac{1}{2}^\circ$
BA	252°	$-1\frac{1}{2}^\circ$	$250\frac{1}{2}^\circ$
BC	349°	$-1\frac{1}{2}^\circ$	$347\frac{1}{2}^\circ$
CB	$167\frac{1}{2}^\circ$	0	$167\frac{1}{2}^\circ$
CD	$298\frac{1}{2}^\circ$	0	$298\frac{1}{2}^\circ$
DC	$118\frac{1}{2}^\circ$	0	$118\frac{1}{2}^\circ$
DE	229°	0	229°
ED	48°	$+1^\circ$	49°
EA	$135\frac{1}{2}^\circ$	$+1^\circ$	$136\frac{1}{2}^\circ$

The readings have been estimated to the nearest $\frac{1}{2}^\circ$, and the bearing of each line has been observed twice, the first letter in the designation of the line showing the station of observation. On examining the values of the observed bearings, it will be seen that only in the case of the line between C and D are the forward and back bearings consistent. It may therefore be taken that stations C and D are *both* free from local attraction. Consequently all bearings observed at C and D are correct, and their values are transferred to the third column. Now, since the correct bearing of DE is 229° , that of ED must be 49° , but, as it was read as 48° , station E is influenced by local attraction, and a correction of $+1^\circ$ must be applied to all

readings taken at E. The corrected bearing of EA is therefore $136\frac{1}{2}^\circ$, and consequently that of AE must be $316\frac{1}{2}^\circ$, giving a correction for station A of $-2\frac{1}{2}^\circ$. On applying this correction, and obtaining the adjusted value $70\frac{1}{4}^\circ$ for bearing AB and $250\frac{1}{4}^\circ$ for BA, the correction at station B is found to be $-1\frac{1}{4}^\circ$, so that bearing BC should be $347\frac{1}{4}^\circ$, which agrees with the reading CB taken at the unaffected station C.

Notes.—(1) If the bearings are expressed quadrantly, care must be exercised to apply the corrections in the proper direction, since the numerical values of bearings increase clockwise in the 1st and 3rd quadrants and counter-clockwise in the 2nd and 4th. Positive corrections are applied clockwise, and negative corrections counter-clockwise. Expressed in quadrantal reckoning, the above table becomes:—

AE	N. 41° W.	$-2\frac{1}{2}^\circ$	N. $43\frac{1}{2}^\circ$ W.
AB	N. $72\frac{1}{4}^\circ$ E.	$-2\frac{1}{2}^\circ$	N. $70\frac{1}{4}^\circ$ E.
BA	S. 72° W.	$-1\frac{1}{4}^\circ$	S. $70\frac{1}{4}^\circ$ W., etc.

(2) If no two bearings agree within the limits of permissible error, the corrections should be made from the mean value of the bearing of that line in which there is least discrepancy between the foresight and backsight readings.

(3) In exceptional cases a large source of attraction may exercise a constant effect at all points on a line, and permit of agreement between the forward and back bearings. If those bearings are regarded as correct, and are used as a basis for the correction of others, then all stations really free from local attraction will be found to require a correction of constant amount. If this occurs, it is better to assume that such constant correction should be zero, otherwise the bearings are corrected to a meridian other than magnetic meridian.

(4) The effects of local attraction can be eliminated only if the bearings of adjacent lines are observed under the same conditions. Bearings for correction should therefore be observed at stations, and not at intermediate points on the lines.

Field Work of Compass Surveying.—The chief difference between compass and theodolite field work lies in the method of observing the bearings and the necessity, in the former case, for reading and booking both the forward and back bearing of each line.

The accuracy possible in taking compass bearings does not warrant great refinement in linear measurement. Ordinary chaining is sufficiently accurate, while the less exact methods of pacing, etc. (Vol. II, Chap. VII), are sometimes employed. The allowable length of offsets may be increased, and in small scale work detail may be located by pacing offsets, by estimating distances, or by intersecting compass bearings from two points on the survey line.

Sources of Error.—The effects of local attraction being assumed eliminated, errors may be classed as instrumental and observational. Accurate work is impossible with a sluggish needle, and this defect is most likely to arise from dulling of the pivot, which must therefore be protected from unnecessary wear. Even when in adjustment (page 104), it is found that most compasses possess an individual error, generally due to non-coincidence between the

magnetic and geometrical axes of the needle, so that different instruments give different values for the bearing of a line. Individual instrumental errors of this kind of anything up to several degrees are quite commonly met with. If the same compass is used throughout, this defect does not influence the accuracy of a survey, but would affect the result of an observation for the value of the magnetic declination.

Observational errors are those of sighting and reading. The former are not peculiar to compass surveying, and are of less importance than the latter. In estimating the reading, parallax should be avoided by bringing the eye into the vertical plane of the needle. In taking important readings it is well to tap the compass box when the needle has come to rest, to overcome pivot friction. A system of double reading may also be adopted by displacing the needle after the first reading and taking a second. The possibility of deflection of the needle by sources of attraction about the person, or by electrification of the glass cover, should not be overlooked.

Limits of Precision.—If the position of the needle can be estimated to the nearest 10 min., the error of a reading should not exceed 5 min. On account of diurnal variation, however, readings can seldom be relied upon to less than 10 min. As this corresponds to a lateral deviation of 1 in 344, free needle surveying is evidently inapplicable to many surveys. The figure also suggests the inconsistency of combining refined taping with compass bearings.

As regards the probable error of displacement, in a compass traverse the error in bearing at one station does not affect the errors in bearing at other stations, nor is there usually any question of an adjustment of bearings. Consequently, for such traverses the probable displacement at the end of n equal legs may be assumed to be given by an expression of the form :

$$R = \pm \left[C^2 \cdot L_1^2 + \frac{L^2}{n} (k^2 + 0.000305 \times r_1^2) \right]^{\frac{1}{2}}$$

where r_1 is the probable error, in degrees, of a single bearing, L_1 is the direct distance between terminal points, L is the total length of the traverse, C is the probable taping error per unit of length which is constant over the whole traverse, and k is a factor such that the probable (accidental) error in taping a single leg of length l is given by $\pm kl$. Generally, if the traverse is more or less straight, the term $C^2 \cdot L_1^2$ will swamp the term $\frac{k^2 L^2}{n}$, so that the latter can be neglected.

CHAPTER V

OFFICE COMPUTATIONS

THE office computations consist of the numerical calculations required to put the results of the field measurements into a form in which they can be used for whatever purpose the survey is required, and they generally form an intermediate stage between the field work and the plotting of the plans. In some classes of work, chain surveying for example, there is very little, if any, computing to be done; but in others, theodolite traversing for instance, the total time spent on the computations may be almost equal to that spent on the field work. In this chapter we will deal first of all with some fundamental points in computing and then more particularly with the computation of traverses. Other special computations will be dealt with in later chapters as the need arises.

In dealing with the field operations, much emphasis has been laid on the necessity for great care at every stage in order to prevent mistakes, and the same considerations apply, perhaps with even greater force, to computing. In a long traverse, the computational work is exceedingly heavy, and, for the most part, monotonous; and only those who have much experience of this class of work know how fatally easy it is to make a mistake, and how difficult it sometimes is to detect or locate it when it has been made. This fact must never be overlooked, and the computer must constantly be on guard to see that mistakes do not occur, or that, if they do, they are detected and corrected at once or before it is too late.

In some computations, automatic checks are available and, whenever this is so, they should always be used. We have had one simple example of this type of check in the case of the computation of bearings, where adding together all the included angles and the initial bearing checks the final bearing. In other computations, simple and effective checks of this kind are not always available, and, in that event, the only real check is at least one complete re-computation.

A re-computation to be effective must be as independent of the original one and as complete as it is possible to make it. It is not sufficient to go over the original work and to tick off figures as being correct. If possible, the check computation should be done by a different person from the one who did the original, and it should start at the very beginning—that is, with the measurements recorded in the field books. All reductions or corrections made

from, or to, the figures recorded in the field books should be re-computed and made anew, and it is only after this has been done that the figures should be abstracted to the forms or sheets on which further calculations are to be made.

Sometimes alternative methods or formulæ can be used, and, if so, full advantage should be taken of them. For instance, if the co-ordinates of two points are given, and the bearing and distance between them required, the distance can be worked out either from the sine or the cosine of the bearing after the latter has been obtained. Both formulæ should be used, and the results compared, although, as will be pointed out later, it may be better, when the two results show a *very small* difference, to accept those from one formula in preference to those from the other.

Neatness and uniformity in method are as essential in computing work as they are in keeping the field books. The use of a special paper, divided by faint blue rulings into squares of one-quarter-inch side, so that each square holds two digits, will do much to assist in keeping work neat; and working as much as possible on standard forms, preferably printed, will help considerably to prevent mistakes being made. All figures should be legible and clearly set out, and, where alterations are necessary, incorrect figures should not be erased but should be crossed out, and the correct figures written above them. Also, it is well to do all important computing work in ink, and not to do it in pencil first, and then ink the figures in later. The different computation sheets should be numbered in order, and references made, from one sheet to the other and to the field books, whenever this is necessary to enable the survey to be followed easily by an independent computer or draughtsman. Finally, when the work is complete and it is desired to preserve it, the different sheets would be put together in their proper order, indexed and filed.

Mathematical Tables and Other Aids to Computing.—The first essential for computing is a book containing tables of the logarithms of the natural numbers and the logarithms of the sines, cosines, and tangents of angles. For a good deal of work, notably the calculation of small corrections, tables of four-figure logarithms are sufficient, and a book of these should be in the possession of every surveyor. For much general work, however, six- or seven-figure logarithms are required, and, for the latter, "Chambers's Seven-figure Mathematical Tables" can be recommended. This book gives, each to seven places, the logarithm of every number from 1 to 108,000, together with the natural sine, cosine, tangent, etc., and their logarithms, of every angle, at intervals of 1 minute of arc, from 0° to 90° . Besides these tables, the book contains others that are useful in astronomical and other kinds of mathematical work. For six-figure work, "Chambers's Shorter Six-figure Mathematical Tables" gives a selection of tables which include those ordinarily used in survey work.

When angular work is taken to seconds of arc, the seven-figure tables in "*Chambers*" relating to the trigonometrical functions are not convenient, as the 1-minute interval is too large. In this case, Shortrede's "*Logarithmic Tables*" are more suitable for logarithmic work. These tables give, to seven places, the logarithms of the trigonometrical functions, at intervals of 1 second of arc, for every angle from 0° to 360° .

For work with a computing machine, tables of natural sines and cosines are essential, and Gifford's eight-figure "*Tables of Natural Sines*" give these, at intervals of 1 second of arc, for every angle from 0° to 90° .

In computing compass and low-order theodolite traverses with fairly short legs, traverse tables are often employed. These are simply tables, conveniently arranged, of the natural sines and cosines multiplied by the figures 1, 2, 3, . . . up to 10. They are usually given to four or five places and at intervals of 1 minute of bearing. "*Boileau's Traverse Tables*" (Nisbet & Co., Ltd.) are to five places with an interval of 1 minute between bearings.

When dealing with calculations relating to errors, or the adjustment of errors by the method of least squares, tables of squares and square roots are extremely useful, if not essential. "*Chambers's Seven-figure Tables*" contains a table of quarter squares, but this is not so convenient for survey calculations and adjustments as, say, the tables given in Barlow's "*Tables of Squares, Cubes, etc.*"

Mechanical computing machines are now used very extensively by surveyors and in computing offices. They are simple to operate and, in the hands of a practised worker, are quicker than the ordinary method of logarithmic computation. Moreover, the fact that tables of the natural trigonometrical functions are used instead of logarithmic tables is often an advantage and makes interpolation easier when small angles, or angles near 90° in value, are involved. In addition, computing machines produce less eye strain and fatigue in the computer. Unfortunately, they are expensive and, when used in the field, liable to get out of order fairly easily.

A description of various types of computing machines and an explanation of their working, by the late Dr. L. J. Comrie, is given in Vol. II, Appendix 1, and this can be consulted if further information is required. It may be said, however, that there are very few survey departments or large computing offices to-day which do not have, as part of their ordinary equipment, several computing machines and one or two adding machines, the latter being useful for long additions, such as the addition of the latitudes and departures in long traverses or of backsights and foresights in levelling.

A slide rule, either of the straight rule or spiral barrel type, is inexpensive and often of considerable use when working out small corrections.

Rounding Off Figures.—When rounding off figures it is well to keep to a definite system so that the laws of chance operate and not

the personal caprice or unconscious bias of the computer. If the computation is to n significant figures, the n th figure should be kept as it is if the figures following it are less than 5 in the $(n+1)$ th place. If the figures following the n th significant figure are greater than 5 in the $(n+1)$ th place, the n th figure should be increased by one unit. Thus, in working to four decimal places, 16.453238 is rounded off at 16.4532, but 16.453261 becomes 16.4533. If, however, a figure like 16.45325 is obtained, the computer might call this either 16.4532 or 16.4533. In such a case, the following is a usual and convenient rule to adopt. When the n th significant figure is an even number followed by a 5, keep it as it is and discard the 5, but, if it is an odd number followed by a 5, add one unit to it and discard the 5. Thus, 16.45325 becomes 16.4532, but 16.45335 becomes 16.4534.

Probable Error of Last Figure.—When a series of numbers, each consisting of $(n+1)$ digits, is rounded off to n digits, in the manner described, it can be proved that the probable (and here also the average) error of the last figure of any one of them is one-quarter of a unit in the last significant place of the rounded off number. Thus, if a series of numbers of five-figure decimals is rounded off to four decimal places, the probable error of the figure in the last place of the rounded off number is ± 0.000025 .

A similar rule holds with regard to reading a scale. For example, if a tape graduated to hundredths is being read to the nearest tenth, any reading between, say, 94.65 and 94.75 would be called 94.7. In that case the probable error of any single reading is ± 0.025 . This means that the real reading is as likely to lie within the interval 94.675 to 94.725 as it is to lie outside it.

In the above two cases, the only errors considered are errors of 0, 1, 2, 3, 4 and 5 in the $(n+1)$ th place; and errors of 1, 2, 3 and 4 occur twice as often as errors of 0 and 5. The maximum error, of course, is ± 5 in the $(n+1)$ th place and the minimum error is zero.

When a number such as a logarithm, a trigonometrical function, etc., which consists of a very large or an infinite number of figures, is given to n significant figures, it is usually "forced," that is, the last digit in the part retained is increased by unity when the first figure in the part not retained is 5, 6, 7, 8, or 9. The digits 0, 1, 2, 3, . . . 9 occur with equal frequency in the omitted part and the error in the forced number may consist of any number whatever, including all decimal or fractional numbers, between -5 and $+5$ in the $(n+1)$ th place. In this case, the probable error of the last figure in the forced number is $\pm 1.947 \dots$ in the $(n+1)$ th place.

Precision of the Last Figure in the Logarithm.—When judging the number of places to which computations involving logarithms should be carried or when estimating the precision of values

computed from logarithms, it is useful to bear in mind the following formula.

Let $\log X$ be the logarithm of a number X and let ΔX be the change in X due to a change of p units in the n th place in $\log X$. Then

$$\frac{\Delta X}{X} = 2.3026 \times p \times 10^{-n} = \frac{p}{0.4343 \times 10^n}.$$

Hence, in working with four-figure logarithms, a mistake or difference of 1 in the fourth place means a mistake or difference of $\frac{X}{4343}$ in the value of X . Again, an error of 27 in the fifth place of logarithms, made when computing the length of a line, represents a fractional error of $\frac{27}{0.4343 \times 10^5} = \frac{1}{1600}$ in the actual length of the line.

If we call the error in any quantity divided by the quantity itself the ratio of precision, we see that, when the quantity is derived from ordinary logarithms, the ratio of precision depends only on the error or difference in the last place of logarithms and not on the quantity itself. This rule does not apply to angular values derived from computations involving the logarithms of the trigonometrical functions.

Choice of Formulæ for Computing.—Many formulæ can be put into several different forms, and, while one form may be suitable for one method of computation, it may not be suitable for some other method, although another form will. To take a simple case, if a and b are given and $\sqrt{a^2 - b^2}$ is required, the expression $\sqrt{a^2 - b^2}$ is not very suitable for logarithmic computation as the solution involves looking up three logarithms and three anti-logarithms. However, if we put it into the form $\sqrt{(a-b)(a+b)}$ it is only necessary to look up two logarithms and one anti-logarithm. On the other hand, if a table of squares which goes to sufficient places, and not a table of logarithms, is available, the original form might be the more convenient one to use.

Nearly all formulæ used in survey work are most conveniently expressed when put into a form suitable for logarithmic computation, and, in this form, provided they do not contain fractional indices, they are usually suitable for computation by machine, though, when a machine is used, it is best to examine the formula and see if it is expressed in the form most suitable for machine computation. (See Vol. II, Appendix 1, page 471.)

Most of the formulæ involving the trigonometrical functions, when derived from first principles, are obtained in a form unsuitable for logarithmic computation, and anybody who has studied trigonometry will be familiar with the transformations necessary to put, say, the formulæ for the solution of triangles into forms that are suitable for computation by logarithms.

In many cases an unsuitable formula can be transformed into a suitable one by the choice of an auxiliary angle. Thus, if we want to compute $A \cos \theta + B \sin \theta$; where A , B , and θ are known or given, we can find an auxiliary angle φ from $\tan \varphi = \frac{B}{A}$ and we then have :

$$\begin{aligned} A \cos \theta + B \sin \theta &= A(\cos \theta + \frac{B}{A} \sin \theta) \\ &= A(\cos \theta + \tan \varphi \sin \theta) \\ &= \frac{A(\cos \varphi \cos \theta + \sin \varphi \sin \theta)}{\cos \varphi} \\ &= \frac{A \cos (\varphi - \theta)}{\cos \varphi} \end{aligned}$$

Here, the expression on the right is easier to handle by logarithms than is the expression on the left. Again,

$$\sqrt{A^2 + B^2} = A \sqrt{1 + \frac{B^2}{A^2}} = A \sqrt{1 + \tan^2 \varphi} = A \sec \varphi$$

where $\tan \varphi = \frac{B}{A}$. The angle φ is found from $\log \tan \varphi = \log B -$

$\log A$ and then $\sqrt{A^2 + B^2}$ from $\log \sqrt{A^2 + B^2} = \log A - \log \cos \varphi$. When A and B are large numbers, this is a much easier method than taking the square root of the sum of their squares by ordinary arithmetic.

In both these cases, φ is an auxiliary angle so chosen as to put the expression into a form suitable for easy computation by logarithms.

Another case where choice of formulæ is important is where angles are either small or else near 90° in value. The logarithm of the sine changes most rapidly and irregularly and that of the cosine most slowly and regularly when the angle is small, and the reverse is true when the angle is almost a right angle. Hence, when a choice of function exists in numerical work, it is usually best to choose the one in which the logarithm is changing least rapidly with change in angle. Thus, the length between two points A and B can be found from either of the formulæ :

$$l = \Delta X \sec a ; l = \Delta Y \operatorname{cosec} a$$

where a is the bearing of the line and ΔX and ΔY the co-ordinates of one end referred to the other end. If a is small, the logarithm of the sine is changing very rapidly and irregularly, while that of the cosine is changing slowly and regularly. Hence, in this case it is better to accept the value computed from the cosine in preference to that computed from the sine. On the other hand, if a is large and near 90° in value, it is better to accept the value computed from the sine. (For an example, see page 265.)

Conversion of Small Angles from Sexagesimal Measure into Circular Measure.—The quantity $\sin 1''$ occurs very often, either as a multiplicand or as a divisor, in formulæ relating to survey work. The sine of one second of arc differs from the circular measure of one second by a very small quantity. Hence, $\sin 1''$ is introduced to convert small angles expressed in seconds into radians or vice versa. Sexagesimal measure is, of course, only an artificial system of measurement, but circular measure is a natural one, and it is circular measure that is used in all purely theoretical work.

To convert a small angle, $\delta\theta$, which is expressed in seconds of arc, into radians, multiply $\delta\theta$ by $\sin 1''$. If $\delta\theta$ is in radians and it is required to be expressed in seconds, divide it by $\sin 1''$.

The difference between the circular measure and the natural sine of an angle only exceeds 5 in the 8th decimal place when the angle is more than about $23'$ of arc in value, and 5 in the 7th decimal place when the angle exceeds $48'$ in value, so that it is at about these values that a change of one unit takes place in the tabulated 7th and 6th places respectively.

Computation of the Logarithms of the Sines and Tangents of Very Small Angles.—When angles are very small the logarithms of the sine and tangent change very rapidly and irregularly while the logarithm of the cosine changes slowly and regularly. If, therefore, the log sine or the log tangent of a small angle is required, direct interpolation from tables by the use of proportional parts is not sufficient for accurate work and special methods must be used.

Five methods are available :

- (1) By interpolation by second or third differences.
- (2) By finding, by direct interpolation, from a table of natural trigonometrical functions, the natural sine or tangent and then taking the logarithm of the quantity so obtained.
- (3) By two means.
- (4) By Maskeleyne's Rules.
- (5) By tables for "S" and "T."

For interpolation to second differences see Vol. II, Chap. I, though it is as well to avoid this method. The natural sines and tangents of small angles increase almost uniformly so that a value may be obtained by direct interpolation and finding the logarithm of this quantity found. The method of two means is described in the introduction to Shortrede but is not suitable for general use. The use of "S" and "T" tables is also explained in the introduction to Shortrede, and tables are given on the last page. Examples of the use of Maskeleyne's Rules are given in "Chambers's Seven-figure Tables" (see "Explanation"; Sections 24 to 28).

For all ordinary work, method (2) will probably be found as convenient as any when a table of natural trigonometrical functions is available. Otherwise, methods (4) or (5) may be used.

Solution of Plane Triangles by Ordinary Sine Rule.—As an example of convenient arrangement of computation and of suitable independent checks, take the case of the solution of a triangle by the ordinary sine rule. Let the data be as follows :

Station.	Observed Angles.			Adjustment.	Adjusted Angles.		
	°	'	"		°	'	"
A	47	15	50	—10	47	15	40
B	68	35	10	—10	68	35	00
C	64	09	30	—10	64	09	20
	180	00	30	—30	180	00	00
Length AB = 4527.27 feet.							

Here, the observed angles exceed the theoretical sum of 180° by $30''$. Hence, in order to adjust the angles, $10''$ is subtracted from each angle and the adjusted angles, which are the ones to be used in the computation, are set out in the last column. The logarithmic computation is now arranged as follows :

$$\log AC = 3.670\ 529 \quad AC = 4683.05$$

$$\log \sin B = 9.968\ 926$$

$$\log \operatorname{cosec} C = 0.045\ 767$$

$$\log AB = 3.655\ 836$$

$$\log \sin A = 9.865\ 965$$

$$\log BC = 3.567\ 568 \quad BC = 3694.61$$

The different logarithms of the known quantities are arranged in the order shown, when the sum of the first three gives $\log AC$, and that of the last three gives $\log BC$. This is better than the following, more common, arrangement :

$$\log \sin B = 9.968\ 926 \quad \log \sin A = 9.865\ 965$$

$$\log \operatorname{cosec} C = 0.045\ 767 \quad \log \operatorname{cosec} C = 0.045\ 767$$

$$\log AB = 3.655\ 836 \quad \log AB = 3.655\ 836$$

$$\log AC = 3.670\ 529 \quad \log BC = 3.567\ 568$$

as it not only saves extra writing but it avoids an extra possibility of copying error, due to having to write $\log AB$ and $\log \operatorname{cosec} C$ down twice instead of once. A very similar arrangement will be found on pages 262 and 263 when working out the latitudes and departures of a traverse.

If the co-ordinates (page 259) of the points A and B are known, those of C will usually be required. In this case, the bearing of the line AB, if not already given, can be computed, and, from this and the given angles, the bearings of the lines AC and BC can be obtained.

These, together with the values of $\log AC$ and $\log BC$ obtained from the solution of the triangle, give sufficient data to enable the co-ordinates of C to be computed from both A and B and the check therefore consists in seeing if the co-ordinates computed from each point are the same. If they are, the solution of the triangle is correct. This, therefore, not only gives an independent check on the solution of the triangle, but it also gives checked co-ordinate values for the point C .

If the co-ordinates of A and B are not known, or if those of C are not required, the solution can be checked by the test :

$$AB = AC \cdot \cos A + BC \cdot \cos B,$$

and we also have :

$$AC \cdot \sin A = BC \cdot \sin B.$$

Thus, in our example :

$\log AC$	$= 3.670\ 529$	$\log BC$	$= 3.567\ 568$
$\log \cos A$	$= \underline{9.831\ 651}$	$\log \cos B$	$= \underline{9.562\ 468}$
$\log AC \cos A$	$= 3.502\ 180$	$\log BC \cos B$	$= 3.130\ 036$
$AC \cos A$	$= 3178.19$		
$BC \cos B$	$= \underline{1349.08}$		
AB	$= 4527.27$	(Check.)	

Similarly, we also see that $AC \cdot \sin A = BC \cdot \sin B$ is satisfied, but the first is the better and more complete test.

RECTANGULAR CO-ORDINATES

In survey work it is usual to define the position of a point with reference to two lines, drawn through some convenient point, at

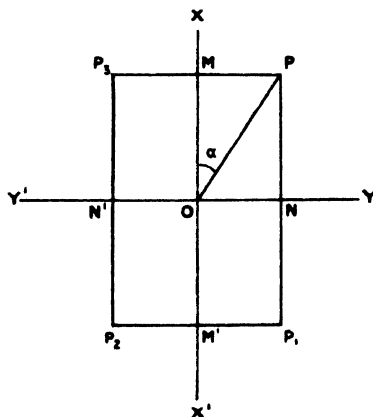


FIG. 189.

right angles to each other. These reference lines are known as the "axis of co-ordinates," and the point of their intersection is known as the "origin of co-ordinates."

In Fig. 189 let OX and OY be the two reference lines and O the origin. Let P be the point whose position is to be defined with reference to O. From P draw PM perpendicular to OX and PN perpendicular to OY. Then the distance OM = PN represents the "X co-ordinate" of P and ON = PM represents the "Y co-ordinate." If these two distances are known, the position of P is completely defined with reference to the origin, because this position can be plotted by laying off OM = NP = X along OX and then drawing MP = ON = Y perpendicular to OM. Distances measured from O in the directions opposite to X and Y, that is along OX¹ and OY¹, are reckoned as negative distances and give negative co-ordinates. Thus, the point P₁ is defined by the distances OM¹ and ON, corresponding to a negative value of X and a positive value of Y.

The position of P can also be defined by means of the length OP = L and the angle XOP = α , because P can be plotted by laying off the line OP at the angle α from OX and then cutting off the distance OP = L. If P lies in the first quadrant, as at P, the angle α lies between 0° and 90°, but if P is in the second quadrant, as at P₁, α lies between 90° and 180°. Following up this line of reasoning we can make the following table.

P. lies in	Sign of Co-ordinates.		Limits of α .
	X	Y	
First Quadrant	+	+	0° to 90°
Second Quadrant	—	+	90° to 180°
Third Quadrant	—	—	180° to 270°
Fourth Quadrant	+	—	270° to 360°

From Fig. 189 it follows that

$$X = L \cos \alpha ; Y = L \sin \alpha$$

$$L = X \sec \alpha ; L = Y \operatorname{cosec} \alpha.$$

These are the equations giving the relations between the co-ordinates X, Y, and the length L and the angle α .

We also have :

$$\tan \alpha = \frac{Y}{X},$$

$$L^2 = X^2 + Y^2.$$

The X axis of co-ordinates is usually taken either as the meridian through the origin O or parallel to some standard meridian, and in that case the angle α becomes the bearing referred to the meridian of origin.

In Fig. 190 we have three points, P, Q, and R, plotted on the plane. The co-ordinates of P are $OM_0 = X_0$, $ON_0 = Y_0$. Those of Q are $OM_1 = X_1$, $ON_1 = Y_1$, and of R, $OM_2 = X_2$, $ON_2 = Y_2$.

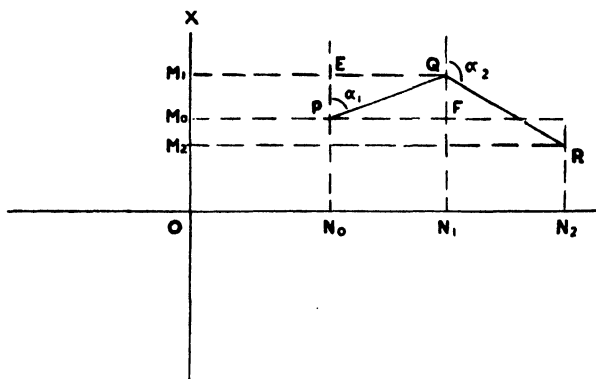


FIG. 190.

If through P we draw lines parallel to OX and OY, we can treat P as a local origin of co-ordinates for the point Q, and the co-ordinates of Q referred to P become $PE = M_0M_1 = OM_1 - OM_0 = X_1 - X_0$; $PF = N_0N_1 = ON_1 - ON_0 = Y_1 - Y_0$. Calling $PE = \Delta X_1$, $PF = \Delta Y_1$, we have

$$X_1 = X_0 + \Delta X_1; \quad Y_1 = Y_0 + \Delta Y_1.$$

Also, if the length of PQ is equal to l , and angle $EPQ = \alpha_1$ so that α_1 is the bearing of the line PQ we have:

$$l \cos \alpha_1 = \Delta X_1; \quad l \sin \alpha_1 = \Delta Y_1$$

$$\tan \alpha_1 = \frac{\Delta Y_1}{\Delta X_1}; \quad l = \sqrt{\Delta X_1^2 + \Delta Y_1^2}.$$

Again, for R:

$$X_2 = OM_2 = OM_0 + M_0M_1 - M_1M_2$$

$$Y_2 = ON_2 = ON_0 + N_0N_1 + N_1N_2.$$

Here, $M_1M_2 = OM_1 - OM_2 = -(OM_2 - OM_1) = -(X_2 - X_1)$ is negative since it falls in the second quadrant with reference to Q. Calling

$$X_2 - X_1 = \Delta X_2, \quad Y_2 - Y_1 = \Delta Y_2$$

we have, when the signs of ΔX_2 and ΔY_2 are taken into account:

$$X_2 = X_0 + \Delta X_1 + \Delta X_2$$

$$Y_2 = Y_0 + \Delta Y_1 + \Delta Y_2.$$

Similarly, for any number of points, we have, when ΔX_1 , ΔX_2 , $\Delta X_3 \dots$, ΔY_1 , ΔY_2 , $\Delta Y_3 \dots$ are given their proper signs:

$$X_n = X_0 + \Delta X_1 + \Delta X_2 + \Delta X_3 + \dots + \Delta X_n$$

$$Y_n = Y_0 + \Delta Y_1 + \Delta Y_2 + \Delta Y_3 + \dots + \Delta Y_n$$

or, if $l_1, l_2, l_3, \dots l_n$ are the lengths and $a_1, a_2, a_3, \dots a_n$ the bearings :

$$X_n = X_0 + l_1 \cos a_1 + l_2 \cos a_2 + l_3 \cos a_3 + \dots + l_n \cos a_n$$

$$Y_n = Y_0 + l_1 \sin a_1 + l_2 \sin a_2 + l_3 \sin a_3 + \dots + l_n \sin a_n.$$

If the points P, Q, R . . . are points on a traverse, the co-ordinate elements $\Delta X_1, \Delta X_2, \Delta X_3, \dots \Delta X_n$ parallel to the axis of X are called the "latitudes" of the different legs and the elements $\Delta Y_1, \Delta Y_2, \Delta Y_3, \dots \Delta Y_n$ are called the "departures." Hence we have the rule :

$$\frac{\text{X Co-ordinate}}{\text{Y Co-ordinate}} \text{ of end point of traverse} = \frac{\text{X Co-ordinate}}{\text{Y Co-ordinate}} \text{ of first point of traverse plus the algebraic sum of all the } \frac{\text{latitudes}}{\text{departures}}$$

Sometimes the X co-ordinate of the end point of a traverse is called the "total latitude" and the Y co-ordinate of the end point the "total departure." Also, the latitudes and departures are sometimes called co-ordinates, but it is better to reserve this term for defining the position of a point with reference to the main axes of co-ordinates.

The signs of the different latitudes and departures are controlled by the quadrant in which the end point of the line joining one point to another lies with respect to the point at the beginning of that line, or, in other words, by the bearing of the line. In fact, we can draw up a table almost exactly similar to that already drawn up for defining the signs of the co-ordinates.

Position of line with respect to initial point of line.	Sign of Latitude.	Sign of Departure.	Limits of α .
First Quadrant	+	+	0° to 90°
Second Quadrant	-	+	90° to 180°
Third Quadrant	-	-	180° to 270°
Fourth Quadrant	+	-	270° to 360°

In the rules given above and in all subsequent work in this chapter we will assume that bearings are reckoned clockwise, from 0° to 360° , from the positive direction of the axis of X. For purposes of computation, however, it will often be convenient, especially when using tables which only give trigonometrical functions for angles from 0° to 90° , to use "reduced bearings," which have already been defined and described on page 207.

Computation of Latitudes and Departures from Bearings and Distances.—The formulæ for computing latitudes and departures have already been derived above and are

$$\text{Latitude} = \Delta X = l \cdot \cos \alpha$$

$$\text{Departure} = \Delta Y = l \cdot \sin \alpha.$$

In order to illustrate the most convenient method of computation we will take a numerical example in which

$$l = 1745.28, \text{ and } \alpha = 164^\circ 15' 20''.$$

Hence the end point of the line lies in the second quadrant so that the latitude is minus and the departure plus. Also, the reduced bearing is $15^\circ 44' 40''$. For logarithmic calculation arrange the work as follows :

$$\begin{array}{rcl} \log \Delta X & = & 3.225 \ 2576 \\ \log \cos \alpha & = & 9.983 \ 3924 \\ \log l & = & 3.241 \ 8652 \\ \log \sin \alpha & = & 9.433 \ 5256 \\ \log \Delta Y & = & 2.675 \ 3908 \end{array} \qquad \begin{array}{l} \Delta X = -1679.80 \\ \Delta Y = +473.58 \end{array}$$

Here, a space is left on top for the addition of $\log l$ and $\log \cos \alpha$. Then $\log l$ is written down in the position shown. Above it is written $\log \cos \alpha$ and below it $\log \sin \alpha$. The addition of the first two logarithms gives $\log \Delta X$, and the addition of the last two gives $\log \Delta Y$. Having found $\log \Delta X$ and $\log \Delta Y$, ΔX and ΔY are written down, with their proper signs, as shown.

This arrangement of the computation is much better than the usual one of writing down $\log l$ twice, with $\log \cos \alpha$ written under one set of figures and $\log \sin \alpha$ under the other. Having to write down $\log l$ twice means that the chances of the occurrence of copying errors are increased. Also, the arrangement shown saves one column in the latitude and departure sheets (page 268) when, as is usual, the computation is done and shown on these. This may be an advantage when, for filing purposes, it is desired to restrict the size of the latitude and departure sheets so that they are no larger, or very little larger, than ordinary foolscap size.

It will be noticed that, following the usual practice in survey work, the negative characteristics in the $\log \sin$ and $\log \cos$ are replaced by 9 and that 10 is subtracted from the sums of the characteristics.

For machine computation, the work is set out as follows :

$$\begin{array}{r} \Delta X = -1679.80 \\ \cos \alpha = 0.962 \ 4815 \\ l = 1745.28 \\ \sin \alpha = 0.271 \ 3471 \\ \Delta Y = +473.58 \end{array}$$

Here, l and $\cos \alpha$ are multiplied together to give ΔX , and l and $\sin \alpha$ are multiplied together to give ΔY .

Check Computation of Latitudes and Departures.—*Auxiliary Bearings.*—Traverse tables may be used to compute or check the latitudes and departures of a compass traverse or of a low order theodolite one, but, apart from giving a rough check against, or to locate, a gross error, they are of little use either for computing or

checking higher class theodolite traverses. The method now to be described, however, although at present little known or used in England, is an extremely valuable one since it is efficient and accurate and is about as independent of the method used for the original computation as it is possible to make it.

It can easily be proved that :

$$l \cos a = \frac{l}{\sqrt{2}} \sin (a+45^\circ) + \frac{l}{\sqrt{2}} \cos (a+45^\circ)$$

$$l \sin a = \frac{l}{\sqrt{2}} \sin (a+45^\circ) - \frac{l}{\sqrt{2}} \cos (a+45^\circ).$$

The method therefore consists in adding 45° to each bearing of the traverse and so obtaining new bearings, which, for the sake of convenience, we will call "auxiliary bearings."

Two quantities, C and S, are then calculated from :

$$C = \frac{l}{\sqrt{2}} \cos (a+45^\circ)$$

$$S = \frac{l}{\sqrt{2}} \sin (a+45^\circ)$$

and we then have :

$$\text{Latitude} = \Delta X = S + C.$$

$$\text{Departure} = \Delta Y = S - C.$$

The signs of the quantities S and C are dependent on the quadrant in which the auxiliary bearing, $a+45^\circ$, lies, and follow exactly the same rules as those for latitudes and departures, as computed from ordinary bearings.

The numerical value of $1/\sqrt{2}$ is 0.707 1068 and its logarithm is $\bar{1}.849\ 4850$. The computation is best done on a printed form, on which, in the case of the logarithmic computation, the logarithm is printed direct in a suitable place. A specimen form of this kind is given later (Traverse Form B).

As an example, take the data already used for the direct computation. Here, $l = 1745.28$, $a = 164^\circ\ 15'\ 20''$, $a+45^\circ = 209^\circ\ 15'\ 20''$, reduced auxiliary bearing $= 29^\circ\ 15'\ 20''$.

$$\begin{array}{lll} \log C & = 3.032\ 0900 & C = -1076.69 \\ \log \cos (a+45^\circ) & = 9.940\ 7398 & \\ \log l & = 3.241\ 8652 & \\ \log \frac{1}{\sqrt{2}} & = 9.849\ 4850 & \\ \log \sin (a+45^\circ) & = 9.689\ 0475 & \\ \log S & = 2.780\ 3977 & S = -603.11 \\ \therefore \Delta X = S + C & = -1076.69 - 603.11 & = -1679.80, \\ \Delta Y = S - C & = -603.11 - (-1076.69) & = +473.58. \end{array}$$

The results, therefore, are exactly the same as those obtained by the direct method.

In this computation, log C is obtained by adding together the three logarithms below the upper line and log S by adding together the three above the lower line. C and S are both negative because the auxiliary bearing $\alpha + 45^\circ$ falls in the third quadrant. Note, also, that the logarithm of $1/\sqrt{2}$ has been written with 9 as a characteristic instead of the negative 1. Consequently, as the same thing has been done with regard to the logarithms of the cosine and sine, 20 is subtracted from the characteristic in each summation.

For machine computation, the work is arranged as follows :

$$\begin{array}{rcl}
 C & = & -1076.69 \\
 \cos (\alpha + 45^\circ) & = & 0.872 \ 4486 \\
 \frac{l}{\sqrt{2}} & = & 1234.10 \\
 \sin (\alpha + 45^\circ) & = & 0.488 \ 7058 \\
 S & = & -603.11 \\
 \\
 S & = & -603.11 \\
 C & = & -1076.69 \\
 \hline
 \Delta X & = & -1679.80 \\
 \\
 S & = & -603.11 \\
 C & = & -1076.69 \\
 \hline
 \Delta Y & = & +473.58
 \end{array}$$

Here, l is multiplied direct by $1/\sqrt{2}$, the result being written down as shown and then used to multiply the cosine and sine in turn. In the computation form, the value of $1/\sqrt{2} = 0.707 \ 1068$ —can be printed at the top of the column used for this computation, so that it is always before the computer as he needs it.*

Computation of Bearing and Distance from Co-ordinates.—If we are given the co-ordinates of two points we can compute the bearing and distance between them, this being the reverse problem to the previous one. The formulæ are :

$$\begin{aligned}
 \tan \alpha &= \frac{\Delta Y}{\Delta X} \\
 l &= \frac{\Delta X}{\cos \alpha} = \frac{\Delta Y}{\sin \alpha}
 \end{aligned}$$

ΔX and ΔY are obtained as the differences of the X's and the Y's of the two points respectively. The quadrant in which the bearing lies depends on the signs of ΔX and ΔY .

As an example, let the co-ordinates of the points B and A be

* See also Appendix III for additional notes on the checking of latitudes and departures.

$$\begin{aligned}
X_a &= 4379.28 & Y_a &= 7849.36 \\
X_b &= 6186.32 & Y_b &= 7764.28 \\
\Delta X &= -1807.04 & \Delta Y &= +85.08 \\
\log \Delta Y &= 1.929 \ 8275 \\
\log \Delta X &= 3.256 \ 9678 \\
\log \tan \alpha &= 8.672 \ 8597 \\
180 - \alpha &= 2^\circ 41' 44''; \alpha = 177^\circ 18' 16''
\end{aligned}$$

$$\begin{aligned}
\log \Delta X &= 3.256 \ 9678 & \log \Delta Y &= 1.929 \ 8275 \\
\log \cos &= 9.999 \ 5192 & \log \sin \alpha &= 8.672 \ 3654 \\
\log l &= 3.257 \ 4486 & \log l &= 3.257 \ 4621 \\
l &= 1809.04 & l &= 1809.10
\end{aligned}$$

$$\text{Bearing } \overline{AB} = 177^\circ 18' 16''; \text{ Bearing } \overline{BA'} = 357^\circ 18' 16''$$

The bearing from A to B must be in the second quadrant because ΔX is negative and ΔY positive in going from A to B.

This example has been chosen purposely to illustrate the point referred to on page 255 regarding the choice of formulae when angles are small, or are near 90° in value. Here, the reduced bearing is a very small angle so that its log tangent and log sine are changing very rapidly and irregularly. In this case, the value to be taken for l is that computed from the cosine—1809.04. In order to obtain better agreement between the values computed from the sine and cosine it would have been necessary to work out the bearing to decimals of a second, instead of to single seconds, but, as the example is taken from an ordinary survey, and not from geodetic work, so that geodetic accuracy is not required, it is hardly necessary to go to such extremes. (See also page 266).

Check Computation of Bearing and Distance by Use of Auxiliary Bearings.—Bearings and distances can also be computed from given co-ordinates by the use of auxiliary bearings. Being given the co-ordinates of the two points, we know ΔX and ΔY . Then :

$$\begin{aligned}
S+C &= \Delta X \\
S-C &= \Delta Y \\
S &= \frac{1}{2} (\Delta X + \Delta Y) \\
C &= \frac{1}{2} (\Delta X - \Delta Y) \\
\tan (\alpha + 45^\circ) &= \frac{S}{C}
\end{aligned}$$

and

$$l = \frac{\sqrt{2} C}{\cos (\alpha + 45^\circ)} = \frac{\sqrt{2} S}{\sin (\alpha + 45^\circ)}.$$

Using the data of the last example :

$$\begin{array}{rcl}
 \Delta X & = & -1807.04 \\
 \therefore S & = & -860.98 \\
 \log S & = & 2.934\ 9931 \\
 \log C & = & 2.975\ 9187 \\
 \log \tan (a+45^\circ) & = & 9.959\ 0744 \\
 \therefore (45^\circ + a) - 180^\circ & = & 42^\circ\ 18'\ 16''; \ a = 177^\circ\ 18'\ 16'' \\
 \log C & = & 2.975\ 9187 \\
 \log \sqrt{2} & = & 0.150\ 5150 \\
 \log \cos (a+45^\circ) & = & 9.868\ 9846 \\
 \log l & = & 3.257\ 4491 \\
 l & = & 1809.04 \\
 \Delta Y & = & +85.08 \\
 C & = & -946.06 \\
 \log S & = & 2.934\ 9931 \\
 \log \sqrt{2} & = & 0.150\ 5150 \\
 \log \sin (a+45^\circ) & = & 9.828\ 0601 \\
 \log l & = & 3.257\ 4490 \\
 l & = & 1809.04
 \end{array}$$

Here, the auxiliary bearing $(45^\circ + a)$ must lie in the third quadrant since both S and C are negative.

It will be seen that, by using auxiliary bearings, the discrepancy between the values of $\log l$ computed from the cosine and sine is eliminated in this particular example. Hence, this suggests that, if in computations of this or of a similar kind the reduced bearing is very small, or is near 90° in value, the disadvantage of the rapidly changing log tangent or log cotangent and log sine or log cosine can be avoided by the use of auxiliary bearings, as, in such a case, these have the effect of giving a working angle of somewhere near 45° in value.

THEODOLITE TRAVERSE COMPUTATIONS

The computations of a traverse involve the following operations :

- (1) Application of all necessary corrections to the measured lengths.
- (2) Reduction and meaning of measured angles in the field books.
- (3) Abstracting measured lengths and angles on to the co-ordinate sheets.
- (4) Computation of bearings.
- (5) Adjustment of bearings if traverse is either a closed surround or ends on a line of fixed bearing.
- (6) Computation of reduced bearings.
- (7) Computation of latitudes and departures.
- (8) Computation of co-ordinates.
- (9) Check computation.
- (10) Adjustment of co-ordinates if traverse is either a closed one or ends on a point whose co-ordinates are fixed and known.

The operations (3), (4), (5), (6), (7), (8), and (10) are all carried out on one form, a specimen of which is given later. The check computation repeats operations (1), (2), (3) . . . (8), and (10) and is carried out quite independently of the original computation on an entirely separate form. If possible, different computers should be responsible for the original and check computations. In any event, owing to the ease with which mistakes are made and repeated, the two computations should be entirely independent and both should start at the very beginning and not be compared until the end is reached.

The examples which will be given to illustrate methods and procedure are taken, unless stated otherwise, from a high class, but not a "precise," theodolite traverse, in which taping was done with a steel tape suspended in catenary, and angles were measured to the nearest 10" of arc with a small micrometer theodolite, the latitudes and departures being worked out by means of seven-figure logarithms, as these are often the most convenient to use when bearings are carried to seconds and tables such as Shortrede's are used. The fundamental principles and operations, however, are the same for all classes of traverse, so that it will not be difficult, with these examples as basis, to adapt the methods or instruments of computation to traverses of lower order. Thus, if angles are only read to single minutes of arc, it will be quite sufficient, when working out latitudes and departures, to use five-figure logarithms, and distances, if measured to two decimal places, can be rounded off to one place, the latitudes and departures also being computed to one place only.

Corrections to Distances and Reduction and Meaning of Angles.—

The various corrections to be applied to the measured distances have been described in Chap. II and the methods of reducing and meaning angles in Chap. IV. These operations are best done and shown in the field books themselves. The only thing to be seen to before proceeding further is that all the work up to this stage has been completely and thoroughly checked, as, of course, any error will affect subsequent work and perhaps render it valueless.

Abstracting Lengths and Angles.—Having made certain that all the data are correct, the distances are entered in the sixth column of the computation form (Traverse Form "A," page 268), and the angles in the third, the station identification numbers being entered in the first and second columns, as shown in the example. In this example, the initial station was station A, and the first backsight was taken to a trigonometrical point, No. 346, to which the bearing from A was known. This bearing is entered as shown in the first line and the fourth column, and the angle at A— $102^{\circ} 22' 20''$ —between point 346 and station B entered in the second line and the third column, the distance from A to B—3937.28—being entered on the same line and in the sixth column. The other station numbers, angles, and

distances are then entered consecutively in their proper positions on the form.

When all the data have been written down, the entries should be very carefully checked. To do this, the computer should get somebody else to take the form and check the entries, while he calls the figures out from the field books, the person checking the form repeating each figure as it is called out.

Computation of Bearings.—This computation is done on the form itself and the bearings entered in the fourth column. The method of computing bearings has already been described in Chap. IV. After the last bearing on each sheet has been obtained, the included angles on that sheet should be summed, and this sum added to the first bearing. This checks the working out of the bearings (see page 216).

Adjustment of Bearings.—If the traverse is a closed one, or ends on a line of known bearing, the difference between the bearing brought forward through the traverse and the known bearing at the last station gives the closing error in bearing, and it is usual, before working out co-ordinates, to adjust the intermediate bearings to make the last one agree with the known bearing at the end. The rule commonly employed to do this is as follows :

Let e be the closing error in bearing and n the number of legs through which this has to be distributed. Then the correction to the first bearing is e/n , to the second $2e/n$, to the third $3e/n$, and so on to the last, which is $ne/n = e$. The same result could, of course, be obtained by adding e/n to each of the observed angles, but it is more convenient to adjust the bearings direct.

In the example given on the form, the closing error at the end of twenty stations was $1' 40''$ or $100''$, which is rather large for the instrument used, the measured bearing being less than the fixed one. Hence, the correction to be applied to the first bearing is

$$+\frac{100}{20} = +5'', \text{ to the second } +10'', \text{ to the third } +15'', \text{ and so on.}$$

These corrections are entered underneath the bearing to which they refer, and the corrected bearings worked out and written as shown.

Computation of Reduced Bearings.—If “Shortrede’s Tables” are to be used for the computation of the latitudes and departures it will not be necessary to work out reduced bearings (page 207), as these tables give the log cos and log sin of every angle from 0° to 360° . If, however, ordinary tables, which only give trigonometrical functions from 0° to 90° are to be used, it will be convenient to work out the reduced bearings, which are entered in the fifth column of the form. The figures in this column should be carefully checked before proceeding further by putting a blank sheet of paper to cover the column and the columns to the right, but leaving the columns

Survey - Guruda Estate, Sarawak Traverse

TRAVERSE FORM A

Origin of Co-ordinates

Point 10000 ft. W.
and 10000 ft. S
of Δ322

Reference Meridian for Bearings Meridian through Δ322

Check Computation File B.34.

Field Books B.28

Date of Survey 16.8.37

Surveyor J. Smoots
Computer M.E. Smith.

From	To	Included Angle	Bearing	Reduced Bearing	Length	log Δ X log cos α log sin α log Δ Y	Latitude		Departure		Co-ordinates		Remarks
							+(N)	-(S)	+(E)	-(W)	X	Y	
A	Δ346		253 16 30								20,114.28	16,748.17	
A	B	108 22 20	355 38 80 + 5 355 38 55	4 21 05	3937.28	3.593 9427 9.998 7463 3.596 1964 8.880 0877 2.475 2841	3925.93	—	—	298.73	24,040.21 + 0.19 24,040.40	16 449.44 - 0.16 16 449.28	A is complete Pillar No. 127 on Traverse B.48.
B	C	194 18 30	9 37 20 + 10 9 37 30	9 37 30	2648.10	3.416 3414 9.993 4070 3.422 9344 9.237 8753 2.660 8097	2608.20	—	457.94	—	2.664.8.41 + 0.33 2664.8.74	16 907.38 - 0.26 16 907.12	
C	D	198 13 20	28 10 40 + 15 28 10 55	28 10 55	3348.17	3.470 0063 9.948 1988 3.524 8075 9.674 1931 3.199 0006	2951.25	—	1581.25	—	29 599.66 + 0.50 29 600.16	18 488.63 - 0.40 18 488.23	
D	E	264 34 10	113 04 30 + 20 113 05 10	66 54 50	1124.98	2.644 5572 9.593 4123 3.051 1447 9.963 7484 3.014 8931	—	441.12	1034.89	—	29 188.54 + 0.66 29 189.10	19 523.52 - 0.44 19 523.08	
E	F	177 14 10	110 19 10 + 25 110 19 35	69 40 25	973.64	2.529 1877 9.540 7093 2.986 3984 9.972 0774 2.960 4758	—	338.21	918.01	—	28 820.33 + 0.60 28 820.93	20 436.53 - 0.48 20 436.05	
F	G	208 27 30	138 46 40 + 30 138 47 10	41 12 50	1007.19	2.879 4766 9.876 3682 3.003 1114 9.818 8009 2.821 9123	—	757.66	663.61	—	28 042.87 + 0.65 28 043.52	21 100.14 - 0.52 21 099.62	
G	H	220 17 50	179 04 30 + 35 179 05 05	0 54 55	1621.18	3.209 7758 9.999 9446 3.209 8312 8.203 4118 1.413 2430	—	1620.97	25.90	—	26 441.70 + 0.73 26 442.43	21 126.04 - 0.59 21 125.45	
H	K	177 47 30	176 32 00 + 40 176 32 40	3 07 20	779.79	2.891 3326 9.999 3549 2.891 9777 8.736 1262 1.628 1039	—	778.63	42.47	—	25 663.07 + 0.77 25 663.84	21 168.51 - 0.62 21 167.89	
K	L	181 52 20	178 44 20 + 48 178 45 08	1 14 55	878.24	2.943 5101 9.999 8969 2.943 6132 8.338 2702 1.281 8834	—	878.03	19.14	—	24 785.04 + 0.82 24 785.86	21 187.65 - 0.66 21 186.99	
L	M	166 29 30 1889 57 20 253 16 30 2143 13 30 1980 163 13 50	163 13 50 + 50 163 14 40		16318.57	9.685.38 4.814.08 4.670.78 20.114.28 24.785.86	4814.62		4738.21 298.73 4439.48 16748.17 21187.65				

to the left of column five open and visible, and then re-working the reduced bearings on the sheet of paper.

Computation of Latitudes and Departures.—The latitudes and departures are computed in the space provided in the seventh column of the form. The method to be used, and the manner in which the computation is made, are described on page 262. The example shows a logarithmic computation but the space can also be used for computation by machine, the only entries in this case being the natural cosine and sine of the angles, as the length of the line is given in the previous column.

As the values of the latitudes and departures are found, they are entered in the next four columns, separate columns being used for positive and negative values. Signs, of course, are controlled by the values of the bearings entered in column four. It would usually save time if the computer, before he works out latitudes and departures, examines the bearings and draws a short line—to show a blank—in each space in the latitude and longitude columns where no entry will occur. This will save him having to think about signs when looking out anti-logarithms or reading off multiplication results from the computing machine.

One very common type of error in traverse computations is to enter a latitude or departure with the wrong sign, or else to enter a latitude in a departure column and vice versa. Errors of this kind can easily be seen by inspection and, as each sheet is compiled, it is well to examine the latitudes and departures to see that all are entered correctly with the correct sign. If the reduced bearing is less than 45° the latitude will be numerically greater than the departure, but if the reduced bearing is greater than 45° , the latitude will be less than the departure. Hence, it is easy to see if a latitude and departure have been interchanged.

Computation of Co-ordinates.—When the latitudes and departures have been calculated, the co-ordinates are easily obtained. If the initial point A has definite co-ordinates, the values of these should be entered in the top line in the last two columns on the form. If the initial point is not a fixed one, so that there are no definite values for it, zero co-ordinate values in X and Y could be assigned to it; but it is better to assign arbitrary positive values—each a multiple of 1,000—so that there will be no negative co-ordinates in the whole survey. This is equivalent to choosing an arbitrary “false origin,” in such a position that it lies to the south and west of any point included in the survey. The values assumed for the co-ordinate values of A then become the co-ordinates of this point referred to axes parallel to the true axes through A, but intersecting at the false origin.

Having written down the co-ordinate values of A in the appropriate columns, the latitude and departure to the point B are now added algebraically to these values and the results are the co-ordinates of B.

These are written down and the latitude and departure from B to C added algebraically to them to give the co-ordinates of C. Proceeding from point to point in this way, we arrive at the co-ordinate values of the last point entered on the sheet.

Before going on to the next sheet the work already completed should be checked. To do this, add together the figures in each of the latitude and departure columns and subtract the sum of the minus latitude column from the sum of the positive column. To the result add the X co-ordinate of the first point on the sheet, when the figure obtained should be exactly equal to the X co-ordinate of the last point on the sheet. Similarly, check the last Y co-ordinate from the algebraic sum of the sums of the positive and negative departure columns. This test is a very simple one, and it should be used for every sheet as soon as the other work on it is completed. The check is clearly shown in the example given.

Check Computation.—When a rough check against gross error only is needed, or it is known that a gross error exists and it is required to locate it, a check by traverse tables, or by some other approximate method, will often suffice; but, if the traverse is an important one, a thorough and complete check, of a standard of accuracy not less than that of the field work, is essential. For example, if the traverse were measured in order to establish the alignment of a long tunnel, the ends of which were not intervisible, an error of only a very few feet might have serious consequences and cause considerable expense.

Assuming that a complete and thorough check is required, by far the most thorough and reliable one that can be used is that of auxiliary bearings. This check computation can be carried out on a form somewhat similar to the ordinary traverse form, the use of which has just been described. The second form, Traverse Form B, is very similar to the first but has an extra column and slightly different headings for some of the others.

The check computation cannot be considered to be complete if data are copied from the original sheets. Accordingly, the latter should be put away and all the data—distances and included angles—entered direct from the field books in the appropriate columns of the form. The initial and closing bearings to be used are now the original bearings with 45° added to each. Bearings are worked out and adjusted as before, and since every bearing, including the fixed initial and final ones, will now differ by a constant amount, 45° , from the original ones, the closing error and the correction for each bearing will be the same as before.

When the new reduced bearings have been worked out they can be compared with the original reduced bearings. The sum or the difference of each corresponding pair of reduced bearings should be either 45° or 135° .

The computation of S and C is now made in the seventh column

Computer *J. Jones.*

From	To	Included Angle	Auxiliary Bearing (45°+)	Reduced Auxiliary Bearing	Length	log C log cos (45°+) log P log $\frac{P}{C}$ log sin (45°+) log S	C S	Latitude (ΔX) = S + C		Departure (ΔY) = S - C		Co-ordinates	
								+(N)	-(S)	+(E)	-(W)	X	Y
A	$\Delta 346$		298 16 30									201116.28	16748.17
A	B	102 22 20	40 38 50 + 5 40 38 55	40 38 55	3937.18	3.324 7823 9.880 0809 3.595 1964 9.849 4850 9.813 8598 3.258 5412	+ 2112.33 + 1813.60	3925.93	-	-	298.73	24040.21	16748.17
B	C	194 18 30	54 57 20 + 10 54 57 30	54 57 30	2648.10 6585.38	2.031 4614 9.759 0420 3.422 9346 9.849 4850 9.913 1432 3.185 5626	+ 1075.13 + 1833.07	2508.20	-	457.94	-	26648.41	16907.38
C	D	198 13 20	73 10 40 + 15 73 10 55	73 10 55	3348.17 9933.55	2.835 6912 9.461 3987 3.524 8075 9.849 4850 9.981 0156 3.355 3081	+ 885.00 + 2266.25	2951.25	-	1581.25	-	29599.66	18488.63
D	E	264 54 10	158 04 50 + 20 158 05 10	21 54 50	1124.98 11058.53	2.868 0587 9.957 4290 3.051 1447 9.849 4850 9.571 9564 2.472 5861	- 738.00 + 296.88	-	441.12	1034.88	-	29158.54	19523.51
E	F	177 14 20	155 19 10 + 25 155 19 35	24 40 25	973.64 12032.17	2.796 3042 9.958 4208 2.988 3984 9.849 4850 9.620 6030 2.458 4864	- 625.61 + 287.40	-	338.21	913.01	-	28820.33	20436.52
F	G	208 27 30	183 46 40 + 30 183 47 10	3 47 10	1007.19 13039.36	2.851 6475 9.999 0911 3.003 1114 9.849 4850 8.819 7546 1.672 3510	- 710.64 - 47.03	-	757.67	663.61	-	28062.66	21100.13
G	H	210 17 50	224 04 30 + 35 224 05 05	44 05 05	1621.18 14660.54 14660.54	2.915 6292 9.856 3130 3.209 8312 9.849 4850 9.842 4353 2.901 7515	- 823.43 - 797.54	-	1620.97 3157.97	25.89 298.73	-	26441.69	21126.02
		1365 48 00 298 16 30 1664 04 30 1440 224 04 30						9485.38 3157.97 6327.41 20114.28 26441.69		4676.58 298.73 4377.85 16748.17 21126.02			

of the form, using the method described on page 262. Here, it will be a convenience if a printed form, with the logarithm of $1/\sqrt{2}$ printed in its appropriate place, as shown in the specimen, is used; if a printed form is not used, this logarithm is written in the form in all the places where it will be required.

The eighth column of the form contains the values of C and S, each written with its proper sign. Signs, of course, are governed by the quadrant in which the auxiliary bearing, as given in the fourth column of the form, falls.

Having obtained the C's and S's, the latitudes and departures are found by addition and subtraction and entered in the columns provided for them. From this stage on, the form is completed in exactly the same manner as the original form. Small differences of a unit in the last decimal place, due to rounding off of decimal places, will occasionally appear in the co-ordinate values, but these are of no importance. A comparison of the two forms shows that the only figure which remains the same in both computations is the logarithm of the length. The equivalent bearings and reduced bearings are entirely different so that entirely different angles are used in the logarithmic work or in the multiplications on a machine. This method of checking traverse computations, if properly carried out, is, therefore, almost fool-proof and is the most efficient one available.*

Adjustment of Traverses.†—If the traverse starts and ends on points whose co-ordinates are fixed and known it will be found that the co-ordinates obtained for the end point will differ slightly from the fixed values. The differences so found are caused by an accumulation of small errors and are unavoidable. It is, however, desirable to adjust the whole traverse so that the differences between the computed and the fixed co-ordinates of the end point are eliminated and the closing error is properly distributed throughout the various stations of the traverse.

A traverse may be adjusted by applying corrections to the latitudes and departures, or directly to the co-ordinates, or else by means of corrections applied to the individual lengths and bearings. Since it is usually the co-ordinates that are ultimately required, and not the distances and bearings from which they are derived, it is generally more convenient to use corrections that can be applied direct to the co-ordinates.

Of the methods available for the adjustment of traverses there is none that is really simple and at the same time theoretically sound. Probably the one most commonly used, especially for the adjustment of compass traverses, is that due to Bowditch. This method depends on the assumption that the probable errors in length and bearing

* See also Appendix III for additional notes on check computations.

† An interesting series of articles on traverse adjustment appears in the *Empire Survey Review*, Vol. IV, Nos. 28, 29 and 30, and Vol. V, No. 31. (April 1938, July 1938, October 1938, and January 1939.)

produce equal displacements at the end of the leg and that the probable errors of the linear measurements are proportional to the *square roots of the lengths of the legs*. From these two assumptions it follows that the probable errors of the bearings must be proportional to the reciprocals of the square roots of the lengths of the legs, and, as we have already seen, neither this, nor the assumption regarding the probable errors of the linear measurements, is a sound one.

From a purely theoretical point of view, the Bowditch rule is more suitable for the adjustment of compass traverses than it is for that of theodolite traverses, because it has been worked out for the case in which bearings are observed directly, as in a compass traverse, whereas, of course, in a theodolite traverse it is angles, and not bearings, that are observed. Moreover, the corrections obtained are corrections to the latitudes and departures and one disadvantage of the rule is that these corrections may cause excessive disturbance to the bearings, though, if co-ordinates alone are needed, this disadvantage does not make itself apparent, nor does it matter much. However, for ordinary work, as opposed to precise traversing for primary framework purposes, any elaborate method of adjustment, which involves a considerable amount of labour, is hardly justified, so that Bowditch's rule can quite safely be, and usually is, employed in nearly all normal cases of traverse adjustment.

The Bowditch rule is :

$$\text{Correction to } \frac{\text{latitude}}{\text{departure}} = \text{Closing Error in } \frac{\text{latitude}}{\text{departure}} \\ \times \left(\frac{\text{length of corresponding side}}{\text{total length of traverse}} \right)$$

the sign varying with the sign of the closing error at the end.

In the example given on the form, let the total length of the traverse be 35,646 ft. and let the fixed and computed co-ordinates of the end point be :

	X	Y
Fixed Values . . .	33,131.26	27,604.93
Computed Values . . .	33,129.48	27,606.36
Closing Error . . .	+1.78	-1.43

Then, according to Bowditch's rule, the correction to the X co-ordinates will be $+\frac{1.78}{356.5} = +0.0050$ ft. per 100 ft., and to the

Y co-ordinates $-\frac{1.43}{356.5} = -0.0040$ ft. per 100 ft.

The first step is to add each length to the sum of the lengths before it, so as to get the total distance of each point from the initial point of the traverse, these total lengths being entered under the length of each leg. In the example, take the leg EF. The total distance of the point F from the beginning of the traverse is 12,032 ft. Rounding

off to the nearest 10 ft., this is 120.3 in hundreds of feet. Hence the corrections to the co-ordinates of F are $+120.3 \times 0.0050 = +0.60$ ft. in X and $-120.3 \times 0.0040 = -0.48$ ft. in Y. These corrections are entered underneath the co-ordinates of F, and the corrected co-ordinates written down, as shown, the same being done for each point of the traverse.

It will be noticed that in this example we have applied the corrections to the co-ordinates direct and not to the individual latitudes and departures. This is by far the more convenient procedure, and it is obvious that it amounts to the same thing as correcting the latitudes and departures, as, in computing the corrections, we have used the total length from the beginning of the traverse up to the point being adjusted, instead of the length of the corresponding leg.

If the corrections are small, they may be derived very quickly either by slide rule or even graphically by the method outlined on page 279 in connection with the adjustment of compass traverses.

The Bowditch rule can easily be amended to fit the assumption that the probable errors of the linear measurements are proportional to the length of the leg and the probable errors of the angular measurements proportional to the reciprocals of the lengths of the legs. In this case, the factor which multiplies the closing error in latitude or departure is the square of the length of the side divided by the sum of the squares of the lengths of all the sides. This, of course, involves more arithmetical work than the ordinary rule, and, considering that the assumption made regarding the probable errors of the bearings is still unsound, it is doubtful if the extra labour is worth while.

Theodolite traverses are often adjusted by making the correction in latitude (or departure) equal to the closing error in latitude (or departure) multiplied by the latitude (or departure) of the leg to be adjusted and divided by the arithmetical sum of all the latitudes (or departures). This rule is purely empirical, and there is no sound theoretical foundation for it.

If it is assumed that the method of adjusting the angles already described does give the most probable values for the angles, and that any further adjustment should not disturb these values, so that the only measured quantities to be adjusted are the lengths, it is easy to obtain formulæ for corrections which can be applied directly to the latitudes and departures. These formulæ are given in various textbooks, such as Crandall's "Text Book of Geodesy and Least Squares" or Jameson's "Advanced Surveying," but they all involve a considerable amount of arithmetical work. The assumption made in obtaining them generally is that the probable errors of the linear measurements are proportional to the square roots of the lengths of the legs, but it is easy to modify them to suit the case where the probable errors are assumed to be directly proportional to the lengths of the legs.

In all probability, the simplest method so far devised of adjusting the bearings and lengths of a traverse to fulfil the conditions of closure in bearing and in position, and which has a reasonably sound theoretical justification behind it, is one described by Mr. H. L. P. Jolly in the *Empire Survey Review*, Vol. IV, No. 28, April 1938. This method involves corrections to each bearing and length so that corrections to the co-ordinates have to be worked out subsequently and are not obtained directly. The labour is therefore much greater than that required for the application of the Bowditch rule, so that, whatever defects this latter rule may have from a theoretical point of view, it probably remains the most practical one so far as the type of traverse considered in this volume is concerned.

Adjustment of a Network of Traverses.—It often happens that the framework for a large survey consists of a network, in which different traverses intersect at common points, and it is desired to determine the most likely values for the points of intersection. The best solution of this problem undoubtedly is one by the method of least squares, in which suitable weights are given to the observed values obtained from each traverse. When great precision is not desired, however, a system of "weighted means" may be adopted as an alternative to the least square solution, and this method will usually yield results that are quite good enough in ordinary practice. The co-ordinates of the more important points of intersection are first obtained, and these values used, where necessary, to determine the co-ordinates of the less important points.

The following is an example of fixing the position of a point in a network when the probable errors of displacement are assumed to be proportional to the length of the traverse. In this case the weights to be assigned to the values obtained from each traverse will be inversely proportional to the squares of the lengths. Let the point P be fixed from four points A, B, C, and D and let the co-ordinates of P, as determined directly from each of the other points, be as follows :

From - Point.	Co-ordinates.		Length/ 10,000 (L).	L^2	$\frac{1}{L^2}$
	X	Y			
A	36945.3	74268.9	0.548	0.3003	3.33
B	36946.6	74267.3	0.748	0.5595	1.79
C	36946.9	74268.1	0.916	0.8391	1.19
D	36944.7	74269.5	1.010	1.0201	0.98
					7.29

Here, the fourth column gives the length of each connecting traverse in units of 10,000 feet, and the last column gives the reciprocal of the square of this number, so that this reciprocal represents the weight to be assigned to the value obtained from each traverse.

Taking the feet and decimals only, the weighted mean of the last two figures in the X co-ordinate of P is :

$$\frac{5.3 \times 3.33 + 6.6 \times 1.79 + 6.9 \times 1.19 + 4.7 \times 0.98}{7.29} = 5.8.$$

Similarly the weighted mean of the last two figures in the Y co-ordinate is :

$$\frac{8.9 \times 3.33 + 7.3 \times 1.79 + 8.1 \times 1.19 + 9.5 \times 0.98}{7.29} = 8.5.$$

Hence, the adjusted co-ordinates of P are :

$$X = 36945.8 ; Y = 74268.5$$

Corrections of Latitudes and Departures for Small Corrections in Bearings and Distances.—If, after the latitudes and departures have been computed, small corrections have to be applied to the bearings and distances in a traverse, and new corrected co-ordinates are required, it is not necessary to re-compute the latitude and departures from the corrected bearings and distances.

On page 235 we obtained the formulæ :

$$\delta x = \delta l \cdot \cos \alpha - l \cdot \delta \alpha \cdot \sin \alpha .$$

$$\delta y = \delta l \cdot \sin \alpha + l \cdot \delta \alpha \cdot \cos \alpha .$$

or, if $\delta \alpha$ is expressed in seconds of arc :

$$\delta x = \delta l \cdot \cos \alpha - l \cdot \delta \alpha \cdot \sin 1'' \cdot \sin \alpha .$$

$$\delta y = \delta l \cdot \sin \alpha + l \cdot \delta \alpha \cdot \sin 1'' \cdot \cos \alpha .$$

Hence, if δl and $\delta \alpha$ are small corrections to the length and bearing of a line, δx and δy will be corrections to be applied to the latitude and departure of the line.

Corrections of Bearings and Distances for Small Corrections in Latitude and Departure.—As we have already seen, Bowditch's rule involves corrections applied to the latitudes and departures or, what is more usual, directly to the co-ordinates, and these corrections mean disturbances to the lengths and bearings. In certain cases, notably in land surveying work, it is often necessary to insert or to tabulate the corrected bearings and distances on the plans. Here, again, it is not necessary to re-compute new bearings and distances from the new co-ordinates, as it is much simpler to work out small corrections to the original values.

By multiplying the first of the equations given in the last section by $\cos \alpha$ and the second by $\sin \alpha$ and adding we obtain

$$\delta l = \delta x \cos \alpha + \delta y \sin \alpha .$$

Similarly, by multiplying the first equation by $\sin \alpha$ and the second by $\cos \alpha$ and subtracting, we get

$$\delta \alpha = \frac{\delta y \cos \alpha - \delta x \sin \alpha}{l \sin 1''}$$

Here, δx and δy are the known corrections to the latitude and departure of the leg, and δl and $\delta \alpha$ (in seconds) are the required

corrections to length and bearing. These corrections are all small, so that it is usually sufficient to work them out by the slide rule.

Correction of the Co-ordinates of the End Point for a Small Evenly Distributed Correction in Bearing.—It sometimes happens that the co-ordinates of the end point or of only two or three intermediate points are required and that, after co-ordinates have been computed, a small evenly distributed correction has to be applied to the bearings. One case where this may occur is on a fairly long traverse where there are only one or two stations marked by permanent beacons, the remainder being marked by wooden pegs, and co-ordinates are computed by means of unadjusted bearings in order to get values from which to compute the convergence at the terminal azimuth station. Here the co-ordinates of the permanent marks only are required, and the final corrections to the bearings cannot be applied until the convergence has been computed. Other cases arise where, for some reason or another, corrections to the terminal bearings only become available after the traverse has been computed.

Let $n\delta a$, supposed small, be the error in seconds which has to be distributed evenly throughout the traverse. Then the corrections to the bearings of successive legs are $\delta a, 2\delta a, 3\delta a, 4\delta a \dots n\delta a$, and, using the formula on page 235, the correction to the X co-ordinate for the correction $r\delta a$ to the bearing in the r th leg will be

$$\begin{aligned} \delta x_r &= -l_r \cdot \sin a \cdot r \cdot \delta a \cdot \sin 1'' \\ &= -\Delta Y_r \cdot r \cdot \delta a \cdot \sin 1''. \end{aligned}$$

Similarly

$$\delta y_r = +\Delta X_r \cdot r \cdot \delta a \cdot \sin 1''.$$

Hence, for the whole traverse up to the m th leg, the total correction to X is

$$-\delta a \cdot \sin 1'' [\Delta Y_1 + 2\Delta Y_2 + 3\Delta Y_3 + \dots + m\Delta Y_m]$$

and the correction to Y is

$$+\delta a \cdot \sin 1'' [\Delta X_1 + 2\Delta X_2 + 3\Delta X_3 + \dots + m\Delta X_m].$$

The expressions in the brackets are very easily found. For the X correction, write down the departures in order in the left-hand column, thus

ΔY_1	$\Delta Y_1 + \Delta Y_2 + \Delta Y_3 + \dots + \Delta Y_{m-1} + \Delta Y_m$
ΔY_2	$\Delta Y_2 + \Delta Y_3 + \dots + \Delta Y_{m-1} + \Delta Y_m$
ΔY_3	$\Delta Y_3 + \Delta Y_4 + \dots + \Delta Y_{m-1} + \Delta Y_m$
.....
.....
.....
.....
ΔY_{m-1}	$\Delta Y_{m-1} + \Delta Y_m$
ΔY_m	ΔY_m

$$\Delta Y_1 + 2\Delta Y_2 + 3\Delta Y_3 + \dots + \overline{m-1} \Delta Y_{m-1} + m \Delta Y_m.$$

Starting at the bottom, form the sums ΔY_m , $(\Delta Y_{m-1} + \Delta Y_m)$, $(\Delta Y_2 + \Delta Y_3 + \Delta Y_4 + \dots + \Delta Y_m)$, $(\Delta Y_1 + \Delta Y_2 + \Delta Y_3 + \dots + \Delta Y_m)$, as shown in the second column, where, for instance, the sum opposite ΔY_2 is formed by adding ΔY_2 to the entry opposite ΔY_3 in the second column. Then it will be seen that the sum of all the entries in the second column gives

$$\Delta Y_1 + 2\Delta Y_2 + 3\Delta Y_3 + \dots + m-1\Delta Y_{m-1} + \Delta Y_m.$$

In using these formulæ, the ΔY 's and ΔX 's may be rounded off to the nearest 10 ft., and the work is particularly easy if an adding machine is available.

No example of this computation is given, as it is one that does not occur very often, and, with the above explanation, the method should be obvious. In forming the summations, the latitudes and departures must, of course, be given their proper signs.

Computation of a Traverse Base.—If the ends of the base are inter-visible, and all angles measured, the figure bounded by the traverse and the straight line joining the ends becomes, so far as the angles are concerned, a closed surround. The first step, therefore, is to adjust the angles so that their sum is the theoretical sum. The length of the line joining the ends could then be found by treating the traverse base as part of the main traverse and computing the length from the co-ordinates so found. Instead of doing this, however, it is better, if the most accurate value possible is to be obtained for the length of the base, to adopt a special system of co-ordinates for the base itself. The co-ordinates should be local ones, based on the initial point as origin and with the imaginary line joining the terminal points as axis of X. The bearings are now referred to this new axis, and the X co-ordinate of the end point gives the length required, while, as a check, the sum of the departures should be very approximately equal to zero.

The advantage of this procedure is that the new bearings all become small angles, and, as the length is now derived from the cosines of small angles, possible errors, due to rapid changes in the values of the trigonometrical functions used in computing, are avoided.

In precise traversing it is also sometimes advisable to compute deviations in this way, though this is hardly necessary in ordinary work, as a simple leg of a traverse need not be so accurately measured or computed as a line which is to be used as a base for triangulation.

COMPUTATION OF COMPASS TRAVERSES

Computation by Traverse Tables.—The easiest and quickest method of computing a compass traverse is by traverse tables, using a modified form of the "Traverse Form A" in which the column for "included angle" is omitted, or not used. The heading of the seventh column is left blank or ignored, the space for the ordinary

computation of latitude and departure being used to include the entries from the traverse table. Also, when the legs of a compass traverse are long, the lengths need only be entered to the nearest foot, but, if they are very short, it is as well to enter the decimals if the linear measurements have been taken to decimals of a foot.

A traverse table simply gives the values of the natural cosines and sines of angles from 0° to 90° , each multiplied by 1, 2, 3, etc., up to 10 units. To find the latitude and departure of a given line, the tabular values for the reduced bearing are extracted for each digit of the given length, and, on properly placing the decimal points, the required co-ordinates are given by summing the parts.

Example.—The following figures are extracted from a traverse table, for bearing $36^\circ 41'$.

1		2		3		4		5	
Lat.	Dep.	Lat.	Dep.	Lat.	Dep.	Lat.	Dep.	Lat.	Dep.
0.8019	0.5974	1.6039	1.1948	2.4058	1.7922	3.2078	2.3896	4.0097	2.9870

If it is required to determine to the first place of decimals the co-ordinates of a line having this reduced bearing and of length 534.1 ft., the additions are :

	Lat.	Dep.
500	400.97	298.70
30	24.06	17.92
4	3.21	2.39
.1	.08	.06
<hr/>	<hr/>	<hr/>
534.1	428.3	319.1

After their numerical values have been determined, the latitudes and departures are given their proper signs and are entered in the appropriate columns, the remainder of the form being completed in exactly the same manner as that already described for the case of theodolite traverses.

Instead of the full Traverse Form A, already given, a modified form, which is more suitable for compass work, may be used, and the following example shows the headings for the first five columns :

Line.	Length	Bearing	Reduced Bearing	Computation	Lat.
					+
AB	612	132 17	47 43	$\begin{array}{r} 2\ 4428 \\ 9\ 8279 \\ 2\ 6149 \\ 9\ 8691 \\ \hline 2\ 4840 \end{array}$	27
BC	275	64 28	64 28	$\begin{array}{r} 862 \\ 302 \\ 22 \\ \hline 1186 \end{array} \quad \begin{array}{r} 180.5 \\ 63.2 \\ 4.5 \\ \hline 248.2 \end{array}$	1

Here, the calculation of the latitude and departure of the first leg is shown as being carried out by logarithms, arranged in the usual way, and that of the second leg by traverse tables. The last six columns, which are not shown, are exactly the same as the last six of Form A.

The method of auxiliary bearings is rather laborious for checking a compass traverse, and this can be done most conveniently by a re-computation from traverse tables, or even in some cases by direct plotting of bearings and distances.

Adjustment of Compass Traverses.—After the co-ordinates have been obtained, a compass traverse can be adjusted by Bowditch's rule, which, from a theoretical point of view, is really more suitable for the adjustment of compass than for theodolite traverses. This adjustment can easily be done graphically as follows.

Set out a line Aa , Fig. 191 (b), equal to the total length of the

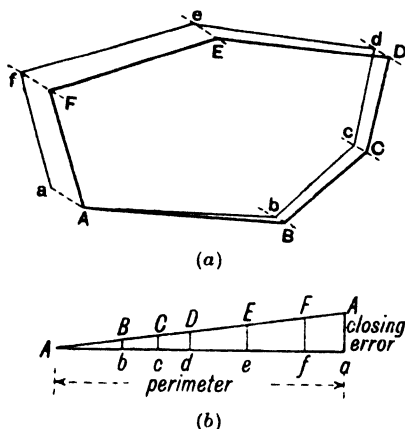


FIG. 191.

traverse and mark off the distances Ab , bc , cd , de , ef , fa equal to the lengths of the different legs. At a , draw aA equal to the closing error in latitude. Join Aa , and at b , c , d , e , and f draw bB , cC , dD , eE , fF , perpendicular to Aa , and to meet the line Aa in B , C , D , E , and F . Then the lengths of the different ordinates give the corrections in latitude to the co-ordinates of the points corresponding to b , c , d , e , and f . Similarly, repeat the process for the departures, using the closing error in departure instead of the closing error in latitude.

If the traverse has been plotted, and not computed, the adjustment can likewise be performed graphically. In Fig. 191(a) let $A b c d e f a$ be a traverse which closes back on itself at A . When the traverse is plotted, the end point will not coincide exactly with A , but will be at the point a , and Aa will represent the closing error.

Set off, as before, the line Aa , Fig. 191 (*b*), equal to the length of the traverse, and at a draw aA perpendicular to Aa , but make aA equal to the total closing error Aa in Fig. 191(*a*). Then the magnitudes of the closing error at b, c, d, e , and f are equal to the ordinates bB, cC, dD, eE, fF . In Fig 191(*a*) draw lines through b, c, d, e , and f parallel to Aa , and along these set off distances bB, cC, dD, eE, fF , equal to the corresponding ordinates on the lower diagram. Then $A B C D E F A$ represents the adjusted traverse.

In the case of lower order compass traverses—e.g. ones in which bearings are measured with a small hand compass instead of with a large one used on a stand—it will be quite sufficient to distribute the closing errors in latitude and departure evenly among the co-ordinates of the traverse, without reference to the lengths of individual legs. Thus, suppose that the closing error in latitude is 150 ft. and that there are 50 legs. Then, the adjustment to the X co-ordinate of the end point of the first leg would be 3 ft., that to the X co-ordinate of the end point of the second leg would be 6 ft., and so on, no notice at all being taken of the lengths of the legs.

Weighting.—Since it may happen that all the measurements have not been performed under the same conditions, allowance may be made for the relative difficulties by distributing the corrections in accordance with the estimated probability of error in the various lines. This operation is termed weighting, and consists in assigning to each line a factor, or weight, representing its relative liability to error as judged from the conditions under which the length and bearing were measured. In estimating the relative difficulty of the linear measurements, consideration is given to roughness of ground and the existence of obstacles, while, as regards the angular work, difficulty of sighting and difference of level between the points observed are kept in view. It is usual to assign unit weight to the line considered least subject to error, the weights of the others being estimated by comparison.

To use Bowditch's rule with weighted lines, each distance is multiplied by its weight, giving the weighted length of the line, then Correction in latitude (or departure) of any line

$$= \frac{\text{Weighted length of that line}}{\text{Sum of weighted lengths of all lines}} \times \text{Total error in lat. (or dep.).}$$

Balancing of Interlocking Nets.—In the case of a number of traverses connected to each other at several points, any attempt at rigorous adjustment would be unwarrantably troublesome and unnecessary for ordinary purposes. It is sufficient that the more important circuits should be adjusted before those in which errors would have less serious consequences. If the traverses cannot be so differentiated, the larger should be balanced before the smaller. Lines common to two or more traverses, when once corrected, must not be subjected to further adjustment.

MISCELLANEOUS PROBLEMS IN RECTANGULAR CO-ORDINATES

Transformation of Co-ordinates.—It occasionally happens that it becomes necessary to transform co-ordinates referred to one set of axes to values referred to another. This case sometimes occurs when a new survey is tied to an old one or it is desired to transform co-ordinates based originally on the magnetic meridian to others based on the true meridian as axis of X.

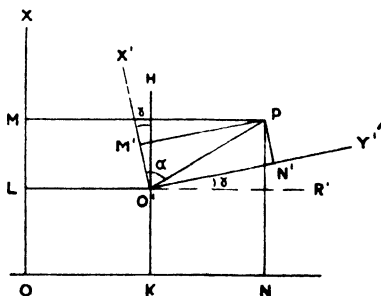


FIG. 192.

In Fig. 192 let $O'X'$, $O'Y'$ be the old axes of co-ordinates and OX , OY the new. Through O' draw $KO'H$ and $LO'R$ parallel to OX and OY respectively. Then angle $HO'X = \gamma =$ angle between old and new axes of X . Through P draw PM' , PN' perpendicular to $O'X'$ and $O'Y'$ and PM , PN perpendicular to OX and OY . Then the old co-ordinates of P are $O'M' = x$, and $O'N' = y$, and the new ones are $OM = X$, and $ON = Y$. Also, let co-ordinates of O' referred to OX and OY be $OL = X_0$ and $OK = Y_0$. Let $X'O'P = \alpha$ be the angle made by $O'P$ with $O'X'$. Then:

$$\begin{aligned} X &= X_0 + O'P \cos (\alpha - \gamma) \\ &= X_0 + O'P \cos \alpha \cos \gamma + O'P \sin \alpha \sin \gamma \end{aligned}$$

But $O'P \cos \alpha = x$, $O'P \sin \alpha = y$,

$$\therefore X = X_0 + x \cos \gamma + y \sin \gamma \quad \dots \dots \dots (A).$$

Similarly

$$\begin{aligned} Y &= Y_0 + O'P \sin (\alpha - \gamma), \\ &= Y_0 + y \cos \gamma - x \sin \gamma \quad \dots \dots \dots (B). \end{aligned}$$

and these are the formulæ required.

If γ is a small angle, expressed in seconds, we can write this :

$$X = X_0 + x + y \gamma \sin 1'' \quad \dots \dots \dots (C).$$

$$Y = Y_0 + y - x \gamma \sin 1'' \quad \dots \dots \dots (D).$$

These approximate formulæ can be used so long as $\frac{x_m}{2} \gamma^2 \sin^2 1''$ and

$\frac{y_m}{2} \gamma^2 \sin^2 1''$ are negligible, x_m and y_m being the maximum values of x and y .

For the reverse problem, multiply equation (A) by $\cos \gamma$ and (B) by $\sin \gamma$ and subtract (B) from (A). Then

$$x(\cos^2 \gamma + \sin^2 \gamma) = (X - X_0) \cos \gamma - (Y - Y_0) \sin \gamma$$

$$\therefore x = (X - X_0) \cos \gamma - (Y - Y_0) \sin \gamma \dots\dots(E).$$

Similarly, by multiplying (A) by $\sin \gamma$ and (B) by $\cos \gamma$ and adding, we get .

$$y = (X - X_0) \sin \gamma + (Y - Y_0) \cos \gamma \dots\dots(F),$$

or, in approximate form, when $\frac{(X - X_0)}{2} \gamma^2 \sin^2 1''$ and $\frac{(Y - Y_0)}{2} \gamma^2 \sin^2 1''$,

where $(X - X_0)$ and $(Y - Y_0)$ are taken at their maximum values, are negligible :

$$x = (X - X_0) - (Y - Y_0) \gamma \sin 1'' \dots\dots\dots(G).$$

$$y = (X - X_0) \gamma \sin 1'' + (Y - Y_0) \dots\dots\dots(H).$$

Computation of the Cut of a Fixed Straight Line on a Grid Line or Sheet Edge.—When work is being plotted on a very large scale, it often happens that a straight line, whose position and bearing are known, or can be computed from the known co-ordinates of the two ends, cuts across several sheets, and it is required to plot the line on each sheet. The best way of doing so is to compute the positions of the points where the line, or the line produced, cuts the edges of each sheet.

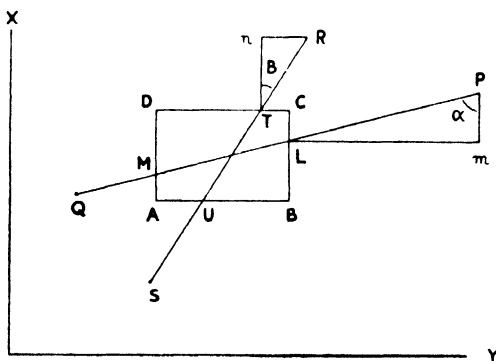


FIG. 193.

In Fig. 193 let ABCD be the sheet, the edges of which are parallel to the co-ordinates axes OX and OY, and let PQ be the fixed line joining the points P and Q. The co-ordinates of one point and the bearing of the line are known, or else the co-ordinates of both points are known, so that the bearing can be computed. Let the

line cut the sheet edges BC and AD in L and M respectively. We require the distances BL and AM, or the co-ordinates of L and M.

Through L draw the line Lm parallel to OY, and through P draw Pm parallel to OX cutting Lm in m . Then angle $LPm = \alpha =$ the known bearing of $PQ - 180^\circ$ and $Pm = Lm \cot \alpha$. But $Lm = Y_r - Y_s$, where Y_r is the Y co-ordinate of P and Y_s that of the sheet edge BC, which is known. Hence $Pm = (Y_r - Y_s) \cot \alpha$. But $Pm = X_r - X_L$, where X_L is the X co-ordinate of L, and therefore

$$(X_r - X_L) = (Y_r - Y_s) \cot \alpha.$$

But the X co-ordinate of B is known, because AB is a sheet edge. Hence BL is known. Similarly AM can be calculated, and therefore a straight line can be drawn through M and L, and this straight line will lie on the straight line PQ.

When the positions of L and M have been calculated by the formulæ just given, the work can be checked by the relation :

$$X_L - X_s = AB \cot \alpha.$$

AB will usually be a round number of thousands, so that, when natural cotangents are used, the multiplication is a very simple and easy one.

Similarly, for the line RS, $Rn = nT \tan \beta$,

$$\therefore (Y_s - Y_r) = (X_s - X_r) \tan \beta,$$

so that CT can be calculated. Also, another similar point U can be found on the sheet edge AB, so that a line joining T and U lies on the line PS. The check on this, of course, is : $(Y_r - Y_v) = CB \tan \beta$.

Graphical Intersection.—The above method of computing the positions of the cuts of lines, of fixed bearing and position, on lines parallel to the co-ordinates axes can often be used in minor triangulation to determine, graphically, the most probable position of a point which is to be fixed by intersection from three or more points.

This case can best be illustrated by an example : A point P has been observed from four points A, B, C and D, and the co-ordinates of each point, and the observed bearings to P, are given in the following table :

Point.	X	Y	Observed Bearing.	Approximate Distance.
A	37346.3	37469.3	212° 22' 10"	16900
B	35180.8	26070.5	168 51 10	12300
C	27361.4	17761.7	111 39 00	11500
D	25679.7	36016.4	251 19 40	8000

The first thing to do is to obtain approximate co-ordinates for the values of the intersected point P, and a preliminary solution

of the triangle BCP, obtained by drawing it on a large scale, showed that the co-ordinates of P were somewhere round about $X = 23120$, $Y = 28450$. These figures are only required to give an indication of the lines to be chosen as grid lines on which to compute the cuts. The grid lines chosen are therefore :

$$\begin{array}{ll} X = 23110 & X = 23130 \\ Y = 28440 & Y = 28460 \end{array}$$

Now compute the cut of the bearing A-P to the lines $X_a = 23110.0$ and $X_b = 23130.0$.

$\begin{array}{r} X_a = 37346.3 \\ X_a = 23110.0 \\ \hline 14236.3 \end{array}$	$\begin{array}{r} 37346.3 \\ 23130.0 \\ \hline 14216.3 \end{array}$
$\begin{array}{r} \log 14236.3 = 4.153\ 397 \\ \log \tan 32\ 22'\ 10'' = 9.802\ 001 \\ \hline 3.955\ 398 \end{array}$	$\begin{array}{r} \log 14216.3 = 4.152\ 787 \\ \log \tan 32\ 22'\ 10'' = 9.802\ 001 \\ \hline 3.954\ 788 \end{array}$
$\begin{array}{l} \therefore Y_a - Y_o = 9024.0 \\ \quad Y_a = 37469.3 \\ Y_a - Y_o = 9024.0 \\ \hline \therefore Y_o = 28445.3 \end{array}$	$\begin{array}{l} \therefore Y_b - Y_o = 9011.3 \\ \quad Y_b = 37469.3 \\ Y_b - Y_o = 9011.3 \\ \hline \therefore Y_o = 28458.0 \end{array}$

(Check : $20 \times \tan (32^\circ 22') = 20 \times 0.6338 = 12.7$. $9011.3 + 12.7 = 9024.0$.)

Hence the line from A, on a bearing of $312^\circ 22' 10''$, cuts the line $X = 23110.0$ where $Y = 28445.3$, and the line $X = 23130.0$ where $Y = 28458.0$. We therefore have sufficient information to plot the points where the line cuts the two grid lines that have been chosen so that they are very close to the final position of P.

Compute the cuts of the other bearings on suitable grid lines and form the following table :

Line.	Grid Line and Position of Cut.	Grid Line and Position of Cut.
A - P	$X = 23110.0$ $Y = 28445.3$	$X = 23130.0$ $Y = 28458.0$
B - P	$X = 23110.0$ $Y = 28449.0$	$X = 23130.0$ $Y = 28445.1$
C - P	$Y = 28440.0$ $X = 23122.8$	$Y = 28460.0$ $X = 23114.8$
D - P	$Y = 28440.0$ $X = 23116.0$	$Y = 28460.0$ $X = 23122.8$

A piece of squared paper of suitable size is now taken, the chosen grid lines plotted on it and the lines corresponding to the lines of observed bearing plotted from the data given in above table. These lines intersect, two by two, in six points and each of these points gives one set of values for P. The most probable position of

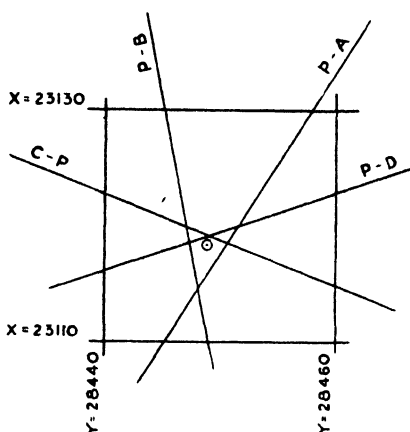


FIG. 194.

P is that point for which the perpendicular distance from it to each of the four lines is directly proportional to the length of the line involved. Hence, the distance of the point from a short line is less than that for a long line. This is because the linear displacement at the end of a given line, for a given error in bearing, is proportional to the length of the line, and it is assumed that the probable error of a bearing is the same for all lines.

From the diagram, the probable values for the co-ordinates of P are taken as :

$$X = 23118.2 \quad Y = 28448.8.$$

The approximate distances from the point P to the points A, B, C, and D, required for estimating the position of P in the graph, can be obtained quite accurately enough for the purposes required by scaling from a fairly large-scale plot of the different points.

This method of graphical solution can also be adapted to the problem of resection—that is when a point is fixed by angles observed from it to three or more fixed points. Observations to three points, involving two angles, are sufficient for a “fix,” but, of course, a better result is obtained if more than three points are used. In that case, an ordinary computed solution is somewhat laborious and the graphical method is much quicker. This method is fully described in Arden-Close and Winterbotham’s *Survey Computations* (H.M. Stationery Office, York House, Kingsway, W.C.2).

Solution of Triangles by Co-ordinates.—It sometimes happens that a triangle has to be solved when two sides and the included angle are given, and, in that case, a solution by co-ordinates is quicker than the usual method given in text books on trigonometry.

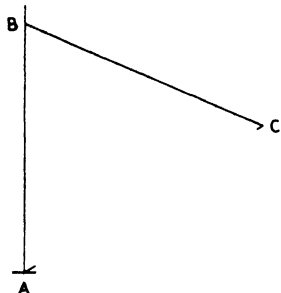


FIG. 195.

In Fig. 195 let the angle A and the lengths of the sides AB and AC in the triangle ABC be given. If the co-ordinates of A and the bearing of either AB or AC are known, the co-ordinates of B and C, and then the length and bearing of BC, can be computed. If the co-ordinates of A and neither of the bearings of AB or AC are known, take A as origin and AB as one axis of co-ordinates. Then the co-ordinates of B are known, and those of C can be computed and the solution completed by computing the bearing and length of BC. When the bearings of the different lines have been found, the angles can be obtained by subtraction.

Omitted Measurements.—If, for any reason, it is impossible to measure all the lengths and bearings of a closed traverse, the values of the missing quantities (lengths or bearings) can be calculated, provided they do not exceed two in number. Since the observed and omitted quantities are parts of a closed polygon, the algebraic sum of all the latitudes and that of the departures are each zero. We have therefore the two independent equations :

$$\begin{aligned} l_1 \cos \alpha_1 + l_2 \cos \alpha_2 + l_3 \cos \alpha_3 + \dots &= 0 \\ l_1 \sin \alpha_1 + l_2 \sin \alpha_2 + l_3 \sin \alpha_3 + \dots &= 0. \end{aligned}$$

where l_1 and l_2 , etc., are the lengths and α_1 and α_2 , etc., the bearings of the lines, and from these equations two unknowns can be obtained.

If only one measurement is missing, a check on the accuracy of the traverse is still available, but in the case of two omissions all errors propagated throughout the survey are thrown into the calculated values of the omitted data, and no check, other than by repeating the field work, is possible. The omission of measurements merely to save field work is not therefore to be regarded as generally allowable.

One important application occurs when a traverse is run between

two points on either side of an obstacle for the special purpose of determining the length or direction of the straight line joining them. A traverse base is one example of this kind of problem and the setting out of a tunnel, where both ends are not intervisible, is another, while a similar one arises in computing check measurements. Six cases may occur, according as the quantities required are :

- (1) Length of one line.
- (2) Bearing of one line.
- (3) Length and bearing of one line.
- (4) Lengths of two lines.
- (5) Bearings of two lines.
- (6) Length of one line and bearing of another.

Calculations of Unknowns.—The solution of cases 1, 2, and 3 have already been dealt with as they are all part of the general problem of computing a bearing and distance from given co-ordinates. So far as case 1 is concerned, however, the solution already given involves the calculation of the bearing, if this bearing is not known or given, and normally this solution is the most convenient one, even when the bearing itself is not required, as it is suitable for logarithmic computation. An alternative method, of course, is to use the formula :

$$\text{Omitted length} = \sqrt{\Delta X^2 + \Delta Y^2}$$

where ΔX and ΔY are the latitude and departure of the length as obtained from the traverse, but, when ΔX and ΔY are fairly large numbers, this is not so convenient a solution as the logarithmic one unless a good table of squares is available.

If the field measurements have been made without error, the calculated value of the omitted length should be such that multiplying it by the cosine and sine of its bearing will give ΔX and ΔY respectively. A check on the field work is therefore provided. On including the calculated line in the table, and computing its co-ordinate is from its observed bearing, the survey may be balanced in the usual manner, but such adjustment is unsatisfactory, since the calculated length necessarily contains the errors made in the other lines.

In case (4), if l_1 and l_2 are the unknown lengths and a_1 and a_2 their known bearings, then

$$\begin{aligned} l_1 \cos a_1 + l_2 \cos a_2 &= \Delta X \\ l_1 \sin a_1 + l_2 \sin a_2 &= \Delta Y \end{aligned}$$

and l_1 and l_2 can be found by solving the simultaneous equations. The solution is independent of whether the unknown sides of the polygon adjoin each other or not. The two equations reduce to one when the unknown lengths are parallel, and this case is indeterminate, except in the unlikely event of the area of the polygon being known, the unknown lengths not being adjacent.

In case (5) the same simultaneous equations can be solved for a_1 and a_2 provided their quadrants are known.

In case (6), on solving the equations two values for the unknown bearing and length are obtained. If either is known approximately, it will, in general, be seen which of the two solutions is wanted.

In all cases but the first it is important to get the signs of ΔX and ΔY right. These can easily be obtained by inspection but, for a closed surround, ΔX and ΔY are equal in magnitude, but opposite in sign, to the respective algebraic sums of the unknown latitudes and departures.

Alternative Method for Cases 4, 5, and 6.—The following solution of the problem is sometimes preferable. In Fig. 196, the incompletely measured lines GH and HA adjoin each other. From the co-ordinates of the survey lines from A round to G, the length and bearing of the line GA are calculated by the ordinary method. The triangle GHA can then be solved for the two unknowns. In case (4) the internal angles of the triangle must be deduced from the known bearings; in case (5) all the sides are known; while in case (6) two sides and the angle opposite one of them are known. If the bearing of GH and the length of HA are measured, the two

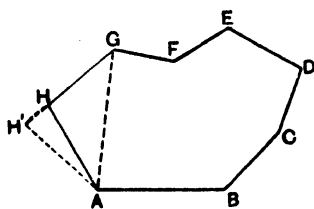


FIG. 196.

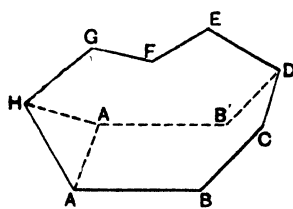


FIG. 197.

solutions are shown by the two possible positions, H and H', of the unfixed station.

When the two incompletely measured sides do not adjoin each other, as CD and HA in Fig. 197, they may be brought into the same triangle by imagining the intervening lines shifted parallel to themselves in a direction parallel to one of the unknowns. In the example illustrated, AB and BC are moved to A'B' and B'D, so that AA' is equal and parallel to CD. On joining HA', the triangle HA'A is formed, the sides HA and AA' of which are the two lines affected by the omissions. HA' is completely known since it is the closing side of a traverse A'B'D'EFGH, of which full particulars are available. The triangle can therefore be solved for the missing data as before. It will be observed that no triangle would be available if CD and HA were parallel.

NOTE ON EXTENDED AND NATIONAL SYSTEMS OF
RECTANGULAR CO-ORDINATES

When local small surveys are tied in to points on the national or government survey, there are one or two matters which require consideration.

It has already been explained that the national surveys of many countries are referred to one or more standard meridians. Each of the standard meridians is taken as the axis of X, with some convenient point on it as origin of co-ordinates. The Y co-ordinates are taken as distances, measured at right angles to the axis of X, from the point on the meridian corresponding to the X co-ordinate of the point whose position is being defined. Usually, if the area involved is only a small one—say a few hundreds of square miles—no trouble occurs, but, when the area runs into some thousands of square miles, as is the case with any state or national survey, the curvature of the earth introduces certain complications.

Suppose that a triangle is drawn on a piece of orange peel, and the peel is then squeezed absolutely flat, so that the triangle, which was originally on a curved surface, now lies on a plane. It will at once be obvious that the material of the peel is strained, and that some distortion of the triangle must take place, with the result that either the sides will alter in length, or the angles will alter, or possibly that both will alter. If the triangle is a very small one, on a very small piece of peel, it will be seen that there is very little distortion of the sides when the peel is squeezed flat.

Exactly the same sort of thing occurs when measurements are taken on the earth's surface. If a small survey is made and then plotted on a plan, this corresponds to the case of the very small piece of peel squeezed flat. Here, there will be very little distortion, either of lengths or of angles, and, for all practical purposes, the survey can be treated as being on a plane. If, however, the area involved is a very large one the lines and angles measured on the earth's surface must become distorted when plotted on the plane. This applies when we try to extend a system of plane rectangular co-ordinates over a considerable stretch of country. As we get further away from the axes of co-ordinates, distances and bearings computed from the co-ordinates no longer agree with the actual distances and bearings between equivalent points on the ground. Consequently, it is impossible to devise any system of co-ordinates that will completely preserve the relative lengths and directions existing on the curved surface of the earth, although it is possible to preserve certain features at the expense of others, the methods of doing this forming the subject of the study of map projections, which is dealt with in Vol. II, Chap. VIII. One system, the Transverse Mercator Projection, which is described in detail in Vol. II, Chap. V, is now in use in Great Britain and in certain of the British Colonies and has the following properties:

- (1) Distances along the X axis of co-ordinates are either the same as distances along the central meridian or are all reduced in the same proportion.
- (2) In the case of points close together, bearings and angles computed from co-ordinates agree with bearings and angles measured on the ground.
- (3) The lengths of lines are distorted, so that lengths computed from co-ordinates do not agree exactly with lengths measured on the ground. This distortion in length varies with the position of the point with reference to the central meridian, but, for a given length, supposed small, and a given position, is independent of the direction of the line.

It will thus be seen that the principal feature of this system is that it produces distortion of length rather than of angle or direction, and, in cases which occur in practice, this distortion may amount to anything up to 1/1000. The exact amount can always be calculated, so that distances computed from co-ordinates can easily be converted into true distances on the ground and vice versa.

From the above it follows that, if a survey starts at one of the national

survey points and ends at another, there may be a very appreciable difference between the values for the second point as obtained by the surveyor and those obtained directly from data supplied by the official survey department. For many purposes, the distortion produced by the indiscriminate use of the official survey points would be far too great, and it becomes necessary to correct for it. In cases such as this, therefore, application should be made to the official survey department, which will usually either provide the surveyor with "local values" to suit his own needs or with the information necessary to enable him to make such corrections as he requires. In this way, the officially established points may still be used as a kind of "yard-stick" that can be applied to the work of the ordinary surveyor, even when the latter is aiming at a relatively high standard of accuracy.

PLOTTING TRAVERSES

Methods of Plotting Survey Lines.—The origin of a survey line being located on the paper, the line can be plotted either by setting off its direction and length, or by fixing the position of its terminal point by co-ordinates. The first system, termed the angle and distance method, involves the laying down of an angle which, according as a reference meridian has or has not been adopted for the field observations, will be either (a) the bearing of the line, (b) the angle, included or deflection, from the preceding survey line. In the co-ordinate system, the direction and length of the line are used in the calculation of co-ordinates, and are not directly plotted.

Angle and Distance Methods.—The more commonly used angle and distance methods of plotting an angle or bearing are :

- (a) By protractor.
- (b) By the tangent of the angle.
- (c) By the chord of the angle.

By Protractor.—The protractor furnishes the most rapid method of laying down angles, but, while the degree of accuracy afforded is sufficient for many purposes, the best results cannot be expected, particularly in plotting lines much longer than the radius of the protractor. With the ordinary simple pattern, of about 6 in. diameter, angles cannot be protracted nearer than to about 10 or 15 min., which accords with the accuracy of free needle readings, but not of theodolite work. Protractors of 9 to 12 in. diameter with vernier reading to 1 min. are more suitable for plotting theodolite surveys, but in using such protractors it is doubtful if full advantage can be taken of the vernier owing to the probability of error in centering. A very useful and inexpensive form of protractor has the graduations printed on cardboard, and may be obtained in diameters of from 12 to 18 in.

By Tangents.—In this method, the angle θ to be plotted is set out by geometrical construction with the aid of a table of natural tangents. From the station point a base length, scaling a round number of units, is set off along the given side of the angle, and

from the end of the base a perpendicular is erected of length = base $\times \tan \theta$. The line joining the station to the point thus obtained includes θ with the given side. The length of base should be such that the hypotenuse obtained exceeds the length of the survey line to be plotted along it. If the angle exceeds one or more right angles, these are laid off by set-square, and the method is applied to the remainder. In constructing angles of nearly 90° it is better to plot the complement of the angle.

With care, the tangent method is capable of higher accuracy than direct protracting.

By Chords.—In this geometrical method, an arc of any convenient radius r , preferably 10 or 100 units, is first swept out with the station point as centre. A second arc, centered at the point in which the first cuts the given side, is then described with radius equal to the chord subtended by the angle to be plotted. The line joining the station to the intersection of the arcs defines with the given side the required angle. The chord length = $2r \times \sin \frac{\theta}{2}$. Various mathematical tables give the lengths of chords corresponding to unit radius. If the angle exceeds a right angle, the construction should be applied only for the part less than 90° , as thereafter the intersections become unsatisfactory. The accuracy attainable is not materially different from that of the tangent method.

Plotting Angles and Bearings.—The manner of applying the above methods to the plotting of a survey line from the preceding one will be evident. Results of angular measurements, noted in tabular form, should always be accompanied by a sketch, to which the draughtsman must make constant reference to avoid setting off angles in the wrong direction.

In plotting bearings, two methods may be distinguished.

(1) A meridian is drawn in a convenient central position on the sheet, and at an assumed point on it the bearings of all the lines to be plotted are set out from it. These directions are then transferred to their proper positions, as required (Fig. 198).

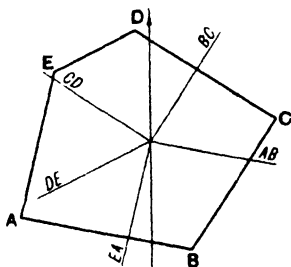


FIG. 198.

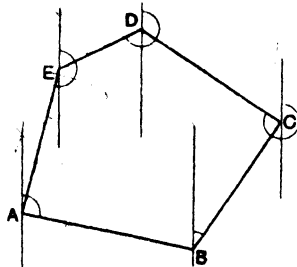


FIG. 199.

(2) Each bearing is plotted from a meridian through the origin of the line and parallel to the reference meridian (Fig. 199).

The first method is the more rapid. By using a circular protractor, all the bearings can be plotted from one setting of the protractor. In the tangent method the same bases can be used for various bearings, while in applying the chord method the several chord lengths can be marked off on one circle. Great care is necessary in transferring the directions: a heavy parallel ruler gives the best results. In the second method there is the same need for accuracy to secure parallelism between the successive meridians.

Plotting by Co-ordinates.—The co-ordinate method is recognised as the most accurate and useful method of plotting traverse lines from bearings. Two systems are used.

(1) Each station is plotted from the preceding one by setting out the latitude and departure of the line between them in directions respectively parallel and perpendicular to the reference meridian.

(2) Each station is plotted independently by ascertaining its position relatively to two assumed co-ordinate axes respectively parallel and perpendicular to the reference meridian.

In plotting closed surveys by either method, the closing error, if appreciable, should first be eliminated by balancing, and the adjusted values of the latitudes and departures used.

First Method.—In this method, direct use is made of the latitudes and departures as shown in Fig. 200. Having assumed a point to represent the first station A and a line ae showing the direction of the reference meridian, B is plotted by setting off in the proper directions the latitude Aa and departure aB of line AB. C is similarly located from B, and so on.

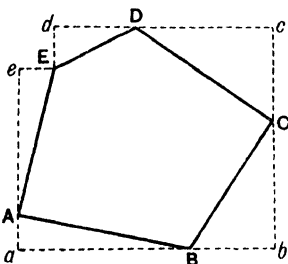


FIG. 200.

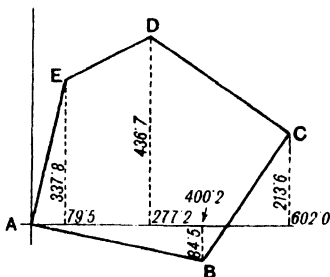


FIG. 201.

Second Method.—By co-ordinate values. The origin may be the origin of the survey or the south-west corner of the sheet. If the latter, the co-ordinates of this corner can be subtracted from the co-ordinates of the different points falling on the sheet, so that the new co-ordinates of these points are referred directly to the sheet edges.

Fig. 201 shows a survey plotted with respect of axes through the most westerly traverse station. Here, there is a negative X co-ordinate for the point B. As a general rule, however, it is better to avoid negative co-ordinates, if necessary by choosing a "false origin" to the south and west of any point likely to fall in the survey.

Precautions in Co-ordinate Plotting.—To obtain good results, pencil lines should be fine, and co-ordinate lengths should be very carefully scaled. Inaccuracy in the directions of co-ordinate lines is the most likely source of error, particularly in the second method, which requires the plotting of longer lines than the first. Accuracy depends largely upon the care taken to secure perpendicularity between the reference axes. Long perpendiculars should be set out by the use of beam compasses rather than by set-square. When a survey covers a large area of paper, so that the station co-ordinates are long lines, it is better to construct a rectangle to enclose the survey. Before using this rectangle, its accuracy should be verified by checking the equality of its diagonals. Stations are then plotted from the nearest sides of the rectangle by subtracting, if necessary, the tabular co-ordinates from the lengths of the corresponding sides of the rectangle. Distances to be scaled can be still further reduced by subdividing the bounding rectangle into smaller rectangles, from the sides of which the plotting can be performed.

Relative Merits of the Two Co-ordinate Systems.—The second method is much more commonly used than the first, but each has advantages. Errors of scaling are revealed in either system by measuring the length of each course as soon as plotted, but this test does not afford an absolute check on the bearings. In the first method, small errors of plotting are carried forward, and in a closed survey the closing error due to inexact plotting is discovered, since the co-ordinates used have been balanced. In the most refined plotting this is an advantage, the cause of the discrepancy being searched for and eliminated. Since small errors are not accumulated in the second method, it is to be preferred for plotting unclosed traverse.

Advantages of Co-ordinate Plotting.—(1) The table of co-ordinates exhibits the extent of the survey, and the position of the point representing the origin of co-ordinates may be selected so that the survey will fall centrally on the sheet.

(2) By calculation of the co-ordinates the closing error can be ascertained, and adjusted if necessary, before plotting commences. In angle and distance methods, plotting must be completed before the closing error is discovered, and then it is not known to what extent the error is due to inaccurate field work or plotting.

(3) The plotting of the stations of a large survey can proceed simultaneously on different sheets or on different parts of the same sheet.

EXAMPLES

1. A, B, C, D are four survey stations. At station B the angle ABC is measured clockwise, and found to be $196^{\circ} 53'$. On setting up at C, the angle BCD, also measured clockwise, is observed to be $142^{\circ} 17'$. If AB is adopted as the meridian, what are the bearings of BC and CD?

2. The following angles, measured clockwise from the back station, were observed on a traverse:

MAB	$136^{\circ} 14' 20''$
ABC	$172^{\circ} 16' 40''$
BCD	$96^{\circ} 37' 10''$
CDE	$217^{\circ} 54' 30''$
DEF	$81^{\circ} 10' 20''$
EFG	$168^{\circ} 46' 50''$

The bearing of the line MA is $327^{\circ} 18' 40''$. What is the bearing of FG? Use the addition test to check your result.

The bearing of the line FG was known from astronomical observations to be $120^{\circ} 18' 00''$. Give the adjusted values of the bearings of the lines AB, BC, CD, DE, EF.

3. The total latitudes or X co-ordinates of two stations on a survey are 8,257 and 7,542 ft., and their total departures or Y co-ordinates are 1,321 and -146 ft. respectively. Calculate the distance between them and the bearing from the first to the second.

Check by the method of auxiliary bearings.

4. Calculate the closing error of the following traverse:

Line.	Length in feet.	Bearing.
AB	520	92°
BC	634	174°
CD	580	220°
DE	1,232	279°
EA	1,348	48°

Check by the method of auxiliary bearings.

5. The following bearings and lengths represent the observed values for the first three lines of a traverse survey. The bearings were referred to magnetic meridian, which was $16^{\circ} 35'$ west of true north at the place. Convert the observed bearings to true bearings. Tabulate the latitudes and departures of the lines derived from the true bearings and the co-ordinates of the stations referred to A as origin.

Line.	Length in feet.	Magnetic Bearing.
AB	738	$33^{\circ} 21'$
BC	560	$349^{\circ} 46'$
CD	893	$15^{\circ} 10'$

6. The bearings of a theodolite traverse have been referred to magnetic meridian at the initial station, and the co-ordinates of a certain line are found to be 796 ft. S. and 368 ft. W. Calculate the co-ordinates of this line with reference to true meridian at the initial station if the direction of magnetic north there is $13^{\circ} 10'$ west of true north.

7. The following are the particulars of a small traverse survey in which the bearings have been referred to magnetic meridian, the value of the magnetic declination being $15^{\circ} 30'$ W. Convert the observed bearings to true bearings, calculate the co-ordinates of the lines, and find the error of closure.

Line.	Length in feet.	Magnetic Bearing.
AB	470	N. $6^{\circ} 2'$ W.
BC	637	S. $82^{\circ} 10'$ E.
CD	432	S. $2^{\circ} 38'$ W.
DA	565	N. $84^{\circ} 40'$ W.

8. The following traverse is carried round an obstruction in a line EF :

Line.	Length in feet.	Bearing.
EG	200	S. 38° 21' E.
GH	580	N. 75° 46' E.
HF	82	N. 11° 8' W.

Compute the angles GEF and HFE and the distance EF.

9. A determination of the distance between two mutually visible points *A* and *E* is required, but cannot be made with sufficient accuracy by direct measurement. Obtain the distance from the following notes of a traverse run from *A* to *E* :

Line.	Bearing.	Distance in feet.
<i>AE</i>	360° 0'	
<i>AB</i>	346° 18'	2,386.4
<i>BC</i>	73° 57'	583.2
<i>CD</i>	296° 33'	401.8
<i>DE</i>	18° 21'	1,156.4
<i>EA</i>	180° 0'	

10. A "measured mile" of 6,080 ft. in the direction of magnetic N. and S. is to be marked off on shore by erecting guide poles at either extremity to form parallel lines. From a point *A* on the line of the first two signals a traverse is run to fix a point *D* on the second line, the notes being : *AB*, 970 ft., 357°; *BC*, 4,632 ft., 331°; *CD*, 5°. The bearings are measured from an arbitrary meridian, the magnetic bearing of which is N. 23° E. Find the necessary length of *CD*. (R.T.C., 1920.)

11. The length and bearing of a line *AB* cannot be observed directly, and the following data are obtained by observations from two stations, *C* and *D* :

Line.	Length in feet.	Bearing.
<i>CA</i>	236	258° 35'
<i>CD</i>	1,142	10° 20'
<i>DB</i>	455	274° 15'

Calculate the length and bearing of *AB*.

12. Particulars of part of a traverse survey are :

Line.	Length in feet.	Bearing.
<i>AB</i>	486	342° 24'
<i>BC</i>	1,724	14° 35'
<i>CD</i>	1,053	137° 20'

Calculate the distance between a point *E* on *AB*, 326 ft. from *A*, and a point *F* on *CD*, 400 ft. from *C*.

13. An underground traverse has been run between two survey stations situated at the bottom of two mine shafts *A* and *B*.

The co-ordinates of *A* are 7,260' N. and 19,340' E. and of *B* 9,760' N. and 21,840' E. A surface traverse has been run from *A* to *B* and the co-ordinates have been calculated to the same axes. The co-ordinates of *A* are 7,260' N. and 19,340' E. as before, but the co-ordinates of *B* now are 9,660' N. and 21,912' E.

Assuming the surface traverse to be correct, what is the error in the bearing of *B* from *A* as determined by the underground traverse ? (Inst. C.E., 1918.)

14. A four-sided traverse *ABCD* has the following lengths and bearings :

Side.	Length in feet.	Bearing.
<i>AB</i>	500	roughly East
<i>BC</i>	245	178°
<i>CD</i>	not obtained	270°
<i>DA</i>	216	1°

Find the exact bearing of the side *AB*. (Univ. of Lond., 1913.)

15. For the following traverse compute the length CD , so that A , D , and E may be in one straight line :

Line.	Length in feet.	Bearing.
AB	340	85°
BC	506	32°
CD	—	350°
DE	622	18°

16. A straight tunnel is to be run between two points A and B , whose co-ordinates are given in the annexed table :

Point.	Co-ordinates.	
	N.	E.
A	0	0
B	3,014	256
C	1,764	1,398

It is desired to sink a shaft at D , the middle point of AB , but it is impossible to measure along AB directly, so D is to be fixed from C , a third known point. Calculate :

- The co-ordinates of D .
- The length and bearing of CD .
- The angle ACD , given that the bearing of AC is $38^\circ 24'$ E. of N. (Univ. of Lond., 1916.)

17. The following traverse is run from A to E , between which there occur certain obstacles.

Line.	Distance in feet.	Bearing.
AB	426	$38^\circ 20'$
BC	518	$347^\circ 55'$
CD	606	$298^\circ 12'$
DE	430	$29^\circ 46'$

It is required to peg the point midway between A and E . Calculate the length and bearing of a line from station C to the required point.

18. It is required to set out a line AC , of length 2,068 ft., at right angles to a given line AB . This is done by traversing from A towards C , the observations being as follows :

Line.	Length in feet.	Bearing.
AB		360°
AD	731	$113^\circ 48'$
DE	467	$81^\circ 18'$
EF	583	$105^\circ 57'$

Compute the necessary length and bearing of FC .

19. From a point C it is required to set out a line CD parallel to a given line AB , and such that ABD is a right angle. C and D are invisible from A and B , and traversing is performed as follows :

Line.	Length in feet.	Bearing.
BA		360°
BE	258.5	$290^\circ 57'$
EF	307.0	$352^\circ 6'$
FC	196.5	$263^\circ 27'$

Calculate the required length and bearing of CD .

20. A and B are two of the stations used in setting out construction lines for harbour works. The total latitude and departure of A , referred to the origin of the system, are respectively $+542.7$ and -331.2 , and those of B are $+713.0$ and $+587.8$ ft. (north latitude and east departure being reckoned as positive). A point C is fixed by measuring from A a distance of 432 ft. on a bearing of $346^\circ 14'$, and from it a line CD , 1,152 ft. in length, is set out parallel to AB .

It is required to check the position of D by a sight from B. Calculate the bearing of D from B. (Inst. C.E., 1925.)

21. A mistake has been made in the tabulation of the numerical values of the co-ordinates of the following closed survey. Eliminate the mistake, and compute the closing error.

Line.	Length in feet.	Bearing.	Latitude.	Departure.
AB	732	360° 0'	732.0	0
BC	328	291° 11'	118.5	305.8
CD	607	253° 39'	170.9	582.5
DE	523	158° 51'	487.8	188.7
EF	441	209° 30'	483.8	217.2
FG	505	67° 22'	194.3	466.1
GA	452	90° 12'	1.6	452.0

22. The following results are obtained on working out the co-ordinates of a closed survey in which the bearings have been satisfactorily checked :

Line.	Latitude in feet.	Departure in feet.
AB	+ 663.5	- 379.0
BC	+ 383.0	+ 402.5
CD	- 476.1	+ 493.6
DE	- 877.2	+ 241.7
EA	+ 404.0	- 785.8

By inspection, estimate where a mistake has probably occurred in the chaining.

23. A and B are two stations of a location traverse, their total co-ordinates in feet being :

	Total Latitude.	Total Departure.
A	34,321	7,509
B	33,670	9,652

A straight reach of railway is to run from C, roughly south of A, to D, invisible from C and roughly north of B, the offsets perpendicular to the railway being AC = 130 ft. and BD = 72 ft.

Calculate the bearing of CD. (R.T.C., 1919.)

24. From a common point A, traverses are conducted on either side of a harbour as follows :

Line.	(1) Length in feet.	Bearing.	Line.	(2) Length in feet.	Bearing.
AB	875	74°	AE	348	192°
BC	320	109°	EF	436	160°
CD	1,064	82°	FG	521	97°
			GH	1,683	89°

Calculate the distance from H to a point K on GH due south of D and the distance DK.

25. The table below shows the positions of three stations in wooded country. It is required to set out the straight line from A to B, and to peg the point on it at which a perpendicular from C meets the line. Calculate the bearing of AB and the distance from A to the foot of the perpendicular.

Station.	Total Latitude in feet.	Total Departure in feet.
A	635.3 S.	1,764.2 W.
B	238.7 S.	4,021.8 W.
C	426.0 N.	3,122.5 W.

26. The following are the latitudes and departures of a shore traverse round a bay, of which soundings are to be taken in lines 200 ft. apart :

Line.	Latitude in feet.	Departure in feet.
<i>AB</i>	-1,040	- 892
<i>BC</i>	- 925	+ 608
<i>CD</i>	- 378	- 413
<i>DE</i>	- 582	+ 779
<i>EF</i>	-1,233	+ 545
<i>FG</i>	+ 150	+1,231
<i>GH</i>	+ 762	+ 813
<i>HI</i>	+1,106	+ 879
<i>IJ</i>	+1,283	- 174

AJ is one of the lines of soundings, and the other lines are to be laid out parallel to *AJ*. Compute, to the nearest foot, the positions on *AB* and *IJ* of the signals marking the line adjacent to *AJ*. (T.C.D., 1930.)

27. The following are the latitudes and departures, in feet, of a series of survey lines :

Line.	Latitude.	Departure.
<i>AB</i>	326 S.	978 W.
<i>BC</i>	641 S.	230 E.
<i>CD</i>	508 S.	392 E.
<i>DE</i>	137 N.	425 E.

A line is to be set out from *E* on a bearing of 342° . Calculate at what distance from *A* it intersects the line *AB*.

28. *M* and *N* are adjacent stations of a large traverse survey, the length and bearing of *MN* being 2,364 ft. and $136^{\circ} 48'$ respectively. From station *M* an unclosed traverse, *Mabcde*, referred to the meridian of the survey, was run out for tracing a stream, and the following are the latitudes and departures, in feet, of this subsidiary traverse :

Line.	Latitude.	Departure.
<i>M a</i>	293 S.	245 W.
<i>a b</i>	506 S.	184 E.
<i>b c</i>	455 S.	217 W.
<i>c d</i>	612 S.	206 E.
<i>d e</i>	307 S.	424 E.

From station *N* a similar traverse *N1234* was run, and the latitudes and departures are :

Line.	Latitude.	Departure.
<i>N 1</i>	356 S.	161 E.
<i>1 2</i>	418 S.	323 W.
<i>2 3</i>	605 S.	172 E.
<i>3 4</i>	431 S.	503 W.

It was subsequently desired to obtain a check closure on this part of the survey by measuring the distance and bearing from *e* to *4*. Compute, to the nearest foot and minute respectively, the values of these quantities from the original measurements. (Inst. C.E., 1931.)

29. A compass traverse is run between two stations, *X* and *Y*, of a theodolite survey, of which the total co-ordinates, in feet, are

X, 20,374 N., 15,896 E. ; *Y*, 22,053 N., 15,140 E.

The notes of the compass route are as follows :

Line.	Length in feet.	Compass Bearing.
<i>X 1</i>	426	$352^{\circ} 0'$
<i>1 2</i>	270	$12^{\circ} 30'$
<i>2 3</i>	588	$311^{\circ} 30'$
<i>3 4</i>	403	$346^{\circ} 0'$
<i>4 Y</i>	316	$6^{\circ} 30'$

Magnetic north is $8^{\circ} 30'$ west of the meridian of the theodolite survey.

Taking the theodolite work as correct, calculate by how much the compass traverse fails to close on Y. (T.C.D., 1928.)

30. In a certain traverse the direction of the first line, AB, was adopted as the meridian, and the adjusted values of the latitudes and departures of the first two lines were, in feet,

Line.	Latitude.	Departure.
AB	1,326 N.	0
BC	689 N.	1,273 W.

At a later date a larger survey, referred to a true meridian, was extended to the station A, and the total co-ordinates of that point were found to be 23,416 ft. N. and 38,572 ft. W., the true bearing of AB being $286^{\circ} 12'$. Calculate, to the nearest foot, the total co-ordinates of points B and C from the origin of the large survey.

31. The following traverse is run from a station P, of which the total latitude and departure are respectively +19,430 and -4,607 ft., to a station Q of total latitude and departure +15,106 and -4,488 ft. respectively.

Line.	Length in feet.	Bearing.
P 1	473	$164^{\circ} 12'$
1 2	1,386	$183^{\circ} 54'$
2 3	721	$158^{\circ} 24'$
3 4	1,207	$190^{\circ} 39'$
4 Q	634	$176^{\circ} 18'$

Taking the given positions of P and Q as correct, calculate the amount by which the traverse fails to close on Q.

32. In the course of setting out a straight portion of a railway, pegs have been driven in the centre line at A and B on either side of a wood. A traverse is carried from A to B along the side of the wood, as follows:

Line.	Length in chains.	Bearing.
AC	6.43	$187^{\circ} 30'$
CD	3.88	$110^{\circ} 48'$
DE	5.21	$159^{\circ} 42'$
EB	4.06	$78^{\circ} 18'$

If A is 393.21 chains from the commencement of the railway, find the corresponding distance of B, which is farther along the railway.

From the traverse station D a line DF can be carried into the wood on a bearing of $61^{\circ} 38'$. Compute the length of DF so that F may be pegged as an intermediate point on AB.

33. A series of traverse lines was run by theodolite at the top of a wooded gorge. A compass traverse was carried along the stream, and was connected to the theodolite survey at both ends, as shown in the following notes. Assuming that the theodolite traverse is correct, find the closing error of the compass work.

Theodolite Traverse.			Compass Traverse.		
Line.	Length in feet.	Bearing.	Line.	Length in feet.	Magnetic Bearing.
AB	361	$98^{\circ} 20'$	AB		112°
BC	514	$136^{\circ} 31'$	A1	186	74°
CD	482	$100^{\circ} 9'$	1 2	225	88°
DE	737	$84^{\circ} 42'$	2 3	380	125°
			3 4	155	196°
			4 5	476	139°
			5 6	303	81°
			6 E	540	119°

34. From a traverse station P, the total co-ordinates of which are 486 ft. S. and 12,374 ft. E., a line is set on a bearing of $49^{\circ} 12'$. From a station Q,

having total co-ordinates 1,026 ft. S. and 15,188 ft. E., a line is set out on a bearing of $325^{\circ} 36'$. Calculate the total co-ordinates of the point of intersection of these lines.

35. It is required to ascertain the distance from A to an inaccessible point B invisible from A. A straight line CAD is run, of which AC = 240 ft. and AD = 190 ft., and angles ACB and ADB are found to be 64° and 72° respectively. Calculate the distance AB.

36. The notes of part of a traverse survey are :

Line.	Length in feet.	Bearing.
AB	4,050	N. $12^{\circ} 25'$ E.
BC	3,763	N. $4^{\circ} 20'$ W.
CD	5,300	N. $2^{\circ} 40'$ E.
DE.	2,911	N. $7^{\circ} 15'$ W.

The bearing from A to an inaccessible point O is N. $35^{\circ} 10'$ W., and from E its bearing is S. $43^{\circ} 55'$ W. Calculate the distance of O from C. (R.T.C., 1919.)

37. From the initial station A of an unclosed traverse the bearing of a distant point X is observed to be $62^{\circ} 18'$. From station K, which is 6,838 ft. N. and 3,016 ft. W. of A, the bearing to X is $91^{\circ} 37'$. Calculate the bearing of X from T, of which the total co-ordinates referred to A are 11,273 ft. N. and 1,419 ft. E.

38. It is required to determine the distance between two inaccessible points A and B by observations from stations C and D 1,000 ft. apart. The angular measurements give $ACB = 47^{\circ}$, $BCD = 58^{\circ}$, $BDA = 49^{\circ}$, $ADC = 59^{\circ}$. Calculate the distance AB.

39. The readings in the annexed table were obtained in a closed compass-traverse on an area subject to irregular local magnetic attraction at each station.

Line.	Bearing.		Length. line.
	Forward.	Back.	
AB	$85^{\circ} 24'$	$268^{\circ} 42'$	536
BC	$164^{\circ} 54'$	$340^{\circ} 0'$	316
CD	$261^{\circ} 6'$	$78^{\circ} 42'$	634
DE	$344^{\circ} 24'$	$170^{\circ} 36'$	140
EA	$8^{\circ} 36'$	$186^{\circ} 30'$	200

Show how to check the accuracy of the readings, and tabulate the bearings you would use for plotting the different lines, in order to obtain as accurate a plan as is possible from these readings. (The plan need not be drawn.)

As the exact direction of magnetic north is of no importance, you may take the bearings observed at any one station—say A—as correct. (Univ. of Lond., 1916.)

40. Distribute the angular error of the following theodolite traverse, and write out the corrected bearings to the nearest $10''$

AM	$205^{\circ} 22' 40''$	GH	$332^{\circ} 17' 30''$
AL	$178^{\circ} 36' 00''$	HE	$49^{\circ} 07' 00''$
AB	$93^{\circ} 14' 10''$	HI	$272^{\circ} 48' 20''$
BC	$161^{\circ} 49' 40''$	IJ	$229^{\circ} 27' 00''$
CL	$251^{\circ} 43' 00''$	JK	$341^{\circ} 06' 40''$
CD	$230^{\circ} 51' 10''$	KL	$52^{\circ} 58' 10''$
DE	$206^{\circ} 10' 20''$	LC	$71^{\circ} 44' 20''$
EH	$229^{\circ} 07' 00''$	LA	$358^{\circ} 37' 20''$
EF	$147^{\circ} 56' 50''$	LM	$343^{\circ} 13' 40''$
FG	$238^{\circ} 36' 00''$	MA	$25^{\circ} 24' 00''$

41. The solution of two triangles gave the following values for the logarithm of a common side—4.317 4632 and 4.317 4875. Using the rule given on page 254 find the discrepancy between the two values expressed as a fraction of the length and check your result by working out the actual lengths given by each logarithm.

42. The following (adjusted) bearings and distances were observed on a traverse :

Line.	Bearing.	Distance.
AB	339° 28' 55"	1,728.63
BC	34° 25' 13"	3,369.17
CD	22° 58' 00"	976.43
DE	40° 53' 13"	1,214.17
EF	97° 11' 01"	2,011.86
FG	105° 39' 54"	1,308.22

The co-ordinates of the point A are $X = 30,425.26$; $Y = 21,642.67$. Work out the co-ordinates of the point G and check your work by using the method of auxiliary bearings. Then, assuming that the co-ordinates of G have the fixed values— $X = 26,034.96$; $Y = 27,373.12$, adjust the traverse and work out corrected bearings and distances.

43. Given that the co-ordinates of two points A and B are :

	X.	Y.
A	37,842.10	46,616.19
B	32,384.74	46,593.72

find the bearing and distance B to A and check by the method of auxiliary bearings.

44. Compute the cuts on the lines $X = 44,000$ and $X = 42,000$ and on the lines $Y = 27,000$ and $Y = 30,000$ by a line drawn on a bearing of $235^\circ 26' 40''$ from a point whose co-ordinates are $X = 46,275.34$; $Y = 36,946.13$. Also, the cuts on the same lines by a line drawn on a bearing of $122^\circ 10' 15''$ from a point whose co-ordinates are $X = 48,240.18$; $Y = 21,894.16$.

45. In the triangle ABC the co-ordinates of the points B and C are :

Point.	X.	Y.
B	23,225.49	14,362.18
C	33,179.18	16,143.74

The observed angles at A, B and C are :

Point.	Observed Angle.
A	$76^\circ 18' 20''$
B	$31^\circ 28' 50''$
C	$72^\circ 12' 35''$

Find the co-ordinates of the point A.

46. The following are the latitudes and departures of a traverse :

Leg.	Latitude.		Departure.	
	+	-	+	-
AB	1,618.97			605.89
BC	2,779.27		1,904.45	
CD	899.03		381.00	
DE	917.92		794.76	
EF		251.58	1,996.07	
FG		353.24	1,259.63	

The co-ordinates of the point A are $X = 5,349.16$; $Y = 23,117.29$. The original bearing of the line FG, to which the bearings of the traverse were adjusted, was $105^{\circ} 39' 54''$ but a subsequent and better determination of this bearing gave a value of $105^{\circ} 37' 30''$. The points D and G are marked by permanent pillars and the co-ordinates of these points are now required on the assumption that the difference of $2' 24''$ has been distributed in the usual way among the six legs of the traverse. What are the old and new co-ordinates of the two points in question ?

47. The co-ordinates of three points, A, B and C are as follows :

Point.	X.	Y.
A	5,091.48	5,902.07
B	19,284.30	5,523.75
C	24,425.10	14,392.97

and the following were the bearings observed from these points to a fourth point, D :

Line.	Observed Bearing.
AD	$34^{\circ} 31' 37''$
BD	$82^{\circ} 03' 59''$
CD	$146^{\circ} 43' 28''$

Find by graphical means the most likely values of the co-ordinates of the point D.

CHAPTER VI

ORDINARY LEVELLING

LEVELLING, or the determination of the relative altitudes of points on the earth's surface, is an operation of prime importance to the engineer, both in acquiring data for the design of all classes of works and during construction.

Definitions.—A *Level Surface* is one which is at all points normal to the direction of gravity as indicated by a plumb line. Owing to the form of the earth, a level surface is not a plane, nor has it a regular form because of local deviations of the plumb line caused by irregular distribution of the mass of the earth's crust. The surface of a still lake exemplifies a level surface.

A *Level Line* is a line lying throughout on one level surface, and is therefore normal to the direction of gravity at all points.

The *Horizontal Plane* passing through a point is the plane normal to the direction of gravity at the point. It is therefore tangential to the level surface at the point, and sensibly coincides with it within ordinary limits of sighting (page 327).

A *Horizontal Line* passing through a point is one lying in the horizontal plane, and is tangent to a level line through the point and having the same direction.

A *Datum Surface (Line)* is any arbitrary level surface (line) to which the elevations of points may be referred.

The *Reduced Level* of a point is its elevation above the datum adopted.

PRINCIPLES

Difference of Level of Two Points.—The simplest operation with the level (page 109) is to determine the difference of level between two points so situated that, from one position of the instrument, readings can be obtained on a staff held successively upon them. The precise situation of the instrument is immaterial, but to eliminate the effects of possible instrumental error (page 322) the two sights should be as nearly equal in length as can be judged. Having set the instrument on firm ground, it is levelled by the foot-screws, and the eyepiece is focused. The staffman is then instructed to hold the staff vertically upon the first point. The observer, having directed the telescope towards it and focussed, notes the reading (page 122), exercising care to ensure that the bubble is

central during the observation. The staff is then held upon the second point, and the telescope is pointed to it. Before sighting, the leveller should examine the bubble to see that it preserves its central position: if it does not, it must be returned to the centre by manipulation of the levelling screws or the tilting screw. The second staff reading is then obtained.

Calculation of Difference of Level.—Let the difference of level between A and B (Fig. 202) be required. The respective staff readings are 7.24 and 2.01 ft., so that A is 7.24 ft. below the first line of sight, and B is 2.01 ft. below the second.

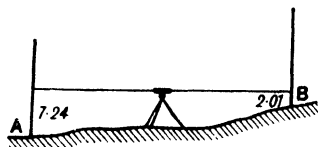


FIG. 202.

But the two lines of sight lie in the same horizontal plane, any want of coincidence between the planes, due to releveling the instrument for the second observation, being insignificant. The staff readings are therefore measurements made vertically downwards from a horizontal plane, and this horizontal plane practically coincides with the level surface through the telescope axis. The difference of 5.23 ft. between the readings is therefore the difference of level between A and B, the smaller reading being observed on the higher point.

Now, let it be assumed that the reduced level of A is known to be 100 ft. above a particular datum, then that of B is obtained as 105.23 ft. above the same datum by application of the difference of level, adding a rise and subtracting a fall. Alternatively, the reduced level of B may be found by referring to that of the lines of sight. By adding the first reading to the reduced level of A, the instrument height, as it is termed, is found to be 107.24. But B is 2.01 ft. below this level, so that its reduced level is obtained as 105.23 by subtraction.

Series Levelling.—The more general case occurs when the two points to be compared are so situated, by reason of their distance apart, their difference of level, or the intervention of obstacles, that it is impossible from any one instrument station to read a staff held successively upon them. In these circumstances, the work is performed in a series of stages, to each of which the previous method is applied. Thus, in Fig. 203, the difference of level between A and D is determined by observing that from A to a convenient point B, and then proceeding similarly from B to C, and from C to D.

The instrument is therefore first set up in such a position as 1, from which a staff held on A can be read and a clear forward view obtained. When the sight on A has been taken, the staffman, proceeding up the slope, selects a firm point B on which to hold the staff. The first stage is completed by noting the reading on B, and the instrument is then transferred to position 2, the staff

meantime being held on B. When the instrument is levelled, the staff on B is again sighted, so that the level of B may be compared

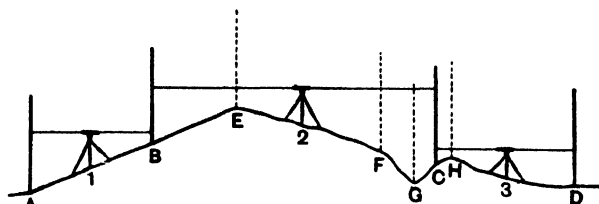


FIG. 203.

with that of a convenient point C in the same manner as before. A third step suffices to reach the point D.

It will be evident that the essential feature of the system lies in the observation of two staff readings on each of the points B and C, one before, and the other after, moving the instrument. When the instrument is being shifted, the staff must not be moved, and, while the staff is being carried forward, the instrument must remain stationary. Points such as B and C, on which the staff is held to permit the transfer of the instrument, are called *change points* or *turning points*. It is unnecessary that they should lie in the line AD. The essential requirements is that they should be unyielding points, to avoid any risk of settlement occurring between the two observations (page 325).

Backsights and Foresights.—The word “sight” is used to denote either an observation or the resulting reading.

A *Backsight* is the first sight taken after setting up the instrument in any position.

A *Foresight* is the last sight taken before moving the instrument.

The first sight on each change point is therefore a foresight, and the second a backsight: every line of levels must commence with a backsight, and finish with a foresight. In Fig. 203 the sights 1A, 2B, and 3C are backsights, while 1B, 2C, and 3D are foresights. The two sets of sights are distinguished because they are differently applied in the calculation of the levels. A *backsight* is always taken on a point the reduced level of which is known or can be computed. By addition of the backsight to that level, the instrument height can be obtained. A *foresight* is always taken on a point of unknown level with the object of ascertaining its level by subtraction of the foresight from the known instrument height.

The terms, *backsight* and *foresight*, are unfortunate because they imply direction. The sights may be taken in any direction, the terms having no other significance than that contained in the above definitions.

Intermediate Sights.—When the backsight has been taken, the observer is in a position to determine the level not only of the next change point, but of any number of points within range. Thus, in the course of the operations of Fig. 203, it may be desired to obtain the elevations of certain points E, F, G, and H. The staff-man on his way from B to C holds the staff on E, F, and G successively, and the leveller takes a single sight on each. The point H is observed from the next instrument station. Sights taken between a backsight and a foresight to ascertain the levels of points are called *Intermediate Sights*.

Datum.—To define the relative altitudes of a series of points, it is sufficient to ascertain their elevations above any one datum surface. A reduced level for one of the points may therefore be assumed arbitrarily, and those of the others are deduced from it. Many advantages, however, accrue from the adoption throughout a country of a standard datum of reduction. Mean sea level, as ascertained by prolonged observation, affords a universal datum, and is that most generally chosen.

The standard datum of Great Britain is that of the Ordnance Survey. This was originally the assumed mean level of the sea at Liverpool. It was obtained from a short series of observations in March 1844, and is believed to be about 0·65 ft. below the general mean level of the sea round the British coast. With the object of establishing mean sea level with greater precision, hourly observations were made at Newlyn, in Cornwall, during the six years, 1915–21. Mean sea level at Newlyn is the new Ordnance datum, and a revision of the levelling of Great Britain has been undertaken with respect to this datum. This revision has now been completed in so far as the transfer of datum is concerned so that at the present time the Ordnance Survey levels for the whole of England and Wales are based on the Newlyn datum.*

The datum for Ireland was fixed on April 8th, 1837, as the level of a particular low tide in Dublin Bay, and is about 8·5 ft. below Ordnance datum in Great Britain.

Bench Marks.—A bench mark is a fixed point of reference, the elevation of which is known. In mapped countries, bench marks are established at intervals throughout the country by the State, and their positions and elevations above the standard datum are published. Surveyors have thus a ready means of expressing reduced levels in terms of standard datum by commencing a series of observations with a reading of the staff held on a bench mark.

Standard bench marks take various forms. In Britain, Ordnance

* "A Description of the Ordnance Survey Large Scale Maps," Ordnance Survey Office, Southampton, 1930. See also Ordnance Survey Leaflet No. 19/33 on "The Newlyn Datum and Ordnance Survey Levels," which can be obtained from the Ordnance Survey Office, Southampton.

bench marks (O.B.M.) are chiselled on buildings, milestones, etc., in the form of an arrow and a horizontal groove (Fig. 204), the centre line of which defines the elevation. The new Ordnance bench marks are cut on bronze tablets let into walls, etc. The position of bench marks and their elevations in feet above Ordnance datum are shown on the 1/2,500 and 1/1,250 Ordnance maps, to the nearest 0.1 ft. in the case of maps which refer to the Liverpool datum and those based on Newlyn which were published before April 1929. Maps published after that date, and referred to the Newlyn datum, give the elevations of bench marks to .01 ft. On these sheets the average difference between the old and new elevations of bench marks is stated to the nearest 0.1 ft. This difference varies throughout the country, since it is due not only to the change of datum, but also to the correction of the old work.

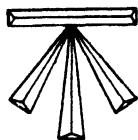


FIG. 204.
ORDNANCE
BENCH MARK.

The surveyor may have occasion to establish private bench marks for his own use. Thus, in the construction of engineering works, repeated levelling between the nearest Ordnance bench mark and the site of the work may be obviated by carefully determining at the outset the elevations of a number of permanent points at the site, these being referred to as required. When levelling in unmapped country, the surveyor should leave marks on rocks, etc., at intervals, for convenience in connecting on to his assumed datum at a future date.

Level Book.—The level book is ruled on the left-hand pages for the entry, in tabular form, of the field observations and for their reduction, the opposite pages being reserved for descriptions and remarks. Various forms of level book, differing in the number and arrangement of the columns, have been proposed with a view to facilitating the entry of the observations and the figures required in reducing, as well as the extraction of quantities to be plotted.

Staff readings are entered in the different forms in one, two, or three columns. The single column method is not recommended, as it necessitates the use of notes or symbols to distinguish backsights from foresights. In using two columns, the first is reserved for backsights, and in the second are entered intermediate sights and foresights, these being similarly applied in reducing the levels. With three columns, the intermediate sights are separated from the foresights, and checking of the reduction is thereby facilitated, so that this system is most generally followed.

Booking and Reduction of Levels.—The observations shown in Fig. 203 may be used to illustrate the method of entering staff readings and reducing the levels. In form A the first three columns

are reserved for staff readings, which are entered in the appropriate columns, in the order of their observation, as shown.

LEVEL BOOK A

Back-sight.	Inter. Sight.	Fore-sight.	Rise.	Fall.	Reduced Level.	Distance.	Remarks.
6-38					50-00		A
5-97		1-17	5-21		55-21		B
	2-10		3-87		59-08		E
	6-35			4-25	54-83		F
	10-20			3-85	50-98		G
1-53		8-22	1-98		52-96		C
	0-90		0-63		53-59		H
		3-76		2-86	50-73		D
13-88		13-15	11-69	10-96	50-00		
13-15			10-96				
0-73 Rise			0-73 Rise		0-73 Rise		

Each point on the ground is represented by a horizontal line, and, since the staff is read twice on a change point, the backsight to it is entered in the same horizontal line as the previous foresight.

The next three columns are used in the reduction of the levels by the "rise and fall" system or method of differences, as outlined on page 304. Each point on the ground, after the first, is compared with that preceding it, and the difference of level, deduced from a comparison of the staff readings, is entered as a rise or a fall. Thus, from A to B we have a difference of 5-21, which is entered as a rise, since the second reading is smaller than the first, A being farther below the line of sight than B. The rise of 3-87 from B to E is obtained by comparing 2-10 with 5-97, since these readings were observed from the same instrument station. No comparison is made of the figures 1-17 and 5-97, since they are common to one ground point and merely show the change of elevation of the instrument from 1 to 2.

On completing the tabulation of rises and falls, the accuracy of the arithmetical work should be checked. The difference between the sum of all the backsights and that of the foresights represents the difference of level between the first and last points, and should equal the difference between the sum of the rises and that of the falls.

To compute the reduced levels, let it be supposed that A is known, or assumed, to be 50-00 ft. above datum. Since the difference of level between consecutive points has been ascertained, it is only necessary to apply these successively, each reduced level being obtained from that of the preceding point by adding the rise or subtracting the fall between them. The calculation of the reduced levels is verified if the difference between the first and last equals the previously obtained total difference.

In Book B the readings are entered as before, but the reduction

is intended to be performed by the instrument height or collimation method, the principle of which was stated on page 304. A column is provided for noting each instrument height or elevation of the plane of sight, from which the reduced levels are to be computed.

LEVEL BOOK *B*

Back-sight.	Inter. Sight.	Fore-sight.	Inst. Height.	Reduced Level.	Distance.	Remarks.
6·38			56·38	50·00		A
5·97		1·17	61·18	55·21		B
	2·10			59·08		E
	6·35			54·83		F
	10·20			50·98		G
1·53		8·22	54·49	52·96		C
	0·90			53·59		H
		3·76		50·73		D

13·88

13·15

50·00

13·15

0·73 Rise

0·73 Rise

The level of the first plane of sight, being 6·38 ft. above A, is evidently obtained by adding the backsight to 50·00, the known elevation of A; and the sum, 56·38, is entered in the instrument height column horizontally opposite the backsight. The reduced levels of all other points on which the staff was read from the first instrument station can be obtained from this elevation by subtracting their readings, so that the reduced level of B is found to be 55·21 by deducting the foresight 1·17. The next instrument height, 61·18, is then obtained by adding the new backsight, 5·97, to 55·21, the level of the change point, and the succeeding intermediate readings and foresight are subtracted as before. Each backsight falls to be added to an already known reduced level to obtain an instrument height, from which intermediates and foresights are subtracted to give reduced levels.

As before, the difference between the sum of the backsights and that of the foresight equals the difference between the first and last reduced levels, but this check verifies the calculation of the instrument heights and levels of change points only. Each reduced level is not obtained from the preceding one, as in the previous method, so that an error made in the reduction of an intermediate sight is not carried forward. Intermediate levels may be checked either by repeating the subtractions or by applying the following rule: The sum of all reduced levels except the first = (each instrument height) \times (the number of intermediate and foresight observations made from it) - (the total sum of the foresight and intermediate readings). In the example, $377·38 = (56·38 \times 1 + 61·18 \times 4 + 54·49 \times 2) - (13·15 + 19·55)$.

LEVEL BOOK C

Back-sight.	Fore-sight.	Inst. Height.	Reduced Level.	Distance.	Remarks.
6.38		56.38	50.00		A
5.97	1.17	61.18	55.21		B
	2.10		59.08		E
	6.35		54.83		F
	10.20		50.98		G
1.53	8.22	54.49	52.96		C
	0.90		53.59		H
	3.76		50.73		D

13.88

50.00

13.15

0.73 Rise

0.73 Rise

The two-column system of booking is illustrated in Book C. Reduction may be performed by either the rise and fall or instrument height method, care being taken, in checking, to add the appropriate figures from the second column.

Notes.—(1) Rapidity in reducing is acquired by practice. In finding rises and falls, the beginner should keep in mind that the second of two figures being compared lies in the lower line, and is either directly below or to the right of the first.

(2) Checking is essential, as slips are easily made. In a long series of levels, each page should be checked before starting with the next. It is best to check from the last entry on the previous page, and this reading, whether a backsight or an intermediate, is included in the backsight summation. If the last reading on the page being checked is an intermediate, it is summed with the foresights, but if a backsight, it is not included.

(3) To minimise the waste of time occasioned by an arithmetical slip, rises and falls should be checked before reduced levels are filled in. With the same object, instrument heights and levels of change points may be worked out and checked before reducing intermediate readings.

(4) It is to be observed that the field work is in no way verified by these checks, which are applied simply to detect and eliminate arithmetical errors in reduction.

Comparative Merits of Reduction Methods.—The instrument height system is economical of figuring, and proves the more rapid method. It is well adapted for reduction in the field, particularly in setting out levels for constructional work. If, as is usually the case, the somewhat tedious check on intermediate levels is neglected, it is open to the objection that a mistake in an intermediate reduction may pass unnoticed. This disadvantage is entirely absent in the rise and fall method, which affords a complete check, only vitiated if mistakes balance. By reason of its greater certainty, the use of the latter method is sometimes insisted upon in important work.

FIELD WORK

The operations of ordinary levelling are :

1. The determination of the difference of level of two points.
2. The running of sections.
3. The setting out or giving of levels for constructional purposes.
4. The running of contour lines (Chap. VIII).

Determination of the Difference of Level of Two Points.—Ordinary levelling for the ultimate purpose of comparing the levels of two points only has sometimes to be performed by the engineer in the manner already described. If the points are in the same vertical line, the use of a level and staff is often preferred to taping. When both points cannot be seen from one set-up of the instrument, it is necessary to run a line of levels between them, observing backsights and foresights only.

Sectioning.—The running of a section is the most common levelling operation, and consists in obtaining a record of the undulations of the ground surface along a particular line, straight or curved, so that they may be represented to scale. This involves observing not only the elevations of a number of points on the line, but also their distances along it. It is important that the points selected should be those at which the inclination of the ground sensibly changes, so that, having plotted them, it is justifiable to assume a uniform slope between each consecutive pair. Thus, in Fig. 205, *a* represents the actual undulations of a piece of ground : if the levels and chainages of the salient points numbered are observed and plotted, a good representation of the slopes is obtained even although the points are joined by straight lines as in *b*. Fig *c* shows the serious misrepresentation produced by omission to observe points 3 and 5. On the other hand, if, as in running a section along a road or a railway, the points of change of gradient are not evident, it is permissible to observe equidistant points, say a chain length apart.

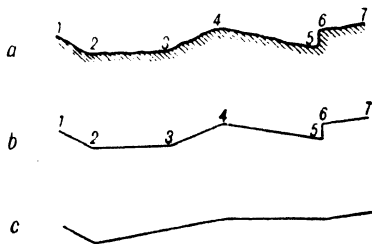


FIG. 205.

Longitudinal and Cross Sections.—Sections are of two kinds : (a) Longitudinal Sections, or Profiles ; (b) Cross Sections.

A longitudinal section is one which follows some predetermined line, usually the centre line of proposed work, *e.g.* a road, railway, canal, or pipe-line. By means of such sections the engineer is enabled to study the relationship between the existing ground surface and the levels of the new work in the direction of its length. In the design of works, it is frequently necessary to run longitudinal sections

along various proposed centre lines and compare the costs of the several schemes.

Since a plotted profile cannot exhibit any particulars as to the character of the ground on either side of the centre line, it does not convey sufficient information for the complete design of works. In the case of those which occupy a very narrow strip of ground, *e.g.* a pipe-line, it will, however, be sufficient to assume that the ground within the limits of the width of the work is level in a direction normal to the centre line. Otherwise, additional information is required, and this is conveniently obtained by means of cross sections, which are sections taken at right angles to the centre line and of sufficient length to embrace the limits of the work on either side.

LONGITUDINAL SECTIONS

Field Party.—The work is most expeditiously performed by a party of four, *viz.* :

The Leveller, who directs the ranging of the line, sees that the staff is being held on suitable points, uses the instrument, and books the staff readings and distances.

The Staffman, who holds the staff and calls out the chainages.

Two Chainmen, who manipulate the chain.

Commonly one of the chainmen holds the staff, the leveller having to wait while the chain is being pulled forward. If only one assistant is available, much time is wasted by the surveyor having to leave the instrument in order to take part in the chaining.

Equipment.—*Essential.*—Level, Staff, Ranging Poles, Chain or Band, Arrows, Chalk, Level Book, and Pencil.

Optional.—Tape and Plumb Line for stepping.

Running the Section.—If necessary, the line of the section must first be set out by locating a sufficient number of points to define the straights and curves, intermediate poles being ranged by eye. In a long section only a sufficient length should be ranged in advance as will occupy, say, half a day to level.

It frequently happens that the section is run along the courses of a traverse survey. If the traversing has been completed previously, it is necessary to recover the stations by reference to the notes describing their positions, and the lines must be ranged afresh. In mapped country it may not be necessary to run a traverse, and the line of the required section is drawn on a map, from which it has to be set out by the levelling party. Its position relatively to buildings, fences, and other definite features represented on the map is obtained by scaling, and the scaled dimensions are reproduced on the ground. In cases where a number of points so located are intended to lie on a straight line, it frequently happens that they

fail to do so exactly on account of errors of scaling and of measurement, but it is usually quite sufficient to range a straight line by eye in such a position as will average out the irregularities. In districts where there is a scarcity of well-defined detail the transfer of the section line from the map to the ground is sometimes troublesome, and it becomes necessary to set it out by theodolite.

To refer the levels to a standard datum, levelling should start from an Ordnance bench mark, and the position and value of that nearest the beginning of the section should be extracted from an Ordnance map before proceeding to the ground. Levelling is conducted from the bench mark, by observing backsights and foresights only, until the instrument can be placed in a position from which the first part of the section can be commanded.

Chaining is now commenced, and the staffman proceeds along the chain, and holds the staff at all points of change of slope. After each reading is taken, he calls out the chainage of the point to the leveller, or he may enter them in a notebook, from which they are transcribed into the "distance" column of the level book at frequent intervals. When the length of sight reaches the maximum permissible for good reading (page 326), or when inequalities of the ground prevent the taking of further intermediate sights, the leveller signals the staffman that he wishes a foresight. The latter selects a firm change point, either on or off the chain line, marks it with chalk, and holds the staff upon it, taking due care as to verticality. Having entered the reading, the leveller carries the instrument forward. He selects his next instrument station on firm ground, and in such a position that he can sight back on the staff, and also obtain an unobstructed forward view along the section. When the backsight observation is completed, the staffman proceeds to give intermediate sights as before, and this routine is followed to the end of the section.

As the various features lying in the section line are reached, such information regarding them as is likely to prove useful in the design of the proposed works should be acquired. Such items include the levels of the beds of streams, flood water levels, if indications of them can be traced, the chainage at which fences, etc., intersect the line, the names and levels of roads and railways crossed, the headroom of bridges, etc. When the section line passes below a bridge, readings are taken with the staff held inverted against the underside of the girders or arch, these being distinguished in booking by being marked with a plus sign or otherwise. The elevation of the road or rail surface on the top of the bridge is ascertained by levelling up the bank.

Notes.—(1) The chainage should be continuous from start to finish of the section.

(2) The use of salient points on the line of section as change points saves time, but accuracy should not be sacrificed by using an unstable point on the line in preference to a firm point on one side.

(3) Time may be wasted by excessive refinement in reading intermediate sights. In ordinary cross-country work it usually suffices to read intermediates to the nearest tenth of a foot only, but the relative importance of the staff points should always be kept in view.

(4) See notes on expedition in levelling (page 329).

Checking.—It is impracticable to check every elevation in a line of levels except by duplicating the whole of the observations. In the great majority of cases, however, it is sufficient to test the quality of the work in so far as the change points only are concerned, or, in other words, to ascertain whether the various instrument heights are correct within allowable limits of error. Elevations obtained by means of intermediate sights from these instrument heights are affected by errors, and possibly by serious mistakes in reading or booking, but the effects are quite local, and cannot be carried forward (page 322).

In a country unprovided with standard bench marks the levelling is checked :

(a) By adopting a special routine at the change points to render the work self-checking.

(b) By levelling back to the starting-point.

In the former system, the most convenient method of limiting the propagation of error and of detecting gross mistakes is to use double change points. At each set-up of the instrument two backsight readings are obtained on the respective change points, so that each instrument height is determined twice. If the discrepancy between them at any stage is no more than is expected to arise from accidental errors, the mean is accepted. Otherwise, repeat sights are taken on the change points. The method is economical of time as compared with the running of a check line of levels, but does not provide quite such a thorough check (Vol. II, Chap. VI).

By the latter method the error of closure is exactly known : if this exceeds the permissible error (page 327), the work must be repeated. A long section should be divided into parts, each to be checked back before the next is levelled. Check levelling may be conducted along the easiest return route or along the original change points, so that mistakes may be discovered and rectified expeditiously. The more prominent change points should be referenced as temporary bench marks for possible future use.

If, however, numerous standard bench marks are available, the work is completed by levelling from the end of the section to an adjacent bench mark, the value of which has been ascertained from the map. Any discrepancy between the known elevation of the bench mark and its value as obtained by reducing the levels may be accepted as the closing error, except in mining districts, where the possibility of irregular subsidence of bench marks makes any check other than levelling back to the starting-point of little or no value. Opportunities of suitably dividing up the section by checking on to the bench marks near the line should not be neglected, and

the positions and values of adjacent bench marks should be noted before proceeding to the field.

Note.—While it is unnecessary to reduce all the levels as the section progresses, backsights and foresights should be kept summed for the purpose of checking on to bench marks.

Plotting the Profile.—On reduction of the levels in terms of Ordnance or other datum, the reduced levels are available for plotting. Having drawn a datum line, and marked off along it the chainages of the points to be plotted, perpendiculars are erected, and the appropriate reduced level is set off on each. A continuous line joining the points so obtained represents the profile of the ground.

Since the horizontal distances involved are in general very much greater than the variations in level, it is usual to plot vertical dimensions to a larger scale than horizontal distances. In this way the irregularities of the ground are made more apparent. The steepness of slopes is exaggerated, and artificial features intersected suffer corresponding distortion. Exaggeration is of value in enabling the relationship between the original surface and the proposed levels of new work to be clearly and accurately shown. The ratio of exaggeration adopted runs from 5 to 15 times and upwards, depending upon :

(a) *The Character of the Ground.*—A greater exaggeration is required to exhibit the irregularities of flat ground than those of rough country.

(b) *The Horizontal Scale.*—The horizontal scale may be chosen arbitrarily, but it is commonly that of the plan upon which the section line is drawn. The smaller the horizontal scale, the greater should be the ratio of exaggeration.

(c) *The Purpose of the Section.*—The vertical scale should be increased in cases where a highly accurate representation of vertical dimensions is required.

If the section has been run in a district at a considerable elevation above sea level, the reduced levels in terms of Ordnance or a similar datum have large values, and to plot them to a sufficiently large scale to exhibit the surface irregularities would require an inconveniently large depth of paper. In such a case, the base line should be assumed to have an elevation of a convenient number of feet above the datum of reduction. The value of the base line may have to be changed several times in a long section, as it is desirable to maintain the lengths of ordinates within the limits of about $1\frac{1}{2}$ and 6 in.

In finishing the profile, the datum and ground lines are drawn in black ink, and the ordinates are shown as thin blue lines. If plotted points are joined by a continuous curve, regard should be had to the character of the ground and the ratio of exaggeration adopted. It is usually safer to join the points with straight lines, rounding off sharp angles unless the exaggeration is great. The value of the

datum line should be given, the reduced levels written against the ordinates, and descriptions entered where necessary.

Notes.—(1) Scaling is obviated by the use of profile paper having printed horizontal and vertical lines. Various rulings are sold, that having 20 horizontal and 4 vertical lines to the inch being most used.

(2) Only points on the line of section are plotted, and caution must be exercised against including any other observations inadvertently.

Working Profile.—When the location of an engineering scheme has been decided, and the design made, a working section is prepared for the use of the resident engineer. This profile incorporates the features of the original ground surface, as well as the levels of the new work, and must exhibit definite information regarding the relationship between the new levels and those of the original ground. All further information likely to be required during construction, and which can be clearly shown, is also included.

The character of this information depends upon the nature of the work. Part of a working profile for a railway is shown in Fig. 206. The new work is represented by two parallel lines, the lower, in red, denoting formation of sub-grade level, i.e. the surface level of the earthwork, and the upper, usually in blue, representing rail level. Ordinates are drawn, in this case at 1 chain intervals, and the datum line is figured to show distances from the commencement of the railway in miles, furlongs, and chains. The figures written

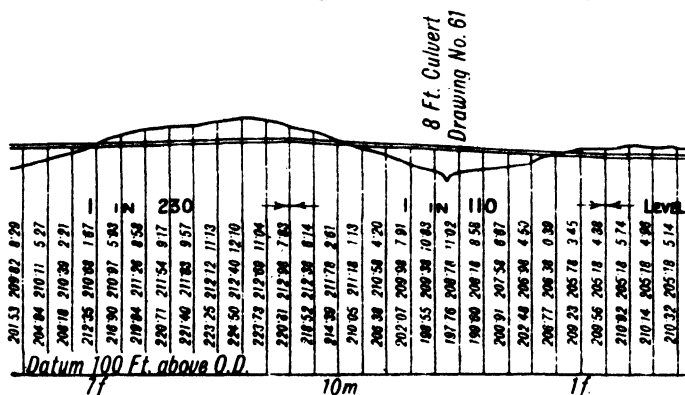


FIG. 206. WORKING PROFILE OF RAILWAY.

against the ordinates represent original ground level, formation level, and depth of cutting or bank. Original levels are written in black, formation levels in red, and depths of earthwork in red or blue, according as they refer to excavation or embankment. The gradients of the new work are figured boldly, and the limits of each clearly shown by arrows against ordinates drawn in red. The positions of bridges, culverts, level crossings, etc., as well as brief particulars of existing features crossed by the line of section, are also entered.

CROSS SECTIONS

These are usually taken during the progress of the longitudinal section. If the best results are desired, the observations may be made with level and staff, particularly if the cross sections are long. More rapid methods by clinometer or hand level are sufficiently accurate for many purposes, and are especially suitable for short sections, the length of which precludes the accumulation of serious error.

Interval between Sections.—The purpose of cross sectioning is to furnish the engineer with sufficient information regarding the levels of the ground on either side of the longitudinal section to enable him to design the intended works and compute the quantities of earthwork, etc., involved. To facilitate estimation of the character of the ground between cross sections, they should be taken at every marked change of slope transverse to the longitudinal section, so that it is valid to assume that the ground surface changes uniformly from one section to the next. This desideratum is often neglected in practice, cross sections being commonly taken at constant intervals of 1 chain or 100 ft.

Setting Out.—The lines of cross sections are in general perpendicular to the longitudinal section line and radial on curves. When for any reason a cross section is run in another direction, the angle it makes with the longitudinal section must be measured, so that its position may be shown in plan. In sectioning a wide area, *e.g.* for a reservoir or a dock, the cross sections may have a considerable length, and their lines should be set out by theodolite, box-sextant, or optical square. In the case of a narrow strip of ground, *e.g.* for a road or a railway, their perpendicularity is usually judged by eye. Since cross sections must be long enough to include the width of the new work, a sufficient margin must be given in cases where the latter is not known exactly.

Cross Sectioning by Level and Staff.—The procedure is similar to that followed in longitudinal sectioning, but in the case of short sections the distances are taped. A chainman remains with the tape box at the point on the centre line from which the section is projected, while the staffman takes the ring and proceeds along the section. The chainman guides the staff holder into the perpendicular, and calls out the measurements of the points observed. If the leveller has only one assistant, he should first mark by arrows or twigs the salient points to be levelled, and note the distances on sketches of the sections.

The levelling of cross sections on flat ground is performed from the instrument stations used for the longitudinal section, but on sidelong ground it may be impossible to complete any cross section

from one instrument station. In these circumstances, the repeated shifting of the instrument renders the progress very slow, particularly if each cross section is finished before the next is begun. The number of instrument stations may be greatly reduced by sighting from each the several points within range on a number of sections. Thus, in Fig. 207, the points *a* may be observed from the instrument

station selected for the sighting of the points *a* of the longitudinal section. On shifting the instrument up the slope, points *b* are levelled, their distances out being measured from marks left on the centre line. Finally, points *c* are observed from a lower position of the instrument.

In booking cross section observations, the staff readings are entered in the appropriate columns in the usual manner. Distances along the cross sections must be clearly distinguished from those of the longitudinal section. This is best secured by having three distance columns, the centre one being reserved for distances on the longitudinal section, and the others for the cross section measurements to right and left respectively. If only one column is provided for distance, the cross section measurements may be entered in it with *R.* or *L.* written after each to signify whether the measurement is to the right or left of the centre line. It is, however, preferable to reserve the single distance column for longitudinal section measurements only, and to enter transverse distances in the remarks column. Care must be exercised in booking cross sections which are observed in groups, as in Fig. 207, and explanatory sketches are often necessary.

The plotting of cross sections observed as above is similar to that of profiles, except that, for the purposes of showing and measuring new work, it is more useful in this case to have vertical and horizontal measurements plotted to the same scale. The scale is commonly that used for the vertical dimensions of the profile. It is usual to arrange the cross sections on a sheet in rows on a series of vertical centre lines. The elevations represented by their datum lines may be frequently altered to keep the ordinates reasonably short. To economise room, datum lines and ordinates are sometimes omitted in inking in, and the reduced level at the centre is written horizontally on the ground line.

Cross Sectioning by Hand Level.—The hand level is held at a constant known height above the ground either by supporting it against a pole or by standing erect when sighting, the height of the eye having been measured. The surveyor stands over the centre point, and observes and notes the readings on a levelling staff or a ranging rod, and books the taped distances. Alternatively, the

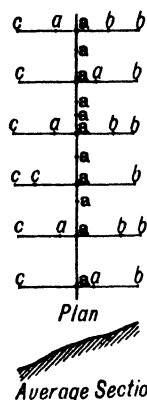


FIG. 207.

staff may first be held on the centre point and a backsight taken on it from a convenient position. It may be necessary to move the instrument in running a section on steep ground, and this is effected by sighting on a change point as in ordinary levelling. Since the level of the centre point is obtained from the longitudinal section, the levels may be reduced, but it is more usual to plot the section by setting down the staff readings from the line, or lines, of sight.

Cross Sectioning by Theodolite.—On steep ground the labour of frequently changing the instrument is obviated by the use of an inclined line of collimation roughly parallel to the ground and of known inclination. This is most accurately given by the theodolite.

In Fig. 208, the theodolite is set over the centre line mark, the level of which is known. Having levelled up, the height of the horizontal axis above the ground is taped and noted. The line of collimation is set roughly parallel to the ground, and the vertical circle is clamped at the nearest whole degree, the angle being booked. The

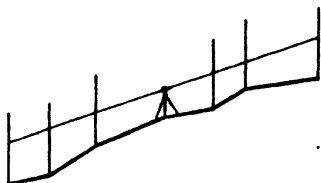


FIG. 208.

inclinations of the line of collimation on either side of the instrument need not be the same, and may differ considerably on rough ground. Where the general slope of the ground is fairly uniform, however, the value of the angle of elevation on the uphill side is reproduced on the circle as an angle of depression on the downhill side in order to simplify the plotting a little. The readings of the staff held vertically at the various salient points are observed. Distances are taped along the line of collimation, the staffman holding the ring of the tape against the staff as nearly at the reading as he can judge, while the measurements are read by the chainman at the horizontal axis of the instrument. In the case of long sections, two tapes should be tied together.

The results of the observations are usually booked in columns, but may be noted on sketches. In the tabular arrangement, the middle column should show the chainage at which the cross section is taken, the height of the telescope axis above the ground, and the elevation of the instrument station, if it is already known. The adjacent columns to right and left are reserved for the vertical angles observed on the corresponding sides of the section, the entries being marked plus or minus, or e or d , to distinguish elevations and depressions. In the remaining columns on either side are booked the staff readings, with the corresponding distances written below. If sketches are preferred, they may be made on the lines of Fig. 208, with the dimensions entered in place. Otherwise, use is made of a survey field book with two rulings down the middle of the page, and in the space between them the chainage, height of axis, etc., are entered as in the tabular method. The vertical angles are figured prominently on the sketches on either side.

To plot the section, the reduced level of the centre point is set up from the datum, and from it is scaled the height of the instrument axis above the ground. From a horizontal line through the point obtained, the line of sight is plotted by protractor. The several distances are then marked off along the line of sight, and from those points verticals equivalent to the staff readings are measured down. A continuous line through their lower ends represents the ground line.

Cross Sectioning by Clinometer.—The Abney or other pattern of clinometer may be used in place of the theodolite. This is the most rapid means of cross sectioning, and the results are sufficiently accurate for many purposes. The clinometer is much used in location surveys.

Three systems of observation are shown in Fig. 209.

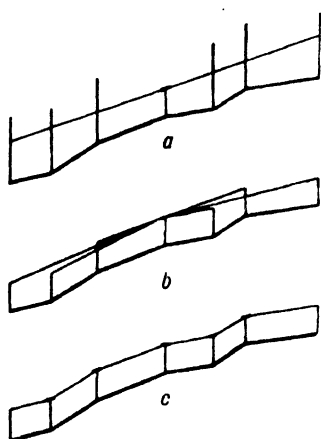


FIG. 209.

(a) The observer, standing over the centre point, holds the clinometer at a known height above the ground with the line of sight roughly parallel to the ground. The angle on each side is noted, readings are taken on a levelling staff or ranging pole, and slope measurements are made as in using the theodolite.

(b) The observer at the centre sights a mark on a ranging pole at the same height as the instrument is held above the ground. The various angles and slope distances are noted.

(c) The angle and length of each slope are separately measured, the observer using method (b) and proceeding along the section.

Of these methods, the first is the most rapid in observation, booking, and plotting, and is to be preferred unless the ground is so irregular that the required staff readings cannot be obtained from two lines of sight. In this case the other methods are preferable. Method (c) is likely to prove slower than (b), but it is useful if the sections are long, as it avoids lengthy tape measurements. If, in the course of using method (a) or (b), a hump occurs on a section, so that the sighting cannot be completed from the centre point, it becomes necessary to move the instrument to a point on which an observation has already been taken in order to continue the section beyond the summit. Circumstances may warrant the combination of two or more methods on one section.

The booking is performed in columns or with the aid of sketches, as described for theodolite sectioning. Sketches are useful in cases where the instrument has to be moved from the centre station, since

the points from which the angles are observed can be indicated clearly. The plotting in the case of method (a) has been described above. In the other methods it is, of course, unnecessary to draw the lines of sight, the points on the section being plotted directly from the angles and distances.

Notes.—(1) If the clinometer is supported against a ranging pole, it is well to hold the latter upside down, as otherwise the pointed shoe will enter the ground a variable distance. An Abney clinometer may be attached to the pole by means of a rubber band. Alternatively, a wooden rod is sawn to such a length that, on placing the instrument on top of it, the line of sight is 5 ft. or other convenient distance above the ground. The sighted rod, used in methods (b) and (c), is cut to this dimension, so that it is a little longer than the other.

(2) The observer may prefer not to have to carry a support for the clinometer, and it is only necessary for him to know the height of his eye when standing erect.

(3) If a levelling staff is used in method (a), it should have an open, bold graduation. If readings are estimated on a long ranging pole, care is required to avoid mistakes of a whole foot.

Setting Out Levels.—The levels to which work is to be built may be shown by driving a peg or making a mark either at the desired level or at a stated distance above or below it. In the former case, the required staff reading when the foot of the staff is at the correct level must first be deduced, and the staff is raised or lowered until this reading is obtained. Instrument height booking will be found the more convenient for such work. In the latter method, an arbitrary point is established, and the staff is read upon it, the difference between its level and that of the construction being communicated to the foreman. There is less likelihood of mistakes on the part of the workmen if this distance is an exact number of feet.

Notes.—(1) The leveller must be constantly on his guard against mistakes and errors, as these may have very serious consequences in setting out. The work must always be well checked against mistakes.

(2) In setting pegs to a required level, time is wasted by driving a peg too far, as it may have to be removed and driven afresh. The latter stages of the driving must be performed with caution, and the reading observed at frequent intervals. The staffman should be told after each reading by how much the peg still requires to be lowered. If, however, a peg is only a small fraction of an inch too low, a nail may be driven in it to the correct level to save time.

Example.—An engineer on works is required to give a number of levels. He observes a backsight on a private bench mark (P.B.M.) of elevation 297.34, and reads 4.06. He then takes a reading of 5.62 at a change point, and transfers the instrument to a position from which he can see the work. His new backsight is 3.81. He is required (a) to give a mark at elevation 295.60, (b) to correct a peg which is roughly indicating a height of 5 ft. above the bottom of an excavation to be taken out to level 291.00, (c) to check the finished level of different points of a concrete foundation which is intended to be at 293.75. In the second case he reads 3.45 when the staff is held on the peg, and in the last his readings are 5.88, 5.86, 5.85, and 5.89.

The height of instrument from which the required observations are taken is $297.34 + 4.06 - 5.62 + 3.81 = 299.59$.

(a) To set out a level of 295.60, the staff must be adjusted until the reading obtained is $299.59 - 295.60 = 3.99$, the mark being made at the bottom of the staff.

(b) The elevation of the peg is $299.59 - 3.45 = 296.14$. As the peg is intended to be at 296.00, it must be lowered by 0.14 ft., or until a reading of 3.59 is obtained with the staff held on it.

(c) If the foundation were at the correct level of 293.75, each staff reading would be $299.59 - 293.75 = 5.84$. The work is therefore too low by $\frac{1}{4}$ in., $\frac{1}{4}$ in., $\frac{1}{4}$ in., and $\frac{1}{4}$ in. at the respective points.

Levels given in this way can be transferred within a limited distance by the workmen by the use of a straight edge and a small spirit level. This is also accomplished by means of T-shaped crossheads or boning rods, which furnish a line of sight whereby, from two given pegs, points at the same level or on the same gradient may be established in their line. In Fig. 210, A and B are

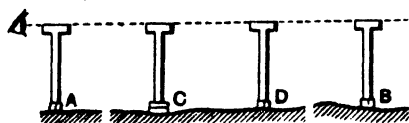


FIG. 210.

pegs set on a particular gradient and, say, 100 ft. apart. The foreman, holding a boning rod on A and looking along the top of it at the top of a similar rod held on B, can direct the adjustment of points such as

C and D into the gradient by judging when the upper surfaces of the boning rods held upon them are in the line of sight.

Setting Slope Stakes.—See page 463.

SOURCES OF ERROR IN ORDINARY LEVELLING

Numerous sources of error may affect the accuracy of a line of levels, but the precautions against them are of a simple nature, so that it is not difficult to obtain good results without delaying progress. It is to be understood that the precautions detailed below refer more particularly to the observation of backsights and foresights, since an error introduced at a change point is carried forward throughout the subsequent work. An error in an intermediate sight affects the recorded level of that point only, and may not prove of much consequence; although it should be recognised that some intermediate sights may be of great importance, especially in engineering work. The various errors and mistakes may be classified as:

- (1) Instrumental Errors.
- (2) Errors and Mistakes in Manipulation.
- (3) Errors due to Displacement of Level and Staff.
- (4) Errors and Mistakes in Reading.
- (5) Mistakes in Booking.
- (6) Errors due to Natural Causes.

1. Instrumental Errors.—*The Level.*—The testing and adjustment of the instrument have been discussed in Chapter I. The important desideratum is that the line of collimation should be exactly parallel

to the level tube axis, so that the line of collimation is horizontal when the bubble is at the centre of its run. The error introduced by non-adjustment is proportional to the length of sight, and is entirely eliminated between change points by equality of the backsight and foresight distances, but intermediate sights, being usually of various lengths, will be thrown into error by different amounts. Errors arising from imperfect estimation of the equality of backsight and foresight distances are compensating, but if the backsights are consistently longer or shorter than the foresights—a tendency to be guarded against on steep slopes—the error becomes cumulative.

A defective level tube may have a considerable influence. If under-sensitive, the bubble may apparently come to rest in the central position although the tube axis is not horizontal, this giving rise to a compensating error. On the other hand, over-sensitiveness in an instrument for ordinary use leads to waste of time in levelling up. Irregularity of curvature of the tube is a serious defect, the influence of which will, however, tend to compensation with equal sights.

The tripod should be examined and loose joints tightened, as instability of the instrument causes waste of time, and leads to erroneous readings.

The Staff.—It is advisable to test the graduation of a new staff by a steel tape or foot-rule, but the error is likely to be negligible in ordinary work. There is greater probability of error through wear of the staff at the joints or by dirt adhering to hinges or sockets. A telescopic staff should be let down gently to minimise wear, and in using the Scotch staff, the ends of the separate pieces should be kept clean to enable them to be pushed firmly home in the sockets. It is also important that the separate parts should all belong to one staff, otherwise very serious errors of graduation may occur at the joints.

It may be noticed that wear at the bottom of a staff is of no consequence since it is unnecessary that the zero of the graduation should be placed at the foot of the staff in order that differences of staff readings may represent differences of level. An exception occurs, however, in obtaining the difference of level between a point below the plane of sight and one above, the staff being held inverted on the latter point. In this case, the difference of level is the sum of the two staff readings, and an error would be produced equal to twice the distance between the zero of the graduation and the foot of the staff.

2. Errors and Mistakes in Manipulation.—*The Level.*—The most serious and common mistake in observing is the omission to have the bubble central at the instant of sighting. In the instruments illustrated in Figs. 92, 98, and 100 the observer obtains, by reflection, a view of the bubble while he is reading the staff. In using a level without a reflector, he should examine the bubble

before sighting, and bring it central if necessary. After reading, he should again glance at the bubble, and, if correct, it may be assumed that it remained so during the observation. After the instrument is levelled, it should not be handled unnecessarily. The tripod should not be grasped, and, in turning the focussing screw or wheeling the telescope, the application of vertical pressure should be avoided.

The Staff.—The staff should be held quite vertical. If held off the plumb, it will be intersected by the line of collimation farther from the foot than it should be, and the reading will be too great (Fig. 211). As the errors caused by a given deviation from the vertical are proportional to the readings, special care must be taken with large readings. Errors are avoided (a) by having a spirit level (Fig. 322) or a pendulum plumb bob attached to the

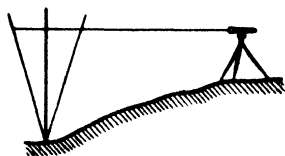


FIG. 211.

staff, to facilitate holding it plumb, or (b) by swinging or waving the staff.

The latter is a most useful and simple method. The staffman, holding the staff on the point in the ordinary manner, inclines it slowly towards and away from the instrument, on both sides of the vertical. The observer sees the reading vary against the horizontal hair, but the smallest reading corresponds to the vertical position, and is that noted. Waving should be performed in the direction of the line of sight only, as the leveller can detect lateral non-verticality by means of the vertical hair or hairs. It is unnecessary and inadvisable to swing the staff if the reading is below about 3 ft., since the bottom of the graduated face is raised appreciably off the ground when the staff is leaning away from the instrument. This difficulty could be overcome by fitting the bottom of the staff with a knife edge or pin placed in the plane of the graduated face, but such a design, although convenient for a precise levelling staff supported on a peg or a plate, is unsuitable for use in ordinary levelling.

Errors due to non-verticality of the staff tend to compensate at change points, but, if the backsight readings are consistently greater or smaller than the foresights, the error becomes cumulative. Thus, in levelling up a slope the observer will read well up the staff in taking backsights and near the bottom for foresights. Careless staff holding increases the former without appreciably affecting the latter, and, in consequence, too great a rise is recorded between change points, and the slope appears steeper than it really is. In levelling downhill, the foresights are the larger readings, and their increase makes the fall between change points appear too great, so that the slope is again exaggerated. In levelling over a hill, therefore, the error accumulated in working up one side is more or less completely neutralised in descending the other side, and the levels may check at the finish, but the observed elevation of the hill is too great.

A further precaution to be observed by the staffman is to see that dirt does not accumulate on the foot of the staff, as this would cause a variable relationship between the zero of the graduation and the foot of the staff.

3. Errors due to Displacement of Level and Staff.—*The Level.*—If the instrument is set up on soft ground, it may gradually settle from the moment of the backsight observation to that of the foresight. This will always make the foresight reading smaller than it should be, giving too great a rise or too small a fall between change points. The error is cumulative, as every settlement of the instrument increases the reduced level of all subsequently observed points by the amount of the sinkage. It follows that the level should, as far as possible, be placed upon solid ground with the legs thrust firmly into the ground, and that time should not be wasted between the backsight and foresight observations. If the engineer must plant the instrument upon staging, he should avoid treading on the planks which support it.

It sometimes happens that the level is disturbed by the tripod being accidentally kicked, but, as the mishap will be noticed, no error need result. If the positions of change points have been marked, it is only necessary to relevel the instrument and again backsight on the last change point, substituting the new reading for the previous one. Any intermediate points taken prior to the dislevelment must have their readings correspondingly altered, or may be observed anew. If change points are not marked, it is necessary to return to the start or to the first change point which can be identified with certainty.

The Staff.—A serious and common error is that occasioned by change of level of the staff at a change point while the instrument is being carried forward. It is commonly caused by choosing unsuitable turning points. Soft ground should be avoided owing to the probability of the staff sinking between the foresight and backsight observations. A flat stone embedded firmly in the ground makes a good support. If only irregular or rounded boulders are available, the staff should be held on the highest point as at *a* (Fig. 212): if held as at *b*, it is difficult to maintain the foot on the point while turning the staff to face the new instrument station. The use of a peg or a foot-plate as a support in soft ground prevents sinkage. Having selected a suitable point, the staffman should first mark the spot with chalk, and should keep holding the staff upon it until the backsight observation is completed.

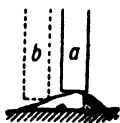


FIG. 212.

Since any change of level of the staff will nearly always be in the direction of sinkage, the error is cumulative. The backsight reading on the settled staff will be too great, and the reduced levels of all subsequently observed points made too high.

4. Errors and Mistakes in Reading.—Small compensating errors occur in the estimation of the decimal part of the readings. The increased size of the image makes estimation easier with short sights than with long ones, and it is desirable that important sights should not exceed about 300 ft., but this limit depends upon the quality of the telescope as regards resolving power, and also upon the character of the staff graduation and the clearness of the atmosphere. Focussing must be carefully performed to eliminate parallax (page 28). The observer should keep moving his head up and down while sighting, and should adjust the focussing screw until no apparent movement of the horizontal hair relatively to the staff can be detected.

In sighting an openly graduated staff, it is sometimes difficult to choose between two possible readings differing by $\cdot 01$ ft. The smaller should be preferred owing to the possibility of non-verticality, especially if the staff is not swung.

Common mistakes made by beginners are : (a) reading upwards, instead of downwards ; (b) reading against a stadia hair ; (c) concentrating the attention on the decimal part of the reading, and noting the whole feet wrongly ; (d) omitting the zero from decimals under $\cdot 10$; (e) reading downwards, instead of upwards, when the staff is inverted.

5. Mistakes in Booking.—These include : (a) entering a reading in the wrong column ; (b) omitting an entry ; (c) noting a reading with the digits interchanged ; (d) entering the wrong distance or remarks opposite a reading.

A fruitful source of erroneous booking occurs when the end of a line is reached and check levelling is to be carried back along it, as the last change point may be used twice. Thus, in Fig. 213,



FIG. 213.

from A, the last position of the instrument for the forward levelling, the reading on the change point B is first observed, and entered as a backsight. When the intermediate readings on C, D, and E have been noted, the reading on B is again observed, and must be entered as a foresight. Uncertainty is avoided by keeping in mind the definitions of the terms, backsight and foresight (page 305).

To detect mistakes in writing down readings, the best method is to read the staff, book the reading, and then sight the staff again to see that the figure booked is the correct reading.

6. Errors due to Natural Causes.—*Wind and Sun.*—(See page 227). If levelling must be performed in a high wind, an endeavour should be made to shelter the instrument, and high readings should be avoided owing to the difficulty of holding a long staff sufficiently steady and plumb.

On hot sunny days the apparent vibration of the staff caused by irregular refraction makes close reading impossible, and, as a

partial remedy, the lengths of important sights should be reduced. Distortion of the instrument by unequal heating and expansion or contraction of the staff produce errors which are negligible in ordinary work.

Curvature and Refraction.—In consequence of the curvature of the earth, the point read on the staff is not strictly at the same level as the horizontal hair of the reticule, since the line of sight is not a level line. In the observation from A to a staff BC (Fig. 214), a difference CD is developed between the horizontal and level lines through the instrument. If the line of sight coincides with AC, the graduation C observed is the distance CD above the instrument height, and the reduced level of B is made out to be lower than it really is by this amount.

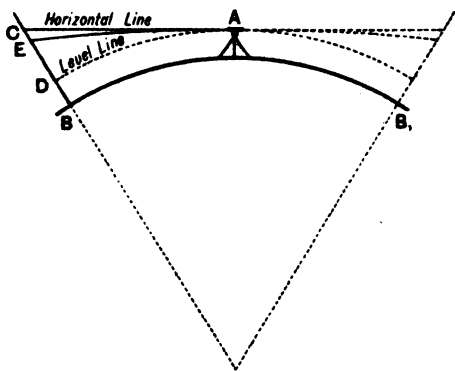


FIG. 214.

Denoting the length of sight by L , then by geometry,

$$CD = \frac{L^2}{\text{Diameter of Earth.}}$$

If L is in miles, $CD = .667 L^2$ ft.

Actually, the line of sight is not straight, but, in consequence of the refraction of light in passing through layers of air of different densities, is, in general, a curve concave towards the earth. It is represented by AE, so that the graduation at E is that actually read. Under normal atmospheric conditions, arc AE may be taken as circular and of radius seven times that of the earth. The effect of refraction is therefore $\frac{1}{7}$ that of curvature, but is of opposite sign, so that the combined error,

$$ED = .57 L^2 \text{ ft.,}$$

by which amount the point sighted is made out lower than it really is.

For ordinary lengths of sights the error is very small. It is eliminated by equality of backsights and foresights, *e.g.* between B and B₁ (Fig. 214), assuming the atmospheric conditions the same for both sights, but will accumulate if the backsights are consistently longer or shorter than the foresights. The error is also eliminated by the method of reciprocal levelling (see below).

Allowable Closing Error.—The total error developed in a line of levels is composed partly of cumulative and partly of compensating

error, but the more refined the procedure becomes, the more nearly will the former be eliminated. In ordinary levelling, adherence to the usual precautions, especially as regards equality of backsight and foresight distances and stability of change points, prevents the propagation of serious cumulative error, and experience shows that the closing error of ordinary careful work may be taken as obeying the law of accidental error. The total error is therefore proportional to the square root of the number of instrument stations. As the number of stations per mile will not vary greatly for a particular kind of country, the error E , in feet, developed in a distance of M miles may be expressed as

$$E = C\sqrt{M},$$

where C is a constant depending upon such circumstances as the observer's experience, the quality of the instruments, the character of the ground, and the atmospheric conditions.

For ordinary levelling on moderately flat ground, $E = .05\sqrt{M}$ represents good work, and is not difficult to attain under favourable conditions. $E = .10\sqrt{M}$ represents fair accuracy under the same conditions, but is regarded as satisfactory on steep ground, and is quite sufficient for many purposes.

Reciprocal Levelling.—When the difference of level between two points has to be determined under conditions necessitating considerable inequality between the sights, the effects of collimation error, as well as of curvature and refraction, may be eliminated by reciprocal levelling. The routine involves two sets of observations yielding two erroneous differences of level, the mean of which is the true result.

Thus, to ascertain the difference of level between A and B (Fig. 215), on opposite banks of a wide river and remote from a bridge,

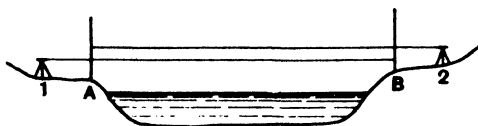


FIG. 215.

readings on a staff held on each point are taken from instrument station 1. The instrument is then transferred to position 2, so that 2B is equal to 1A, and the staff is again observed on A and B. If, due to the combined effect of instrumental error, curvature, and refraction on the long and short sights, the difference of level between A and B, as determined from one of the instrument stations, is made too great, the other determination evidently makes it too small by the same amount. The mean of the two differences of level so obtained is therefore the required difference.

Note.—In levelling of the highest precision it would be recognised that, while the elimination of instrumental errors and the error of curvature is complete, that of the refraction error is not so, owing to the possibility of change of atmospheric conditions during the transfer of the instrument (see Vol. II. Chap. VI).

Expedition in Levelling.—The rate of progress possible depends greatly upon the character of the ground. If a line of levels intersects deep wooded gorges, the necessity for numerous instrument stations on the slopes, combined with difficulty of sighting, makes progress much slower than in open country. Attention to the following items prevents unnecessary delay.

(1) The surveyor should endeavour to select instrument stations from which he will be able to command as much ground as possible, and in particular should avoid setting the instrument too high or too low to read the backsight.

Note.—To test whether the staff can be sighted, plant two of the legs, point the telescope, and bring the bubble roughly to the centre of its run by moving the third leg, retained in the hand. A glance through the telescope will then show if the instrument is at a suitable height. It is better if the surveyor, while looking through the telescope, is warned by a chainman when the bubble is central.

(2) To avoid delays arising from mistakes on the part of an inexperienced staffman, he should be warned of the importance of his share of the work, particularly with regard to change points, and should be instructed not to remove the staff from a point until signalled to do so.

(3) Misunderstandings are largely obviated by using a code of signals, such as :

A quick upward movement of the right hand—Observation completed.

Both hands above the head—Hold on a change point.

Right hand waved up and down—Swing staff.

Right (left) hand up and moved to left (right), and left (right) hand down and moved to right (left)—Plumb staff as indicated.

Right (left) arm extended—Move staff to right (left).

(4) It is sometimes permissible to adopt expedients which are not allowable when the best results are required. Thus, when the staff, held for an intermediate sight, is a little below the line of sight, it can be raised a foot or two off the ground against the divisions of a ranging pole, the reading booked exceeding the staff length. If a high wall has to be crossed, a circuitous route may be avoided by continuing the levels across with the aid of taping. Levelling may be carried across a sheet of water by taking advantage of the fact that the surface of still water is level. The results are quite good if the obstruction is a still pond or lake, but appreciable error may be introduced in the case of a river, unless the points on the water surface are directly opposite and on a straight reach with a symmetrical channel.

PLANE AND GEODETIC SURVEYING

EXAMPLES

1. The following staff readings were observed successively with a level, the instrument having been moved forward after the second, fourth, and eighth readings :

1-67, 8-32, 4-05, 1-39, 9-83, 4-70, 5-45, 1-23, 7-36, 2-87, 5-43.

The first reading was taken with the staff held upon a bench mark of elevation 75-40. Enter the readings in level book form, and reduce the levels.

2. A line of levels has been run from a bench mark of elevation 63-47, and ends on one of elevation 63-50. The sum of the backsights is 49-28, and that of the foresights is 49-29. What is the closing error of the work ?

3. Starting from a bench mark of elevation 123-46, a surveyor runs a longitudinal section in the course of which he takes an intermediate sight of 4-42 with the staff held on a bench mark of elevation 158-60. The sum of the backsights from the start to this point is 120-74, and that of the foresights is 81-13. What is the error of closure on the second bench mark ?

4. It was required to ascertain the elevations of two points, A and B, and a line of levels was run from A to B. The levelling was then continued to an Ordnance bench mark of elevation 127-30, the readings obtained being as shown. Obtain the reduced levels of A and B.

B.S.	I.S.	F.S.	R.L.	Remarks.
3-92				A
1-46		7-78		
7-05		3-27		
	2-36			B
4-81		0-85		
8-63		2-97		
7-02		3-19		
		4-28	127-30	O.B.M.

5. Starting from a point A of elevation 303-46, levels were taken for a section which extended to a point X, the reduced level of which was found to be 322-00. Check levels were carried back along the shortest route from X to A, the readings being given below. Find the error of closure on the starting-point.

B.S.	F.S.	Remarks.
3-68		X
2-43	10-02	
5-90	9-17	
8-16	1-23	
2-39	4-05	
5-97	11-12	
2-36	6-28	
	7-51	A

6. The following staff readings were observed at a road bridge over a railway for the purpose of measuring the clearance between the rails and the girder on the side at which rail level is the higher. The readings marked + were taken with the staff held inverted on the underside of the girder. Reduce the levels, and state the minimum vertical distance between the girder and a rail.

B.S.	I.S.	F.S.	R.L.	Remarks.
1-22			68-40	O.B.M.
0-54		11-45		
	7-92			Rail a
	+ 5-99			Girder above a
	8-07			Rail b
	+ 5-94			Girder above b
	7-94			Rail c
	+ 5-88			Girder above c
	8-06			Rail d
	+ 5-83			Girder above d
12-16		0-32		
		2-15		O.B.M. 68-40

7. Find the error of reading of a level staff if the observed reading is 12.00 ft. and at the point sighted the staff is 6 in. off the vertical through the bottom. If the bubble tube of the level has a sensitiveness of 20 seconds per 2 mm. division, find the error in the staff reading at a distance of 300 ft. caused by the bubble being one division out of centre.

8. The following level book figures show the readings observed on the first few of a series of pegs 100 ft. apart from which the levels of a sewer are to be worked. The reduced level of the invert of the sewer at A will be 90.50, and it will fall with a gradient of 1 in 300 from A. Draw up a list of the depths from the several pegs to invert level, giving the measurements to the nearest $\frac{1}{4}$ in.

B.S.	I.S.	F.S.	R.L.	Remarks.
5.71			89.43	O.B.M.
7.46		2.85		
3.09		1.93		
	4.21			Peg A
	5.63			B
	6.07			C
	4.83			D
	5.78			E
3.51		4.62		F
	5.58			G

9. A gradient of 1 in 120 falling from elevation 202.34 was set out by driving pegs at 100 ft. intervals with the tops of the pegs on the required gradient. After a time it was suspected that some of the pegs had been disturbed, and the following observations were taken in checking their levels. Draw up a list of the errors of the pegs.

B.S.	I.S.	F.S.	R.P.	Remarks.
5.28			192.40	O.B.M.
7.93		2.17		
5.89		4.25		
	2.80			Peg 1
	3.59			2
	4.49			3
	5.27			4
	6.08			5
	6.91			6
2.07		7.78		7
	2.86			8
	3.70			9
	4.55			10
1.84		3.64		
		5.17		
				O.B.M. 192.40

10. A, B, and C are successive points of change of gradient on a portion of an existing railway, the positions and rail levels of these points being:

		Distance in chains.	Rail Level.
A	..	2,032.00	386.34
B	..	2,095.00	439.65
C	..	2,171.00	462.45

In a regrading scheme it is proposed to raise rail level at A by 4.50 ft., and to improve the existing 1 in 78 gradient from A to 1 in 100. Calculate the greatest change of rail level which will be involved and the chainage of the point where the new gradient meets the existing gradient between B and C. (T.C.D., 1928.)

11. The following readings were taken along the centre line of a length of road.

B.S.	I.S.	F.S.	R.L.	Distance in feet.
12.32			252.10	0
	12.80			100
	7.95			200
11.67		0.68		300
	5.80			400
5.06		1.55		500
	2.90			600
	2.15			700
	3.05			800
	3.30			900
12.35		0.42		1,000
9.77		3.15		1,100
	4.20			1,200
	1.05			1,300
8.49		1.40		1,400
		3.45		1,500

It is proposed to improve this portion of road by regrading it to a 1 in 30 gradient passing through the existing surface at 1,100 ft. on the section. Calculate the greatest difference of level between the new and existing surfaces which will occur in excavation and in embankment.

12. Two pegs, A and B, are driven into the ground about 300 ft. apart. A dumpy level is set up near A and levelled, and observations are taken upon a staff held on A and then on B, giving the readings :

on A, 5.48 ; on B, 5.53.

The level is then placed near B, and sights of the same lengths as before are taken with the staff held on the same points. The readings are now

on A, 5.31 ; on B, 5.26.

State whether the instrument is in adjustment or not. If the reduced level of peg A is 100.00, what is that of B ?

CHAPTER VII

PLANE TABLE SURVEYING

PLANE tabling is a method of surveying the peculiar feature of which is that the field observations and plotting proceed simultaneously. It is admirably adapted for the survey of detail between theodolite stations, and is extensively employed for recording topography in government and engineering surveys.

The plane table and the various ways in which it is used in surveying are dealt with in this chapter. Special features of plane table surveying when applied to the small scale mapping of extensive areas are described in Vol. II, Chap. VII.

THE PLANE TABLE

The plane table (Fig. 216) consists of a drawing board, which carries the sheet, and is mounted on a tripod in such a way that the board can be (a) levelled, (b) rotated about a vertical axis and

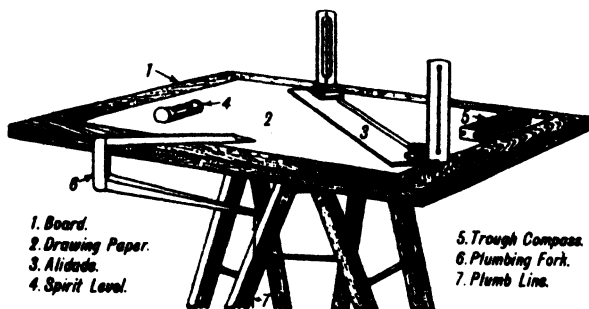


FIG. 216.—PLANE TABLE.

clamped in any position. A sight rule, or alidade, which is simply a straight-edge carrying a line of sight, is used for sighting objects to be located and for recording on the paper the directions in which they lie. The parts may be considered under four heads :

- (1) The Board.
- (2) The Tripod and Horizontal Movement.
- (3) The Alidade.
- (4) Accessories.

The Board.—The board, ranging in size from about 15 in. \times 15 in. to 30 in. \times 24 in., is made of thoroughly seasoned wood, and is constructed in such a manner as to counteract the effects of warping as much as possible. The upper surface must be plane, and parallel to the surface on which the horizontal movement is made.

The Tripod and Movement.—The tripod is generally of the open frame type, combining rigidity with lightness, and may be made to fold for convenience of transportation.

In the simplest and lightest forms of plane table, levelling of the board is effected simply by manipulation of the tripod legs. Otherwise a levelling screw head or ball and socket joint is fitted. Heads differ in detail as regards the manner of permitting horizontal movement of the board about a vertical axis of rotation, but the fitting must afford plane motion without play, and should be as light as is consistent with rigidity. A simple form with levelling screws is illustrated in Figs. 217 and 218. The board is rotated

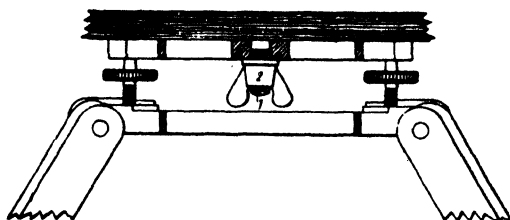


FIG. 217.—SIMPLE MOVEMENT—SECTION.

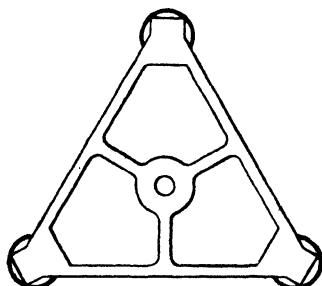


FIG. 218.—SIMPLE MOVEMENT—PLAN.

about the axis 1, and can be clamped in any desired position by the nut 2. The underside of the board either bears directly upon the triangular frame or is fitted with a flat ring of brass to give a smoother motion.

Fig. 219 shows the Gurley ball and socket movement. The ball 1, supporting the board, fits into the socket 2, which is screwed to the head of the tripod. The two spherical surfaces are held together by the spiral spring 3 acting on the clamp piece 4. When the board

is levelled, it is clamped by the nut 5, but can be rotated horizontally on the flange of 1, and, when oriented, is secured in position by the screw 6.

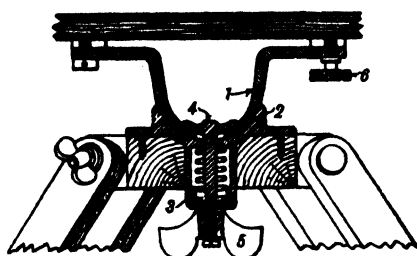


FIG. 219.—GURLEY MOVEMENT.

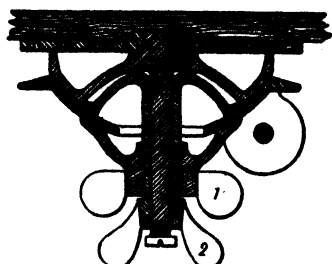


FIG. 220.—JOHNSON MOVEMENT.

The Johnson movement is illustrated in Fig. 220. Levelling is performed as in the last case, and the table is clamped in the horizontal position by the nut 1. On releasing the nut 2, the board can be rotated about the now vertical axis, and, when oriented, is fixed by tightening the same nut.

Fig. 221 shows the United States Coast and Geodetic Survey movement. Levelling is performed by means of three foot screws, and stability is promoted by supporting the table on a wide V-shaped annular surface. A clamp and tangent screw are fitted.

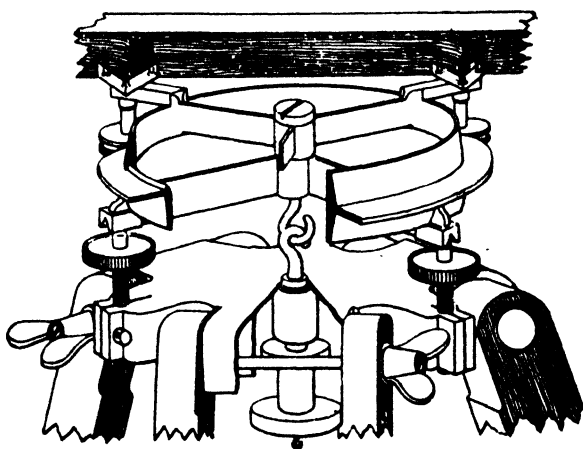


FIG. 221.—U.S. COAST AND GEODETIC SURVEY MOVEMENT.

The Alidade.—In its simplest form, the alidade consists of a wooden or brass ruler of length equal to the smaller dimension of the board and furnished with a pair of sights (Fig. 216). One of the vanes is provided with a narrow slit, while the other is open

and carries a hair or wire. It is essential that the plane of the sights should be perpendicular to the under surface of the ruler, but it is unnecessary that the line of sight should be in the same vertical plane as, or even parallel to, the fiducial or ruling edge of the alidade, provided the horizontal angle between the two lines remains constant. The sights are commonly placed in the vertical plane of the fiducial edge, but they are sometimes mounted midway across the breadth of the ruler, and either edge can be used in drawing. The alidade plate is sometimes constructed as a parallel ruler with the straight-edge used in drawing connected by two links to the base plate. The ruler may be moved out from and parallel to the latter, and, in consequence, rays can be drawn through a plotted point without the alidade being set exactly against it.

The alidade with plain vanes provides a sufficiently definite line of sight for many purposes, but, as ordinarily constructed, it is

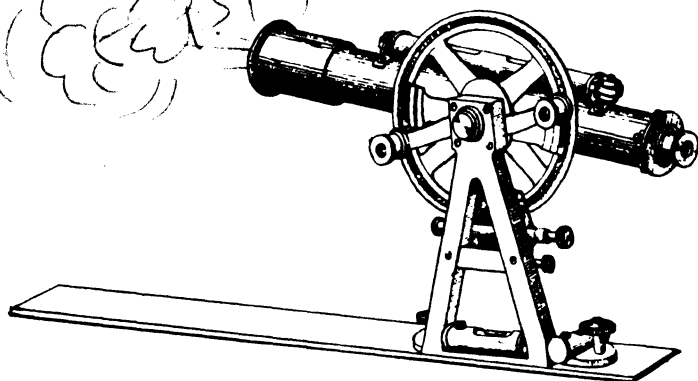


FIG. 222.—SIMPLE TELESCOPIC ALIDADE.

somewhat inconvenient for work in hilly country owing to the limited range of inclination possible in the line of sight. By stretching a thread tightly between the centres of the tops of the vanes, highly inclined sights can be taken.

The accuracy and range of sighting are considerably increased by the use of the telescopic alidade, and, if the telescope is adapted for tacheometry by the provision of stadia hairs and a vertical circle or arc, the scope and utility of the plane table are greatly enhanced. The telescope is usually equal to that of a small theodolite, and is mounted on a short horizontal axis on which it can be rotated, in some forms completely, and in others only partially, but no lateral motion relatively to the ruler is possible. The details of construction of telescopic alidades vary considerably. The horizontal axis may be overhung from one support (Fig. 222), so that the telescope can be transited, or the axis may be mounted on two

short standards at the top of a solid column, which serves as a convenient handle.

The requirements of the telescopic alidade with respect to the levels, the line of sight, and the horizontal axis are similar to those of the theodolite.

For a description of some modern telescopic alidades see Appendix I, pages 611-613.

Accessories.—The remaining features of the instrument are (1) a spirit level (if not fitted to the alidade); (2) a trough compass; (3) a plumbing fork; (4) a waterproof cover.

The spirit level need not be very sensitive. The table is levelled by placing it on the board in two positions at right angles and getting the bubble central in both positions. A circular level is quite suitable.

The trough compass is commonly about 6 in. long, and is used when orienting the table to magnetic meridian and to facilitate orienting to any other meridian.

The plumbing fork (Fig. 216) is used in large scale work for setting the table so that the point on the paper, representing the instrument station being occupied, may be brought vertically over the mark on the ground. When the table is level, the point of the leg resting on the board is vertically over the centre of the plumb bob. The point of the leg is therefore placed at the plotted station point, and the position of the table is adjusted until the plumb bob hangs over the ground mark. Since the plane table is seldom employed for large scale work, the use of a plumbing fork is unnecessary in the great majority of plane table surveys.

Paper.—For other than rough work, the quality of the paper should receive attention. Paper is very sensitive to changes in the humidity of the atmosphere, which produce expansion and contraction of different amounts in different directions, and not only alter the scale, but distort the map. It has been found that if two sheets are mounted with their grains at right angles and with a sheet of muslin between, the effects of a change of atmospheric conditions are practically negligible. Such double-mounted sheets are sold by some dealers. If a single-mounted or an unmounted sheet is to be used, it may previously be seasoned for about a week by exposing it alternately to a damp and a dry atmosphere, such treatment reducing its tendency to distort. For work in damp climates, sheets of celluloid and of zinc have been used.

Attachment of Paper to Board.—The paper must be held very firmly on the board to prevent the possibility of any displacement by the friction of the alidade. Ordinary drawing pins may work loose, and the projecting heads interfere with the placing of the alidade. The principal devices used are :

(1) Pasting down the edges of the paper, or having the linen sheet, on which the paper is mounted, larger than the board, and pasting the margin to the underside of the board.

(2) A wooden frame fitting tightly round the board, the upper surface being flush with that of the board when the paper is clamped (Fig. 216).

(3) Two rollers carrying the paper, one on either side of, and below, the board.

(4) Screw clamps round the sides of the table.

(5) Screw tacks entering hollow brass screws in the board. When the tacks are screwed in, and hold the paper, their heads are flush with, or a little below, its surface.

(6) Spring clips round the sides of the board.

Of these methods, the first is probably the best.



TESTING AND ADJUSTMENT OF THE PLANE TABLE

The Board.—(1) *The upper surface of the board should be a perfect plane.*

Test.—Apply a straight-edge in all directions.

Adjustment.—If necessary, reduce high parts by planing or sand-papering.

(2) *The upper surface of the board should be perpendicular to the vertical axis of the instrument.*

Test.—Place a spirit level on the table, and bring the bubble to the centre of its run. Turn the table through 180° . Place the level at 90° to its previous position, and repeat. If the bubble remains central on reversal, the adjustment is correct.

Adjustment.—If not, correct half the apparent error by rubbing down or packing between the underside of the table and its support.

The Alidade.—(1) *The fiducial edge of the ruler should be a straight line.*

Test.—Draw a fine line along the edge. Reverse the ruler, place it against the ends of the line, and again draw a line. If the two lines coincide, the ruler is true, except in the very improbable case when one part is convex and the part equidistant from the other end has a concavity of exactly the same size and shape.

Adjustment.—If the two lines do not coincide, bring the edge true by repeated rubbing and testing.

(2) *The alidade spirit levels should have their axes parallel to the base of the ruler.*

Test.—Place the alidade on the table, and bring the bubble of either level central by the levelling screws of the table. Mark the position of the ruler, lift and reverse it through 180° , and replace it within the marks. If the bubble remains central, the adjustment is correct.

Adjustment.—If not, eliminate half the error by the level adjusting screws and the remainder by the levelling screws, and repeat until the test is satisfied. Examine and adjust the second level tube in the same way.

If the spirit level is not attached to the alidade, the test by reversal is performed in the above manner, but there may be no provision for adjustment.

(3) *The sights of the plain alidade should be perpendicular to the base of the ruler.*

Test.—Having levelled the table, observe whether the sighting slit and hair appear parallel to a plumb line suspended in front of the instrument. Alternatively, test them with a set-square held on the board.

Adjustment.—If wrong, adjust by filing or packing the base of the sights.

The Telescope.—(1) *The line of collimation should be perpendicular to the horizontal axis of the telescope.*

Proceed as in adjustment 2 of the transit or wye theodolite, according to the manner in which the telescope is mounted (pages 93 and 100).

(2) *The horizontal axis should be parallel to the base of the ruler.*

Proceed as for adjustment 3 of the transit theodolite (page 96), if there is provision for making the adjustment.

(3) *The axis of the telescope level should be parallel to the line of sight.*

Proceed as in the “two peg” adjustment of the dumpy level (page 124).

The Index Frame.—*The vertical circle should read zero when the line of sight is horizontal.*

Proceed as for adjustment 5 of the transit theodolite (page 97).

FIELD WORK

Advantages and Disadvantages of Plane Tabling.—*Advantages.*—

(1) Since the map is plotted in the field, there is no danger of omitting necessary measurements.

(2) The correctness of the plotted work can be readily verified by check observations.

(3) Notes of measurements are seldom required, and the possibility of mistakes in booking is eliminated.

(4) By virtue of the foregoing, no great skill is required to produce a satisfactory map, and the work may be entrusted to a subordinate.

(5) The method is particularly useful and rapid when detail can be sketched in by estimation, as the surveyor has the ground before him, and need take only such observations as are strictly necessary, while the double work of booking sketches and reproducing them on the plan is avoided.

(6) The subsequent office work is confined to finishing up the drawing, so that, in ordinary circumstances, plane tabling proves one of the most rapid methods of surveying.

Disadvantages.—(1) The advantages of the plane table are most evident in open country. It is inferior to the compass in densely wooded ground.

(2) It is unsuitable for work in a wet climate, and is awkward in high wind.

(3) The plotting is trying to the eyes in a bright sun, unless the paper is shaded by an umbrella, or tinted paper or tinted eye-glasses are used.

(4) The instrument is heavy and cumbersome, and the various accessories, being loose, may get lost.

(5) The absence of field notes is sometimes inconvenient if quantities have to be calculated, or if the survey has to be replotted to a different scale.

Setting Up the Table.—In setting up the table at a station, three requirements have to be met :

(1) The table must be levelled.

(2) The table must be oriented.

(3) The point on the paper representing the station being occupied should be vertically over the point on the ground.

Levelling.—The legs should first be spread to bring the table approximately level and the board at a convenient height for working—preferably not above the elbow. The table is then placed over the station to fulfil requirements 2 and 3 approximately, and the levelling is completed by means of the levelling screws, by tilting the board by hand if the instrument has a ball and socket head, or simply by adjusting the legs if there is no levelling head.

Orientation.—The table is said to be oriented when it is so placed, with respect to its vertical axis, that all lines on the paper are parallel to the corresponding lines on the ground. This is obviously an essential condition when more than one instrument station has to be occupied, as otherwise the board would not be kept parallel to itself at the various stations, and the result would be equivalent to using a different meridian at each.

The manner of orienting is analogous to that of theodolite surveying in making use of a backsight. Thus, if the table be set over a station B, represented on the paper by a point *b* which has been plotted by means of a line *ab* drawn from a previous station A, the orientation will consist in bringing *ba* on the paper over BA on the ground. The edge of the alidade is therefore placed along *ba*, and the board is turned in azimuth until the line of sight bisects the signal A, when the horizontal movement is clamped.

Orientation may also be effected (independently of a backsight) by the employment of the trough compass. At the first station, after the table has been levelled and clamped, the compass box is

placed on the board—preferably outside the limits of plotting—so that the needle floats centrally, and a fine pencil line is ruled against the long side of the box. At any subsequent station the compass is placed against this line, and the table is oriented by turning it until the needle again floats centrally. The accuracy of compass orientation is dependent upon the absence of local attraction, but is suitable for work in which speed is of greater importance than accuracy. The compass, however, often proves a valuable adjunct in enabling a rapid approximate orientation to be made prior to the final adjustment.

Further methods of orienting the table will be considered later (page 344).

Centering.—It has been assumed that b is set vertically over B by use of the plumbing fork, so that ba is brought into the same vertical plane with BA . If b happened to lie in the vertical axis of the instrument, its position would be unaffected by the movement of the board in orienting, but otherwise b will be shifted relatively to the mark on the ground. The operations of orienting and centering are therefore interrelated, and, if circumstances require that the plotted station point shall be exactly over the ground point, repeated orienting and shifting of the whole table are necessary. Commonly, however, accurate centering is a needless refinement (see page 355).

Systems of Plane Tabling.—Methods of surveying with the plane table may be classed under four distinct heads, viz. Radiation, Traversing, Intersection, and Resection. In the figures illustrating these, points on the ground are indicated by capital letters, and the corresponding points on the sheet by the corresponding small letters, the size of the board being shown greatly exaggerated.

Radiation.—Select an instrument station, O , from which all points to be surveyed are visible (Fig. 223). Set up and level the

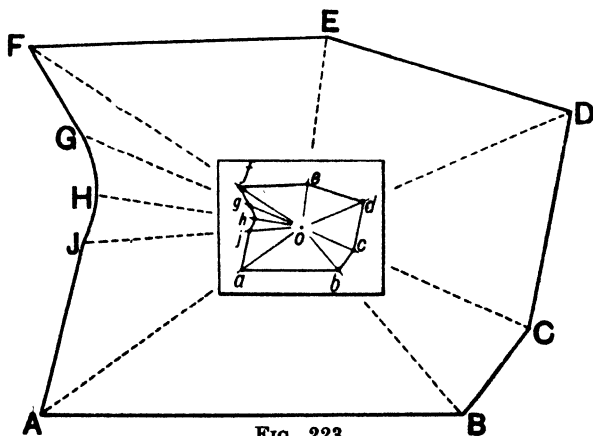


FIG. 223.

table, and clamp the horizontal movement. Select a point o on the sheet to represent the instrument station, and, with the alidade touching o , sight the various points A, B , etc., to be located, drawing radial lines towards them. Measure distances OA, OB , etc., set them off to scale, and join the points a, b , etc., so obtained.

Note.—This method is of very limited application for making a complete survey, but in large scale work is useful, in combination with other methods for surveying detail within a tape length from a station. The method has a wider scope if the distances are obtained tacheometrically. The work can be satisfactorily checked only by comparing distances such as AB with the corresponding plotted lengths ab , etc.

Traversing.—This method is used for laying down the survey lines of a closed or unclosed traverse, and corresponds to theodolite traversing. The detail may be located by offsets in the usual manner.

Having selected a system of stations A, B, C, D, E (Fig. 224), set up over one of them, say A , and, having selected a on the paper,

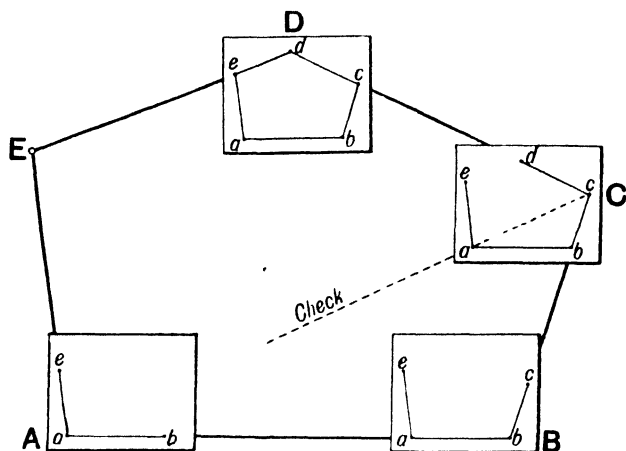


FIG. 224.

bring it over A . Clamp the board, and, with the alidade touching a , sight E and B , and draw rays ae and ab . Measure AE and AB , and scale off ae and ab . Set up at B , with b over B , and orient by laying the alidade along ba , turning the table until the line of sight strikes A , and then clamping. With the ruler against b , sight C , and draw bc to scale. Proceed in this manner at other stations, in each case orienting by a backsight before taking the forward sight.

Notes.—(1) The error of closure at e is determined at D in the case described, and E need not be occupied. As in theodolite surveying, intermediate checks should be taken whenever possible. Thus, if A is visible from C , the work up to C may be checked there by sighting A with the ruler against c and noting if the edge touches a .

(2) If E is not sighted from A on starting, either on account of its not being

marked, or because the traverse is unclosed, A need not be occupied by the table, *ab* simply being drawn in a convenient position on the sheet. If this is done for the traverse illustrated, the closing error is found on setting up and orienting at E.

(3) Plane table traversing becomes analogous to compass traversing if each orientation is made entirely by compass. This is less accurate than back sight orientation, especially if there is a possibility of local attraction. Since compass errors tend to compensate, however, the method is useful for a lengthy traverse of short courses, and proves rapid when the table is set up at alternate stations only. A small and light form of table with a trough compass recessed into the board is sometimes used for such work, and is known as a traverse plane table.

Intersection or Triangulation.—This method is largely used for mapping detail, but is also available for plotting the positions of points to be used as subsequent instrument stations. The only linear measurement required is that of a base line.

Lay out and measure a base line AB (Fig. 225). Plot *ab* in a convenient position on the sheet. Set up at A with *a* over A, and

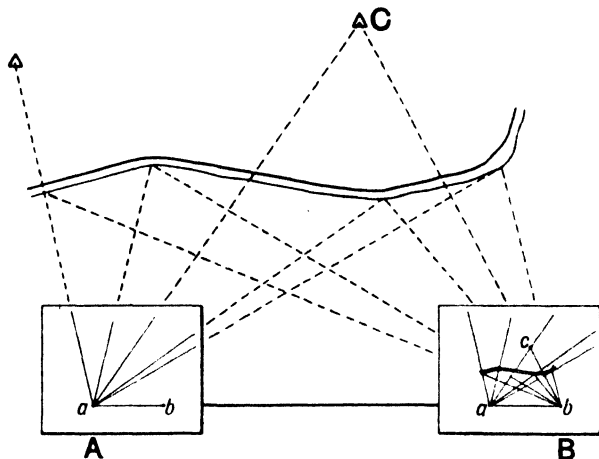


FIG. 225.

orient by laying the alidade along *ab* and turning the table until the line of sight cuts B. Clamp, and, with the ruler touching *a*, sight the various points defining the surrounding detail and points selected as future instrument stations, drawing a ray from *a* towards each. Proceed to B, set up with *b* over B, and orient by backsighting on A. Through *b* draw rays towards the points previously sighted. Each point is located by the intersection of the two rays drawn towards it. Before leaving B, draw a series of first rays towards other points not sighted from A, and then proceed to C, orient on A or B, and obtain a new series of intersections.

Notes.—(1) An extended survey should, whenever possible, be based upon a system of points whose relative positions have been obtained by theodolite triangulation or traversing, and plotted on the sheet. No base line is then

required, and not only can such stations be occupied by the table, but, on setting up at other sites, the orientation can be verified by reference to them, and accumulation of error is avoided. An intersection survey uncontrolled by theodolite points is termed a graphic triangulation, and, if extensive, demands constant precaution against the propagation of error.

(2) In any case, greater refinement is called for in plotting points to be used as plane table stations than in fixing detail. No station can be considered as satisfactorily located unless it has been intersected from three others. Triangles should be well-conditioned in order to yield definite intersections: it is desirable, particularly in graphic triangulation, that intersection angles should not be less than 45° .

(3) Since the length of the base line influences only the scale of plotting, it is evident that a graphic triangulation can proceed without the preliminary measurement of a base. The scale can then be determined when the survey includes two points whose distance apart has been determined astronomically, trigonometrically, or by direct measurement. Such procedure is useful in exploratory and reconnaissance surveys.

Resection.—This method can be used for the location of station points only. As in the preceding system, one linear measurement is required. The simplest case is as follows.

Measure a base line AB (Fig. 226), and plot ab in a convenient position. Set up at A with a over A, orient on B, and through a

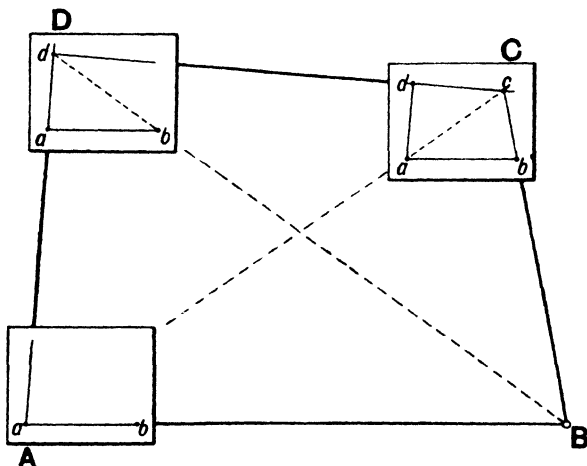


FIG. 226.

draw a ray of definite length towards D. Set up at D with the estimated position of d over D, and orient by backsighting on A. Place the alidade against b , and sight B. The point in which the edge of the ruler cuts the line previously drawn from a towards D is the required point d . From d draw a ray towards C. Set up at C with the estimated position of c over C, and orient by backsighting on D. Place the alidade against b and sight B, obtaining the intersection c . Check c by sighting A with the ruler touching a .

Notes.—(1) Errors of centering are inevitable, but, since resection is usually confined to small scale work, the accuracy of the plotting is not appreciably affected thereby.

(2) The characteristic feature of resection is that the observations at any station after the first are directed to plotting the point occupied by the instrument, the surrounding detail then being located by radiation or intersection. The back ray method of orienting, as described, necessitates a ray being drawn from a preceding station to that being occupied, and therefore involves the previous selection of the instrument station. This is only a particular case of resection. Solutions for cases where no ray has been drawn to the instrument station are given below.

Of the various cases where resection, or the plotting of the station occupied, can be performed, that in which the data consist of two visible stations, their plotted positions, and a ray from one to the station occupied has just been described. In the more usual case in which no such ray has been drawn, the data must consist of either :

(a) Three visible points and their plotted positions. (The Three-point Problem.)

(b) Two visible points and their plotted positions. (The Two-point Problem.)

Of these, the former is the more important and useful case.

The Three-point Problem.—The problem may be stated : Given three points A, B, and C, visible from a station P over which the table is set, and a , b , and c , their plotted positions. Required to plot p .

If the table were oriented at P, so that ab and bc are respectively parallel to AB and BC, it would only be necessary to draw rays through a and b with the alidade directed towards A and B respectively. These would intersect at p , the position of which could be checked by seeing whether a ray through c towards C also passes through p . The problem therefore resolves itself into that of orienting the table at P, so that Aa , Bb , and Cc will pass through one point p .

Strength of Fix.—The relative position of A, B, C, and P has an important influence on the accuracy with which the position of p can be determined.

If P lies anywhere on the circumference of the circumscribing circle through A, B, and C, its position is indeterminate, since p may lie anywhere on one of the segments of the circumference of the circle through a , b , and c , and still subtend the correct angles APB and BPC with ab and bc respectively. No matter how faulty the orientation is in such a case, the three rays, Aa , Bb , and Cc , will always meet in a single point. For other situations of P they will not pass through one point unless the table is oriented, but will either form a triangle, called the triangle of error, or two of them may be parallel and intersected by the third. In such cases the accuracy with which P is fixed varies with different positions of P relatively to A, B, and C.

The fix is good when :

- (a) The middle station B is much nearer than the others.
- (b) P is within the triangle ABC.
- (c) One angle is small (becoming zero when two points are in range) and the other is large, provided the points subtending the small angle are not too near each other.

The fix is bad when :

- (a) Both angles subtended at P are small.
- (b) P is near the circumference of the circumscribing circle.

Trial and Error Solution.—It is evident that, unless p is indeterminate, its position can be obtained by repeated trial of the orientation until the table is so placed that the rays Aa , Bb , and Cc all pass through one point. Unnecessary expenditure of time is avoided by making a preliminary orientation by compass or by a range when the line of two well-defined objects already plotted

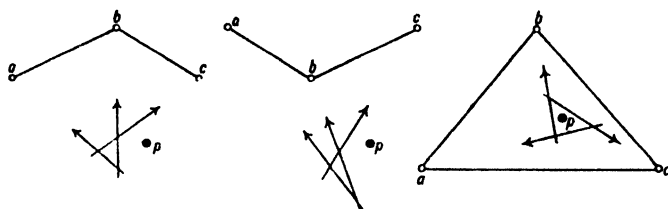


FIG. 227.—EXAMPLES OF TRIANGLE OF ERROR.

passes through or near P . If this is done, the triangle of error will be small, and will be eliminated after one or two trials. The adjustment in orientation is, however, facilitated by the application of Lehmann's rules for estimating the position of p from the triangle of error (Fig. 227).

Lehmann's Rules.—(1) The point p is always distant from each of the three rays Aa , Bb , and Cc in proportion to the distance of A , B , and C from P , and, when looking in the direction of each of the distant points, it will be found on the same side, right or left, of each of the rays.

Note.—It follows from this rule that p lies within the triangle of error only when P lies within the triangle ABC .

Although rule (1) suffices for the solution of the problem, two subsidiary rules are of assistance.

(2) When P is outside the circumscribing circle ARC , p is always on the same side of the ray to the most distant point as the intersection of the other two rays.

(3) When P falls within either of the three segments of the circumscribing circle ABC , formed by the sides of the triangle

ABC, the ray towards the middle point lies between p and the intersection of the other two rays.

Having estimated the position of p by applying the rules, the table is oriented by laying the alidade touching p and one of the points a , b , or c , and turning the table to sight the corresponding object. If the three rays now intersect in one point, the estimate has been correct. If not, a new position of p must be assumed with reference to the new triangle of error, and the orientation is repeated.

Mechanical Solution.—(1) Clamp the table, fasten a sheet of tracing paper on the board, and on it mark a point p' approximately over P .

(2) With the alidade touching p' , sight A, B, and C successively, and draw a ray of indefinite length towards each. These rays will not pass through a , b , and c , except in the unlikely event of the table being correctly oriented with p' coinciding with p .

(3) Unfasten the tracing paper, and move it over the drawing until the three rays simultaneously pass through a , b , and c , and then prick through the position of p' . The point so obtained is the required point p .

(4) Remove the tracing paper, unclamp, and, with the ruler along pa , orient the table by turning it until the line of sight strikes A. Bb and Cc should now also pass through p . If, owing to careless manipulation or unequal stretching of the tracing paper, a small triangle of error is formed, it can be eliminated by the trial and error method.

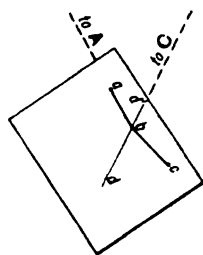


FIG. 228.

Graphical Solution.—Of the various graphical methods which have been proposed, Bessel's solution, by the inscribed quadrilateral is the simplest, and it only will be described. The table is set up and levelled at P , and the process of orienting it may be divided into three steps.

(1) (Fig. 228) With the alidade along ba , turn the table until A is sighted, a being towards A. Clamp, and through b draw a ray bd' towards C.

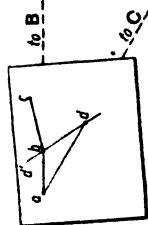


FIG. 229.

(2) (Fig. 229) Unclamp, and, with the alidade along ab , sight B, b being towards B. Clamp, and through a draw a ray ad towards C, meeting the previously drawn ray at d .

(3) (Fig. 230) Unclamp, and, with the alidade along dc , turn the table to sight C, and clamp. The table is now oriented, and p must lie on dc , produced if necessary. But p must also lie on Aa and Bb . Therefore, with the edge of the alidade on a , sight A, and draw ap meeting

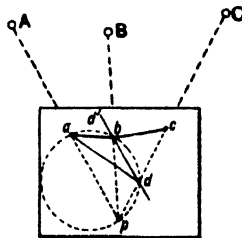


FIG. 230.

cd in p ; and check the orientation by testing whether Bb also passes through p .

Proof.—Assuming that the distance of P from A , B , and C is sufficient to render the effect of centering error negligible, then—

In Fig. 228, $abd' = APC$.

In Fig. 229, $bad = BPC$,

and $bda = abd' - bad = APB$.

In Fig. 230, $bpc = BPC$,

$\therefore bpc = bad$, and consequently $abdp$ can be circumscribed by a circle.

$\therefore bda = apb$, since both are subtended by chord ab , so that $apb = APB$.

$\therefore p$ simultaneously subtends with a , b , and c the required angles APB and BPC .

Notes.—(1) If d comes off the paper, ab and bc may, for the purpose of orienting, be decreased proportionally by drawing through them a parallel to ac . If d comes very near c , ab and bc should be increased. The point d obtained is used with the new c in effecting the final orientation.

(2) In the first two steps, instead of sighting through a and b and drawing rays towards C , any two of the points may be used and the rays drawn towards the signal corresponding to the third, which is then sighted in the final step.

The Two-point Problem.—The problem may be stated: Given two points A and B , visible from a station P over which the table is set, and a and b , their plotted positions. Required to plot p .

This problem appears indeterminate unless the orientation is effected by compass, and a solution cannot be obtained, without the use of the compass, by occupying P alone. Two instrument stations are required. The most convenient solution is as follows (Fig. 231).

(1) Select a fourth point C , such that angles PAC and PBC are not too small for good intersections at A and B . Set the table at C , and orient approximately by estimation or by compass. Sight A and B , and draw rays through a and b respectively to intersect at c' . C is only approximately represented by c' , since the orientation was only approximate. Through c' draw a ray towards P .

(2) Set up at P , and assume a point p' on the last drawn ray to represent P by estimating the distance PC . Bring the table to the same orientation as at C by placing the alidade along $p'c'$ and sighting C . Now, with the alidade against p' , sight A and B , and draw rays intersecting $c'a$ and $c'b$ in a' and b' respectively.

(3) These points have been obtained by ordinary intersection from C and P , so that $a'b'$ is parallel to AB . But $a'b'$ does not coincide with ab because: (1) the orientations at C and P , although consistent, are only approximate: (2) $p'c'$ represents PC by estimation only. In other words, $a'b'c'p'$ is similar to $ABCP$, but is of erroneous size, and is wrongly placed on the sheet. The error of orientation is the angle between ab and $a'b'$.

(4) To orient the table, it must be turned through this angle until ab becomes parallel to the present direction of $a'b'$. The best method is to place the alidade along $a'b'$, and erect a distant pole in that line. Transfer the alidade to ab , and turn the table until the pole is again sighted, clamp, and the orientation is made.

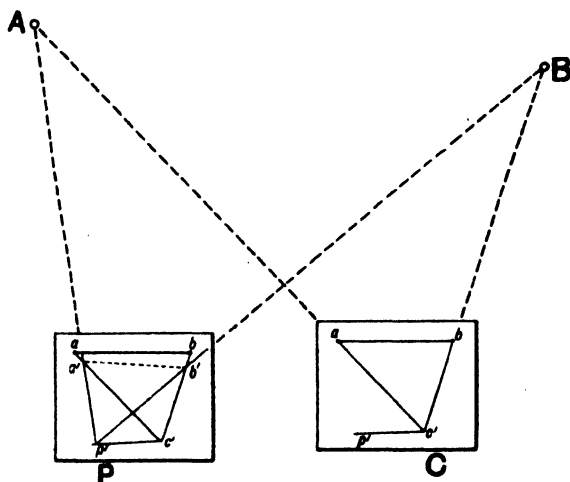


FIG. 231.

(5) P is now plotted by sighting through Aa and Bb and obtaining the intersection p .

Note.—If C can be selected in line with AB, the problem is simplified, since the table can be oriented at C by bringing ab in line with AB. Having done so, draw a ray towards P, and, on setting up at P, orient by means of this ray and a signal left at C.

Application of Resection.—The practical utility of the three-point and two-point problems, particularly the former, is greater than may appear at first sight. These methods of resection have the great merit that the topographer is saved the necessity of selecting forward stations in advance. He has entire liberty to set up solely at points favourable to the taking of detail, and is dependent only upon the visibility of suitable trigonometrical stations and not upon his previous or succeeding plane table fixings. The advantages of resection are most apparent, and the method is greatly used, in the small scale mapping of open country, when detail is taken entirely by sketching with reference to the plane table stations.

Field Party.—For small surveys, the surveyor does not require more than two men to perform chaining, mark stations, etc. In

large surveys, the work may be expedited by the surveyor having a qualified assistant, who can proceed with the plotting while the chief reconnoitres the forward ground. The number of men required depends on the nature of the ground as well as the method of surveying. In tacheometrical radiation, it will usually be possible to keep two staffmen fully occupied, and more if the points sighted are widely spread, as in small scale work in open country. It may be necessary to have a man to carry the table between stations, hold the umbrella, etc.

Equipment.—The amount of apparatus to be carried depends upon the nature and magnitude of the survey. A full equipment consists of :

Plane table, tripod, and alidade.

Spirit level, trough compass, plumb-bob, fork, cover, and field glass.

Scales, one or two set-squares, pencils, rubber, sand-paper, ink, colours, drawing pen, and small note-book.

Portfolio or cylindrical case for sheets.

Poles and flags.

For tacheometry : Staff and stadia reduction tables or diagram.

For traversing, etc. : Chain or band and tape.

For trigonometrical levelling : Curvature and refraction table.

For barometric levelling : Aneroid.

Vertical Control.—The widest application of plane table surveying has been in the preparation of contour maps, for which the instrument is admirably adapted, principally on account of the great advantage that the topographer has the terrain in view while representing its form.

With Telescopic Alidade.—If a telescopic alidade with vertical arc is available, the necessary elevations may be obtained either by tacheometric or trigonometric levelling. Tacheometry (Chap. XI) is well adapted to close contouring when the sights do not exceed 1,000 ft. Trigonometrical levelling is most suitable for small scale work involving long sights. The methods are discussed in Vol. II, Chap. VI. The vertical angle to a located point, of which the elevation is required, is observed, and from the known distance the difference of level is computed. In determining the elevations of points to be used as future instrument stations, observations are necessary from at least two stations whose elevations have been found with corresponding care, and account must be taken of the effects of curvature and refraction. In the resection method, the elevation of the instrument is determined from vertical angles to two or more plotted points of known elevation.

With Plain Alidade.—Since a plain alidade can be used for horizontal control only, elevations must be determined by ordinary level, hand level, or one of the various forms of clinometer. Direct

levelling is very much slower than clinometric levelling on account of the large number of points of observation required, and its use is justified only in the close contouring of small areas.

The Indian clinometer (Fig. 235) is specially adapted to plane tabling, and is extensively used. The instrument is placed upon the board, and, when the table has been set level by estimation only, the clinometer is levelled by the levelling screw and spirit level shown. The folding sight vanes are 8 in. apart. The front one is graduated in degrees and natural tangents, and the eye-hole of the rear vane is horizontally opposite the zero of the scales when the instrument is levelled. The level tube is provided with adjusting screws, the testing and adjustment of the instrument being performed by means of reciprocal observations, as in the case of the Abney clinometer (page 131).

In the latest Survey of India model the front vane also carries a small eye-hole alongside the zero mark of the scale, and both vanes are held rigidly vertical by pivoted collapsible struts. The front eye-hole is for easy adjustment of the level tube; for the mean of two readings to a levelling staff read through the two eye-holes, first with the front vane pointing to the staff and then in the reversed position, gives the direction of a horizontal line of sight, and the bubble can then be adjusted to the centre of its run by the level adjusting screws when the line of sight is directed to this reading.

The elevation of the station occupied can be determined by observation to a point of known level already plotted. The surveyor places the clinometer in the direction of the distant point, levels it, and, with the eye a few inches from the sighting hole, observes the graduation on the tangent scale opposite the point sighted. The difference of level is the tangent times the distance as scaled from the map, and to this result will be applied the height of the line of sight from the ground under the table. It is preferable to make the determination from two or more points and adopt the mean result. From the now known elevation of the station occupied, those of surrounding points can be obtained in the same manner.

Field Methods.—In the execution of a plane table survey, the general system and the details of routine to be adopted both for

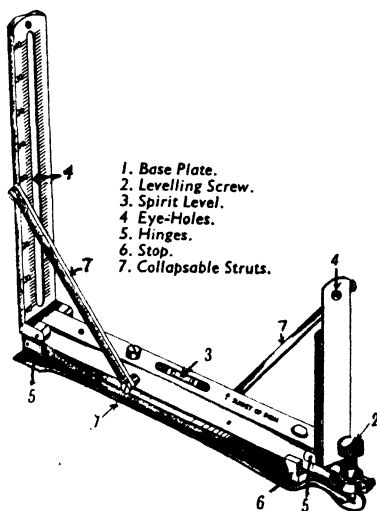


FIG. 232.—INDIAN CLINOMETER.

(Reproduced by the kind permission of the Surveyor General of India.)

horizontal and vertical control depend on several factors, such as : (a) Scale of plotting ; (b) Degree of accuracy required ; (c) Extent of survey ; (d) Character of ground ; (e) Time at disposal.

The following outlines of field methods will serve to suggest the routine suitable for any particular case.

Scales greater than 1/2,500.—*Horizontal Control.*—Traversing, Radiation, and Intersection.

Plane table stations in large scale work should be located by traversing. If the survey is extensive, and a high degree of accuracy is required, a controlling theodolite traverse or triangulation may be undertaken so that a number of the plane table stations may be plotted beforehand. The detail is fixed by offsetting and by radiation, the distances being taped. Points so situated with respect to the stations as to yield good intersections may be located by intersection. As the importance of the detail and the scale decrease, tacheometric radiation and pacing may be employed, and the same attention need not be paid to the quality of intersection fixes.

Vertical Control.—Direct Levelling, Tacheometry, Indian Clinometer.

If the contour interval is as small as 2 ft., and the contours have to be located with sufficient accuracy for earthwork measurements, the best results are obtained by locating points on the contours by ordinary levelling and surveying them by plane table. The method of using the telescopic alidade for direct levelling is slow and inconvenient owing to the necessity for frequent shifting of the table. In ordinary contouring for location surveys, the tacheometric method is most suitable.

Scales of 1/2,500 to 1/10,000.—*Horizontal Control.*—Intersection, Resection, and Traversing.

The survey may either be made by plane table throughout, or may be based upon a system of points established by theodolite triangulation or traversing. The theodolite stations and their plotted positions are used in four ways : (a) Some or all of them are occupied by the plane table ; (b) They serve to verify the orientation of the table at any station, and are used in locating stations by resection ; (c) The distances between them serve as so many base lines, as required in the intersection system ; (d) From their known altitudes the elevations of all other stations are determined.

The detail in the larger scales is commonly located by tacheometric radiation. As the scale decreases, the amount of sketching allowable increases, and the controlling points are most expeditiously fixed by intersection.

Vertical Control.—Tacheometry, Indian Clinometer, Alidade Vertical Angles.

Tacheometric levelling may still be employed where good contour

delineation is required, particularly in flat country. When the distances become too great for tacheometric reading, elevations of intersected points are obtained by clinometer or by measurement of vertical angles by the alidade. In all cases, points surveyed should be characteristic points on ridge and valley lines.

Scales of 1/10,000 to 1/50,000.—*Horizontal Control.*—As above.

The framework is generally fixed by theodolite triangulation. For plotting plane table stations, resection is now used where practicable, in preference to intersection, to avoid the marking of stations and much of the reconnoitring necessary in the intersection method. Where traversing is adopted owing to the presence of woods or the flatness of the country, the distances may be obtained with sufficient accuracy tacheometrically or by wheel (Vol. II, Chap. VII). When long traverse lines are possible, it is an advantage to have an alidade adapted for movable hair tacheometry, so that observations may be made on conspicuous targets fixed on the staff. Points taken to guide the sketching of detail are usually intersected, but few are required. Since $\frac{1}{100}$ in. on the sheet corresponds to from about 8 to 42 ft. on the scales under consideration, much of the detail may safely be sketched by estimation.

Vertical Control.—Indian Clinometer, Vertical Angles.

The number of observations, other than those required to fix the elevation of the instrument stations, is reduced as the scale decreases, and increasing care is necessary in the selection of the points to be located. As a guide in the sketching of contours, the topographer should estimate the surface slopes.

Scales less than 1/50,000.—*Horizontal Control.*—Resection and Intersection.

The plane table fixings are almost exclusively made by resection from triangulation points. Traversing may still be found necessary in confined situations, but should be reduced to a minimum. The detail is commonly sketched entirely by estimation, particularly on the smaller scales and the surveyor is constantly called upon to decide what features should be shown and what must be omitted because of the smallness of the scale.

Vertical Control —Vertical Angles, Aneroid, Indian Clinometer.

On the smaller scales, the elevations of the plane table stations only are determined, and the contours are sketched entirely by estimation. The production of a satisfactory map in the least time calls for a high order of skill on the part of the topographer, who must have a good eye for country.

Preparation of Field Sheets.—When plane tabling is employed for the mapping of detail between control stations, the surveyor must plot these on the sheets before proceeding to the field. In surveys of moderate extent, the plotting is performed by rectangular co-ordinates, as described on page 292. In extensive small scale

surveys, however, the points are plotted by geographical co-ordinates, and it is then necessary to construct a graticule of meridians and parallels (Vol. II, Chap. VIII), on which the stations are plotted in terms of their computed latitudes and longitudes.

The detail surveyor is furnished with a description of the stations, and, before commencing to map, he should test the accuracy of their plotted positions. Having set up over one of them, and oriented by sighting another, he will be able, by successively sighting the other visible stations, to verify the accuracy of the corresponding plotted points.

Junction of Sheets.—When detailing between plotted stations, no trouble is presented in changing from one field to another, since each has its system of points from which fixings are made. The graticules on adjacent sheets are, however, plotted to afford a marginal overlap by means of which a satisfactory junction can be effected between the features on adjoining sheets.

In large scale work, especially if the survey is executed entirely by plane table, very little overlap may be required. When a new sheet is commenced, the manner of securing continuity with the finished one will depend upon the system of observation being employed. The method of using a long joining line, common to both sheets, and pricking through sufficient points on to the new sheet will suggest itself in any case.

General Suggestions for Plotting.—(1) Lines must be drawn as fine as possible. The pencil should be 4H or harder, and the point should be kept very sharp by frequently touching it up on sand-paper. A chisel point may be used for drawing rays, but for marking points a conical point or a needle pricker is required.

(2) Lines must be drawn close to the edge of the ruler. When drawing a ray from a point, the pencil should be held upright and exactly on the prick mark, and the ruler is laid against the pencil. The pencil should then be maintained at a constant inclination while drawing the ray.

(3) Unnecessary complication of rays must be avoided in using the intersection method. The possibility of confusion is lessened by adopting the following suggestions :

(a) Instead of making the first rays from a station point continuous lines draw only a short length of each ray sufficient to contain the point sought. Draw also a reference mark, or *repere*, which is a short length of the ray at the margin of the sheet. A brief description of the point written against the reference mark serves to identify it. In the case of rays likely to be used for subsequent orientations, reference marks at opposite margins of the paper facilitate accurate setting of the alidade.

(b) Intersections should be obtained, without actually drawing the second rays, by marking the points at which the edge of the ruler cuts the appropriate first rays, but reference marks should be made for rays to be used for orientation.

(c) Do not take more sights than are really necessary. In particular, avoid drawing rays to distant objects which can be located with shorter rays from other stations.

(d) Erase rays and reference marks, with soft rubber, as soon as they can be dispensed with.

(4) The drawing should be kept as clean as possible by paying attention to the cleanliness of the underside of the ruler, the plumb-fork, and the compass, by lifting, rather than sliding, the alidade into position, and by keeping the paper free from moisture and dust. Do not erase lines from damp paper. In small scale work, the part of the sheet not being used should be covered with a piece of tracing paper. Ink in each day's work the same evening.

Errors in Plane Tabling.—It is impossible to state generally the degree of precision to be expected of the plotted map, because it depends not only upon the character of the survey, the quality of the instrument, and the system adopted, but also to a considerable extent upon the degree to which accuracy is deliberately sacrificed for speed. Sources of error may be classified as :

- (1) Instrumental Errors.
- (2) Errors of Manipulation and Sighting.
- (3) Errors of Plotting.

(1) The influence of residual errors of adjustment may be regarded as quite negligible.

(2) Errors of manipulation and sighting include :

(a) *Non-horizontality of Board.*—As in theodolite observations, the effect of dislevelment is most marked when there is a considerable difference of level between the points sighted. The discussion of the error caused by defective levelling in theodolite work (page 225) applies here whether the alidade is telescopic or plain. In the latter case dislevelment causes both vanes to be inclined to the vertical. If the table is not horizontal in a direction at right angles to the alidade, the line of sight is parallel to the fiducial edge only for horizontal sights, and makes an increasing angle with the fiducial edge the more inclined is the sight.

(b) *Error of Centering.*—Since the essential requirement of correct orientation necessitates repeated trial in accurately centering a point not on the vertical axis of the table, the surveyor may frequently avoid unnecessary expenditure of time in setting up by having a proper conception of the extent of error introduced by inexact centering.

In Fig. 233, let P represent the instrument station, and p the corresponding plotted point. The angle APB is erroneously plotted as apb , and the angular error ($APB - apb$) will depend upon the distances PA and PB as well as upon the error of centering.

If the angles PAP and PBP are denoted by α and β respectively, evidently $(APB - apb) = \alpha + \beta = \sin^{-1} \frac{PC}{PA} + \sin^{-1} \frac{PD}{PB}$.

Assuming 1 ft. as the greatest value of PC and PD at all likely to occur in practice, the extent of the resulting angular error for equal lengths of sights is indicated by the following figures :

Distance in Feet.	Angular Error.
10	$11^{\circ} 29'$
100	$1^{\circ} 9'$
500	$14'$
1,000	$7'$
5,000	$1\frac{1}{2}'$

Of more practical consequence is the error in the positions of a and b . Point a should be at a' to the left of a , where $aa' = ap \times a$ approximately : similarly for b . These displacements will not affect the accuracy of the plan if they are too small to be plotted, and this evidently depends upon the scale.

If s = the fractional scale,
and D = the distance PA in ft.,

then plotted length $pa = Ds$ ft.

Actual displacement $aa' = Dsa$,

but $a = e/D$, where e = PC in ft. (Fig. 233),

$\therefore aa' = se$ ft.

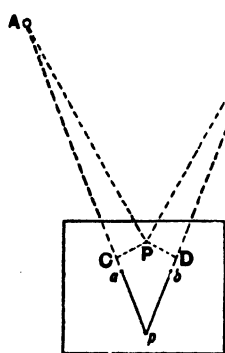


FIG. 233.

If aa' is not to exceed $\frac{1}{100}$ in., assumed as the smallest quantity which can be shown, then e must not exceed $1/1,200s$. Centering must therefore be performed with care in large scale work. For a scale of 10 ft. = 1 in., the horizontal distance pP should be kept under one inch, but for scales smaller than about 200 ft. = 1 in., it is sufficient to have the ground point within the limits of the board. Greater care is desirable in the case of very short rays, to obviate serious error in direction, particularly if such rays are to be used, with the aid of marginal reference marks, for orientation.

(c) *Defective Sighting*.—The plain alidade with open sights is much inferior to the telescopic alidade in the definition of the line of sight. The resulting errors will tend to compensate, and are, in general, relatively unimportant.

(d) *Movement of Board between Sights*.—This should not occur if the clamp has been firmly applied, but it is always advisable to check the orientation at the conclusion of the observations from a station.

(e) *Defective Orientation*.—The above errors contribute towards erroneous orientations and consequent distortion of the survey. To limit the propagation of error, the orientation of the table should be checked at as many stations as possible by reference to a distant prominent object already plotted.

(3) Errors of plotting are a common source of inaccuracy, and can only be minimised by constant care in drawing and in the use of scales, and by using a paper so mounted as to eliminate distortion errors as far as possible.

Survey Adjustment.—Traversing.—A considerable length of traverse may have to be run out from a known point without the surveyor having an opportunity of checking his orientation by observation on a triangulation point. If, in addition, the measurement of distance is not very refined, there is every probability of appreciable error being generated. Most commonly the error is determined, not by a return to the starting-point, but by making a resection fix when the opportunity occurs. The adjustment of the traverse between these fixed points is then made by the graphical method described on page 279.

Triangulation.—Considerable error may likewise be propagated in graphic triangulation, but if it is conducted between points of known position, the results may be adjusted. A rigorous adjustment is uncalled for, and the following arbitrary method will suffice.

In Fig. 234, let $Abc \dots m$ be the plotted result of a triangulation conducted between stations A and M and originating from a base line Ab . If M is the correct position of the terminal point, the error mM may be distributed by treating $Abd g k n m$ as a traverse and adjusting the points by the previous method. The route $Ac f h l m$ is similarly treated. Lines ce and ef are not included, since they do not influence the position of m , and e is replotted from the adjusted positions of c and f .

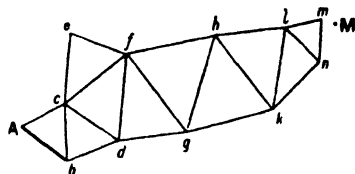


FIG. 234.

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CHAPTER VIII

CONTOURS AND CONTOURING

Representation of Three Dimensions.—The value of a plan or map, whether of large or small scale, is greatly enhanced if the altitudes of the surface features are represented as well as their relative positions in plan. One way in which the conformation of the ground may be presented on the plan is by delineation of the surface slopes on some conventional system of shading, intended merely to convey an impression of relative relief, but without indicating actual elevations. These systems of representations (Vol. II, Chap. VIII) are sometimes used in geographical mapping on small scales, but they do not impart sufficiently definite information to be of service for engineering purposes.

In plans for location or construction work, vertical dimensions should be represented at least as accurately and clearly as horizontal distances, and this points to two requirements in delineation :

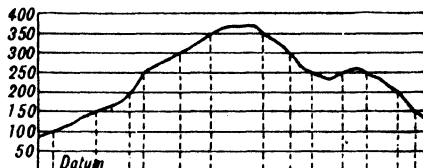
(a) The reduced levels of numerous points in terms of a known datum should be shown.

(b) These should be arranged in such a manner that the form of the surface can readily be interpreted.

The first requirement may be met by the plotting of spot levels, the elevation of each being written over a dot representing the position of the point. The utility of spot levels is, however, limited, as they can convey only a vague idea of the form of the ground. Both conditions are fulfilled by contour lines, a method of representation of great value, as it is equally suitable in flat as in mountainous country, and for large as well as small scale plotting.

Contour Lines.—A contour line, or contour, may be defined as the line in which the surface of the ground is intersected by a level surface. It follows that every point on a contour has the same elevation—that of the assumed intersecting surface. If the contour lines determined in this way by several equidistant level surfaces are imagined to be traced out on the surface of the ground and surveyed, the resulting plan will exhibit the contours in their proper relative positions, and will portray the character of the ground. Thus, in Fig. 235, *a* represents a hill which is shown intersected by a series of level surfaces at elevations of 100, 150, etc., ft. above datum. The contours may run as shown on the plan *b*, which serves to depict the shape of the hill.

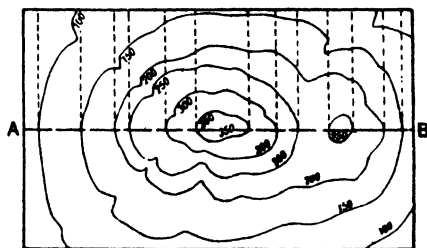
It follows from the definition that the water mark of still water is a contour line, and the student may at first find it helpful to conceive contours as shore lines of a water surface which can be adjusted to any desired level. Regarded in this way, Fig. 235*b* shows the varying outline of an island when the level of the water stands at 100, 150, etc., ft. In the case of a valley or other depression, the contours may similarly be considered as indicating shore lines of a lake.



a

The Contour Interval.—

On the assumption that the contours of Fig. 235*b* are correctly located and plotted, full information is available regarding the surface along each but not about the intervening ground. Considerable irregularities may occur between adjacent contours, but, having escaped intersection, they are



b

FIG. 235.

not represented on the plan. The number of available level surfaces is, however, infinite, and the representation of the ground can be carried to any degree of refinement by sufficiently increasing the number of contours. The constant vertical distance between the contours in any case is called the *Contour Interval*. The most suitable interval to adopt on a survey is a matter to be decided at the outset from consideration of the following items :

(a) *Time and Expense of Field and Office Work.*—The smaller the interval, the greater is the amount of field work, reduction, and plotting required in the preparation of the plan.

(b) *The Purpose and Extent of the Survey.*—Close contouring is required in cases where the plan is intended to be utilised for the detailed design of works or for the measurement of earthwork quantities. In general, the area included in such a plan will be comparatively small, so that it may be quite practicable to locate contours with an interval as small as one foot. In location surveys for lines of communication, reservoirs, and their drainage areas, etc., since a rather large area may be involved, a wider interval must be made to suffice. The requirements of geographical mapping are met by the use of still greater intervals.

(c) *The Nature of the Country.*—An interval which would be sufficient to show the configuration of mountainous country would be quite inadequate to portray the undulations of comparatively

flat ground. The representation by contour lines of a stretch of very flat ground is made possible only by the adoption of a very small interval.

(d) *The Scale of the Plan or Map.*—The interval should, *ceteris paribus*, be in inverse ratio to the scale of the map. By using a close interval on a small scale map, the detail is obscured, and, unless the ground is very flat, the result is confusing.

Values of the contour interval adopted for various purposes are as follows :

For building sites : 1 or 2 ft.

For reservoirs, landscape gardening, and town-planning schemes : 2 to 5 ft.

For location surveys : 5 to 10 ft.

For general topographical work : from 10 ft. upwards, depending upon the scale and the character of the ground ; for average country a common rule is

Contour interval = 50 ft. \div no. of inches per mile.

Whatever interval is adopted, it should be constant within the limits of a map. If the regularity of the intervals is interrupted by the interpolation of additional contours for the better definition of flat ground, these should be distinguished in drawing by dotting or otherwise. Unless this is done, the general appearance of the map is misleading, the closely contoured ground appearing steeper than it really is.

Characteristics of Contours.—By virtue of the fundamental property of a contour, that every point on it has the same elevation, a contour map with constant interval portrays elements of configuration in a characteristic manner. A knowledge of the more important attributes of contours facilitates the interpretation of a map, and, in plotting, enables the draughtsman to avoid possible misrepresentations false to nature. The following items should be kept in view in map reading and plotting.

1. Since the rise or fall from a point on one contour to any point on the next above or below is the constant contour interval, the direction of steepest slope is that of the shortest distance between the contours. The direction of steepest slope at a point on a contour is therefore at right angles to the contour.

2. Steep ground is indicated where the contours run close together ; flat ground where they are widely separated. Uniform distance between contours indicates a uniform slope, and a series of straight, parallel, and equally spaced contours represents a plane surface.

3. Contour lines of different elevations can unite to form one line, only in the case of a vertical cliff.

4. Two contour lines having the same elevation cannot unite and continue as one line, nor can a single contour split into two

lines. This is evident from the conception of contours as water marks. The single line would indicate an impossible knife-edge ridge or hollow.

5. There are conceivable circumstances in which two contours of the same elevation may touch at a point, giving them the appearance of crossing, but this occurs very rarely. Contours of different elevations can cross each other only in the case of an overhanging cliff or a cave penetrating a hillside. At the intersections, two or more points of different levels on the ground are represented by one point in plan.

6. A contour cannot end anywhere, but must ultimately close on itself, although not necessarily within the limits of the map. This property is obvious from the idea of water marks. A series of closed contours indicates either a hill or a depression without an outlet, according as their elevations increase or decrease towards the centre of the series.

7. The same contour appears on either side of a ridge or valley, for the highest level surface which intersects the ridge and the lowest one which intersects the valley do so on both sides. It is impossible for a single higher or lower contour to intervene between two of equal value.

8. Contour lines cross a watershed or ridge line at right angles. They curve round it with the concave side of the curve towards the higher ground.

9. In valleys and ravines, the contour lines run up the valley and turn at the stream so that the convexity is next the higher ground. They are intersected at right angles by the stream. If the scale allows of both banks being shown, the contours should be stopped at the edge of the water unless they have been located across the bed of the stream.

METHODS OF CONTOURING

Since in the location of contours both vertical and horizontal measurements are involved, the field work may be executed in various ways according to the instruments used. Field methods may, however, be divided into two classes—*Direct*, and *Indirect*.

Direct methods comprise those in which the contours to be plotted are actually traced out in the field by the location and marking of a series of points on each. These points are surveyed and plotted, and the appropriate contours are drawn through them.

Indirect methods are those in which the points located as regards position and elevation are not necessarily situated on the contours to be shown, but serve, on being plotted, as a basis for the interpolation of the required contours. This system is used in all kinds of survey, and proves less laborious than the first.

In both methods, but particularly in the case of the latter, the accuracy of the resulting map will greatly depend upon the number

and disposition of the selected points. The careful location of numerous points may be out of the question owing to the time required in the field, and for some purposes sufficient accuracy is attained by interpolating between widely spaced points. In such work, horizontal positions may be determined by intersection, and elevations by barometric or trigonometric levelling, but this class of field work belongs more properly to the subject of topographical and geographical surveying, and is dealt with in Vol. II, Chaps. VI and VII. The field methods to be considered here are those directed to the production of a map suitable for location or construction work. Variation in procedure will depend upon which of the following instruments are used :

Vertical Control : Dumpy or Wye Level, Hand Level or Clinometer.

Horizontal Control : Chain and Tape, Theodolite or Tacheometer, Compass, Plane Table.

Combined Control : Tacheometer, Plane Table with Tacheometric Alidade or Clinometer.

Direct Methods.—Direct contouring is necessarily very slow, and on this account it is comparatively seldom adopted on large surveys. It has the merit of superior accuracy, and is suitable for the close contouring of small areas where considerable precision is required. The field work consists of two steps : (1) the location of points on the contours ; (2) the survey of those points. These operations may be conducted nearly simultaneously if performed by two parties, one levelling and the other surveying, but in dealing with a small area the pegging out of the contours may be completed before their survey is commenced.

Although the field routine is largely dependent upon the instruments employed, two principles of general applicability are to be observed.

1. The degree of accuracy required in the location on contours should depend upon the scale to which they are to be plotted, and, for a given scale, upon the use to which the plan is to be put.

2. Since the contours are to be drawn through the plotted points, only such points should be located that the contours are nearly straight between them. At places of sharp curvature more locations are necessary than elsewhere. Salient points on ridge and valley lines are of special importance, and should never be omitted.

Direct Vertical Control.—1. *By Level and Staff.*—Having levelled from the nearest bench mark to the site of the survey, set the instrument to command as much ground as possible. From the known instrument height deduce the readings to be observed when the staff is held on the various contours within range of the level. Taking one contour at a time, direct the staffman up or down hill until the required reading is sighted. On receiving the signal to mark, the staffman should insert a lath or twig split to receive a

piece of paper on which is noted the reduced level of the contour. Having located one contour over the length visible from the instrument, proceed in the same manner for the others with the new staff reading, until a fresh instrument station is required.

Fig. 236 shows a possible arrangement of the points fixed from one set-up, the arrows showing the route followed by the staffman.

If the instrument height were 86.37, the staff readings would be 1.4, 6.4, and 11.4 in locating points on the 85, 80, and 75 contours respectively. It is unnecessary to read to two places of decimals.



FIG. 236.

Notes.—(1) It is advisable that the levelling from the B.M. to the first control station should be checked before contouring is commenced. A temporary B.M. should therefore be established on the site of the survey, and checked first of all. If, however, there are one or more Ordnance B.M.'s on or near the ground, this step is unnecessary.

(2) In walking forward, the staffman should try to avoid going up or down hill. It is an advantage to have the assistance of an experienced staffman, who, when remote from the instrument, can be relied upon to select suitable points, but the surveyor should nevertheless be on the lookout for possible omissions at salient points. A special signal should be used to indicate that the staffman has to begin on a different contour.

(3) Fixing adjacent points on several contours occasions loss of time and confusion with the marking tags; the method of working along each contour as described, is much preferable.

2. By Hand Level.—The hand level may be of any type, an Abney clinometer clamped to zero proving suitable, and the instrument may be used in conjunction with a levelling staff or simply a ranging pole marked off in feet. By levelling from a B.M., first locate a point on one of the contours, preferably one having an elevation about the mean of those to be traced. This contour is to be pegged out first in order to provide points of known elevation from which the remaining contours can be located. To trace the contour, stand on the initial point, and direct the staffman until the point on the staff or pole corresponding to the height of the instrument above the ground is in the line of sight. The level should be held against a pole, so that the reading is a whole number, say 5 ft. Locate as many points on the contour as can be conveniently sighted from the instrument station, and then move forward to the last point marked.

When a sufficient length has been set out to form the basis of the day's work, the fixing of points on the contours on either side can be commenced. Let it be assumed that the contour interval is 2 ft., and that the instrument is held 5 ft. above the ground. Send the staffman uphill until a reading of $(5-2) = 3$ ft. is obtained, and fix a few suitable points with that reading. The next series gives a reading of 1 ft., but to locate the higher contours the surveyor must take up a position over one of these points and start anew. In setting stakes on the downhill side of the reference contour, the

readings from it will be 7, 9, etc., ft. If a sufficiently long rod is not available, the staffman can hold his pole on the reference contour, while the surveyor places himself approximately on the lower contour and shifts about until he sights 3 ft. as before. Having in this manner located portions of the several contours with respect to the initial point, the other points on the reference contour are utilised in the same manner.

Notes.—(1) Greater accuracy is attained if the preliminary work of levelling from the B.M. and setting out the reference contour is performed by dumpy level. A reference contour may be dispensed with by establishing a number of temporary B.M.'s over the area by means of ordinary levelling.

(2) If the contour interval exceeds the height of the instrument above the ground, the contour above the reference contour cannot be located directly from it, but the levelling must be carried up from the reference points in a series of stages.

Direct Horizontal Control.—The survey of the positions of the stakes defining the contours is a somewhat tedious operation. The system to be adopted must be decided upon from consideration of the size and shape of the area involved and of the accuracy required. For small areas, chain surveying may prove suitable, but in general it is necessary to execute a traverse, either by theodolite, compass, or plane table. If contouring is confined to a narrow strip of ground, a single traverse can be run approximately along the centre line, and offsets taken to the stakes on either side. Some of these offsets may be longer than is generally desirable, but this is not a serious objection, and one or two 100 ft. tapes should be carried. When the width is too great for effective control by a single traverse, a network, or framework, must be laid out.

The survey should be conducted as soon as possible after the location to avoid possible errors due to displacement of the stakes. If only one surveyor is in the field, he should therefore locate and survey on alternate days, or employ the forenoons for locating and the afternoons for surveying. A considerable saving of time may be effected by the use of the tacheometer, which, although best adapted for indirect contouring, can be utilised in the direct system for horizontal control only. As soon as the staffman has been placed on a contour by the leveller, an observation is taken for the bearing and distance of the staff from the tacheometer set over a traverse station. In this way the need for marking located points is eliminated.

However the points are surveyed, a note must be taken of the contours on which they are situated, this information being obtained by reading the tag on the stake when offsetting, or from the leveller in the above tacheometric system. When the points are plotted, the contours are drawn through them as curved lines, due attention being paid that the fundamental characteristics of contours are nowhere violated.

Indirect Methods.—In these methods, the points to be completely located may be either (1) situated along a series of straight lines

set out over the area, or (2) scattered spot heights or representative points. In (1), the straight lines simply constitute lines of sections, and in general no cognisance is taken of the ground between these sections. In (2), more particular attention can be paid to the salient features upon which the topography depends.

(1a) *By Cross Sections.*—Suitably spaced sections are projected from traverse lines, the observations being made in the usual manner by level, clinometer, or theodolite. The sections should be spaced more closely than usual at places where the contours curve abruptly, as on spurs and in ravines, and in the latter case it is expedient to run a section approximately along the line of the stream, taking its direction by compass. The configuration may in places suggest the running of a number of sections radiating from a point.

The sections need not be plotted if an ordinary level has been used, so that the reduced levels of the various points are known. To draw the contours, the points levelled are first marked off along the section lines, and the elevation of each point is written against it. On the assumption of uniform slope between adjacent points, the contours are then interpolated by estimation as shown in Fig. 237.

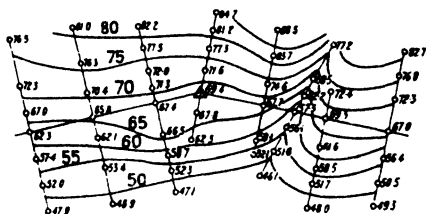


FIG. 237.

If the sections are plotted, the interpolation may be performed more quickly and mechanically by the following method, which must be used when the sectioning is performed by clinometer. Rule on a sheet of tracing paper a series of equidistant parallel straight lines, the distance between which represents the contour interval to the scale of the sections. Place this sheet over a section in such a position relatively to its datum or its ground line that the

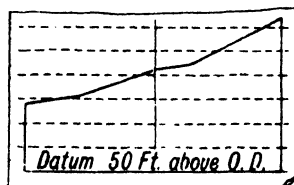


FIG. 238.

ruled lines coincide with the intersecting planes defining the required contours. In Fig. 238, the lines on the tracing paper are shown dotted, the interval between them being 5 ft. The tracing has been placed so that one of the lines coincides with the datum line of the section, and, reckoning upwards, it will be observed that the ground surface is intersected by lines of elevations 65, 70, 75, and 80. The distances from the centre line to those intersections on right and left are transferred to the section line on the plan, and the contours are then drawn as in the direct method.

Notes.—(1) If the base lines of the cross sections have not been drawn, it is necessary to subdivide a portion of the vertical centre line on the tracing

paper into feet. The tracing paper can then be placed with the horizontal lines in their proper position relatively to the centre line level marked on the cross section.

(2) The transfer of the right and left distances from the section to the plan is expeditiously performed if the work is shared by two men, one using the sections and calling out the distances and elevations of the various contours, the other plotting the points on the plan and drawing the contours through them.

The cross section method of contouring does not fall far short of the direct method in point of accuracy, provided always that additional sections are run where called for, and that the ground is fairly uniform in slope between the points located. It has the merit that the sections can be employed in connection with estimates and construction.

(1b) *By Squares.*—If the area to be contoured is not very extensive, it may be divided up into a series of squares, the corners of which define the points to be levelled. Bounding lines at right angles to each other are set out to enclose the area, and ranging poles are placed at regular intervals along them. If the ground is not too rough, the pegging out of each separate square may be obviated by providing sufficient poles (as in Fig. 239) to enable the staffman to place himself at each point. To prevent misunderstanding, every fifth line may be flagged, and, for

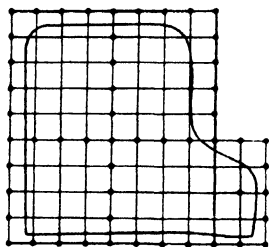


FIG. 239.

referencing, one set of lines should be lettered and the other numbered. If the site is one where it is proposed to carry out earthwork, the enclosing lines should be permanently pegged for convenience in running levels for progress measurements.

The interpolation is similar to that of the previous case, and is exhibited in Fig. 240. It is to be observed that the assumption of uniform slope between the located points is not so justifiable here as in the cross section method, but the smaller the squares the more valid will be this hypothesis.

The sizes employed range from 10 ft. to 100 ft. side, small squares being used for close contouring or on rough ground. In ground of varying character, the squares need not be of the same dimensions throughout, but can be reduced on rugged parts, or, alternatively, a few additional observations on points within the squares can

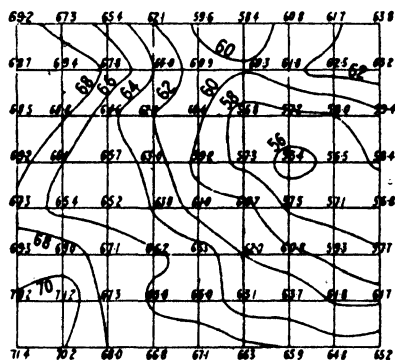


FIG. 240.

be located on the diagonals or by other measurements from the corners.

(2) *By Spot Heights or Representative Points.*—If spot heights only are observed, the number required for contour location is considerably reduced, but the facility of horizontal control possessed by the previous two methods does not obtain. Notwithstanding, this system is the most popular, particularly on large surveys, because of its suitability for tacheometrical methods, which, since they are designed to furnish both horizontal and vertical control, are specially adapted for contouring. This branch of surveying is described in Chapter XI.

Contour Drawing.—The contour lines, having been drawn in pencil, are inked in, either in black or brown, the latter being preferred as being less likely to lead to confusion where roads, buildings, etc., appear on the plan. A drawing pen gives a better line than a writing pen, and French curves should be used as much as possible. The elevations of the contours must be written against them in a uniform manner, either on the higher side or in a gap left in the line. The figures should be normal to the contours, and should lie either in a straight line or along a curve intersecting the contours at right angles. Several rows of figures are required on a large sheet. In small scale mapping it is sufficient to figure every fifth contour, which should be distinguished by a bolder line, but the values of intermediate contours must be shown where there is a possibility of misinterpretation.

Interpolation of Contours.—When contours have been directly located with a rather wide interval, it is a common practice to draw intermediate contours by interpolation. These should be distinguished by dotted lines. It is usually amply sufficient to interpolate by estimation. On the supposition of uniform slopes, interpolation between plotted contours, or between given points, can also be performed :

(1) By calculation of the intermediate positions. This method is unwarrantably tedious.

(2) Mechanically.

(a) By superimposing a piece of tracing paper on which are ruled a number of equally spaced radiating lines (Fig. 241), and moving it about until an imaginary line as nearly as possible normal to both contours, or the line between the points, is suitably divided by the rays. The points of division are then pricked through.

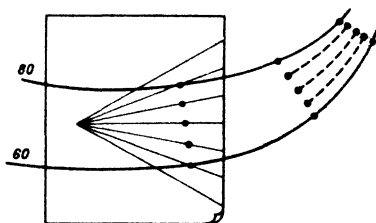


FIG. 241.

(b) By marking off a number of small equal divisions on a flat rubber band, which can then be placed on the drawing and stretched

so that the appropriate number of divisions is intercepted between the given contours or points. The intermediate points are then ticked off.

Contour Gradients.—A line lying throughout on the surface of the ground, and preserving a constant inclination to the horizontal, is called a contour gradient. The path of such a line can be traced in the field or on a map (page 370), if its inclination is known, and it is specified to pass through a given point.

In the field, the location is most quickly performed by means of a clinometer, theodolite, or gradiometer (page 504). The instrument is first set up at the given point, and the line of sight is laid at the given inclination. A man, carrying a level staff, or simply a graduated ranging pole, proceeds in the estimated direction of the contour gradient to a greater or less distance from the instrument, according as the ground is of uniform slope or undulating. He holds up the rod, and is directed by the instrument man to move up or down hill until the rod reading equals the height of the instrument above the ground. The line from the instrument station to the point on which the rod is held is then parallel to the line of sight, and forms a part of the required contour gradient. The point so obtained is pegged, and is used as the instrument station for the location of the next point in the same manner.

Alternatively, a level may be used, and in this case the instrument need not necessarily be set up over the contour gradient. Since the line of sight is horizontal, it is necessary to measure out from the starting-point, or from the last point pegged, the distance, say 100 ft., to the point to be fixed. The required staff reading is computed from this distance, the gradient, and the elevation of the instrument. Thus, if a down gradient of 1 in 75 is being traced, and the line of sight is at 4·2 ft. above a peg on it, the reading on a staff 100 ft. from that peg should be 5·5 ft. The staffman, holding the end of a 100-ft. tape or chain stretched from the peg, is moved up or down hill until the required reading is observed.

Notes.—(1) In the clinometric method, the instrument has to be moved forward after each observation, unless the ground surface is plane. A hand clinometer is therefore the most convenient of the instruments available. In the method of levelling, the same instrument station may be used for the pegging of several points.

(2) From any point there will usually be two or more directions in which a given gradient may proceed, so that an infinite number of contour gradients may be laid out from a given origin. There is, however, less difficulty than might be supposed in deciding which of the directions should be followed, since the gradient required in practical cases is that which is least winding.

(3) If the gradient as set out has to be surveyed, it is usually sufficient to take compass bearings from peg to peg and note the distances. Otherwise, offsets may be taken to the pegs from a traverse with longer courses.

The tracing of contour gradients is a common field operation in the location of road and railway routes. When the locating engineer has to find the best route for, say, a road over a range of hills, it is

generally impossible to adhere, even roughly, to the straight line route between the controlling points on either side because of the steepness of the ground. By laying off a contour gradient equal to the steepest slope which is considered allowable on this part of the road—the limiting or ruling gradient—he discovers a route over the difficult ground which, if it were followed, would enable the road to be constructed without excavation or embankment along its centre line.

The contour gradient located is generally that passing through the lowest point on the ridge near the straight line route, and it is best set out from the ridge down both sides of the hill. It affords a most useful guide to the location of the road, but its adoption as the actual centre line would result in a very tortuous route except where the ground is of uniform slope. In crossing a gorge, for example, the contour gradient runs up the gorge, and crosses the stream in a manner similar to a contour line. The road, when constructed, will necessarily be carried across the gorge at a point downstream from the contour gradient, as it must be above the stream. The engineer, instead of locating the true contour gradient at such a place, therefore examines the ground for the most suitable site for the road crossing, and projects the contour gradient across in the air. It is likewise unnecessary for him to trace the true gradient round small hollows and ridges, at which it would have a sharp curvature. At every place, however, where the finally adopted route for the road deviates from the contour gradient of the same value as the road gradient, earthwork will be required on the centre line. Deviations downhill necessitate embankments, while those to the uphill side involve excavations.

USES OF CONTOUR PLANS AND MAPS

1. Drawing of Sections.—A section along any line, straight or curved, on the map can be drawn by marking off a datum line the distances along the line of section at which the various contours are intersected. The respective contour elevations are then set up as ordinates from these points. Thus, in Fig. 235, *a* is the section AB of the hill represented in the plan *b*.

2. Determination of Intervisibility.—Whether two points of given position and elevation are intervisible or not can be ascertained from the map without having recourse to drawing the section between them. Let it be required to test the points A and B (Fig. 242) in this respect. Join AB. From its length and the known levels of A and B, determine by calculation, or other method of interpolation, the position of the points on it which have the same elevation as the contours. These are marked with

sloping figures. Determine by inspection whether these points are above or below the ground. Examining the point of 90 ft. elevation on AB, it is seen to fall on the lower side of the 90 contour, and therefore is above ground, since a vertical line through it would cut the ground at an elevation of less than 90 ft. Similarly, the points at 80 and 50 ft. are clear, but those at 70 and 60 are below the surface, and in consequence A and B are not mutually visible. C and D, the points at which AB and the surface are estimated to have the same levels, about 72 and 57 respectively, show the limits of the obstruction. The determination of intervisibility, of importance in military work, is sometimes of service to the surveyor. The above method of solution does not apply to very long sights on account of the effects of curvature and refraction.

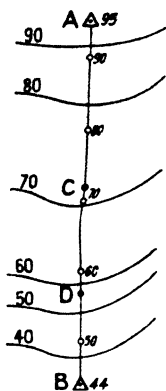


FIG. 242.

3. Tracing of Contour Gradients.—The method of locating on a map the route of a given contour gradient through a given point is illustrated in Fig. 243, in which it is required to trace the upward course from the point A of a contour gradient of 1 in 30. It will be sufficient to locate the points at which the gradient intersects the given contours, and, since the contour interval is 10 ft., the horizontal distance between the successive required points must be

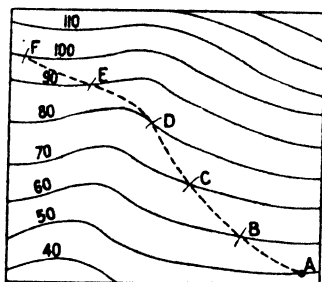


FIG. 243.

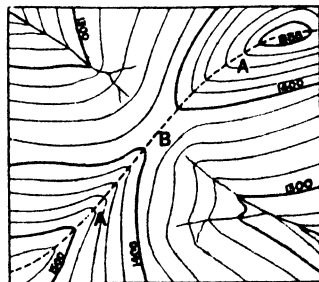


FIG. 244.

300 ft. With centre A on the 50 contour, describe an arc of this radius to cut the 60 contour at B, and, with centre B, describe a similar arc to cut the next contour, and so on. Join these points with a curve. This line represents the path of the contour gradient with sufficient accuracy for most practical purposes notwithstanding that the lengths intercepted between adjacent contours are not now 300 ft. precisely. The actual line would be more tortuous than is shown.

Note.—Each of the arcs described in Fig. 243 will cut its appropriate contour at two points, so that the contour gradient drawn is only one of those which

can be marked out from A. The others which could be shown within the limits of the diagram have a zigzag course.

4. Measurement of Drainage Areas.—For water supply and irrigation purposes the extent of these can be estimated by measurement of the area contained within the watershed line separating the basin from those adjoining. This line is to be traced on the map in such position that the ground slopes down on either side of it, i.e. it lies along ridges, as at A (Fig. 244), and on cols or passes, B. Care must be observed to ensure that the whole area draining into the valley under consideration is included.

5. Intersection of Surfaces.—If two intersecting surfaces are each represented in plan by a system of contours, then the points in which the contours of one surface cut those of equal elevation

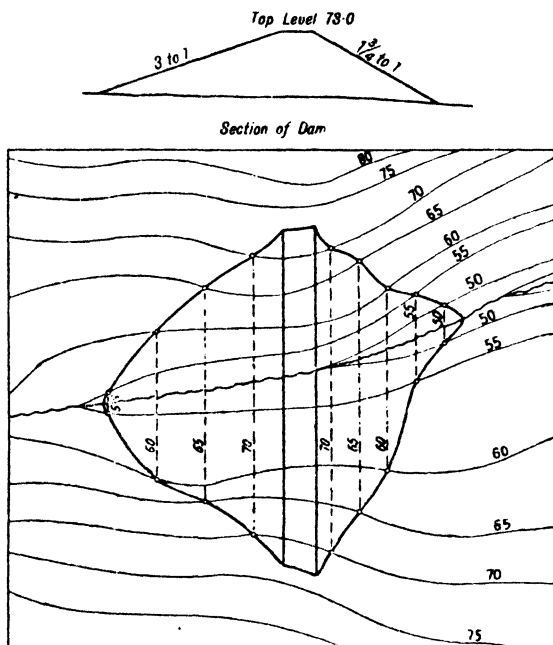


FIG. 245.

belonging to the other are situated on the line of intersection of the two surfaces. Such points are common to both surfaces, and the line of intersection can be drawn by joining up successive points. This construction is of practical utility in affording a rapid method of determining on a plan the boundary of proposed earthwork, which is simply the line in which the earth slopes cut the original surface of the ground.

Thus, in Fig. 245, let it be required to draw the plan of an earth

dam, of the given dimensions, to be built across a valley. Having drawn the top surface in the proposed position, set off the contours of the sloping sides for the same elevations as the contours representing the original ground. The new contours are shown dotted, their positions being obtained from the given side slopes and the top level of the dam. The line joining the points of intersection of the equivalent contours of the new and original surfaces shows the extent of the earthwork in plan.

In the above case, since the top of the dam is level, the contour lines along the side slopes are parallel to the top edges. Fig. 246

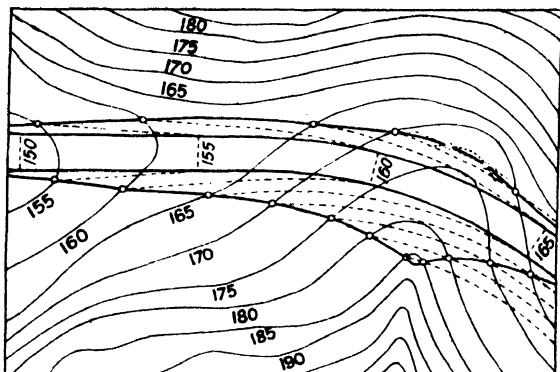


FIG. 246.

illustrates the method of finding the limits of an excavation for a road which has an upward gradient of 1 in 22 towards the right. From a known formation level the positions of contours 150, 155, etc., 110 ft. apart, are obtained on the formation surface, and, on the assumption that this surface is horizontal transversely, the contour lines are drawn straight across it. Along the side slopes, which are to be $1\frac{1}{2}$ horizontal to 1 vertical, the contours will deviate from the sides of the formation surface at the rate of $7\frac{1}{2}$ ft. in 110 ft. The intersection points of the two sets of contours are obtained and joined up as before.

A similar application of the principle to earthwork occurs when it is desired to show where excavation and filling respectively will be necessitated in grading a piece of ground. This problem is illustrated in Fig. 271, where the solid lines represent the contours of the original ground, and the dotted lines the altered surface. The line *aaa*, obtained as above, marks the boundary between cutting and filling; the ground within it must be excavated, and that outside filled within the limits of the diagram.

In tracing these intersection lines, there may at times be some doubt as to the direction of the line between the points located on it. In such cases, the interpolation of a few intermediate contours

on both series will yield additional intersections and fix the line more definitely. Thus, in Fig. 246, two points are obtained on the upper intersection line by the interpolation of small parts of the $172\frac{1}{2}$ contours.

6. Measurement of Earthwork.—See pages 387-410

CHAPTER IX

MEASUREMENT OF AREAS AND VOLUMES

In this chapter are given the methods employed for the measurement of areas and volumes. Needless to say, such measurements are commonly required in civil engineering practice, and form an important branch of office work.

Necessity for System in Calculation.—At the outset it is necessary to emphasise the advantage of a methodical arrangement of calculations. The aim should be to set out the work in such a manner that the result of each step can be seen at a glance, and so that all the intermediate work can be followed and checked. The calculations should be performed in books of about foolscap size, one being used to show the whole of the figuring, and the other to record the results of the various steps and the final quantities. As in all other operations in connection with surveying, verification of the results is necessary before they can be employed for any purpose. If the checking is performed by the person who made the original calculations, the most reliable check is afforded by working out the result by a different method, when possible. If the method first adopted is considered preferable to any other, care should be taken to ensure that the checking embraces every part of the work involved in the first determination. In office work, verification by another person is preferred, and the checker should adopt the same routine, and should work quite independently from beginning to end.

Accuracy of Results.—Arithmetical slips being eliminated, the degree of accuracy of the final result will depend upon (a) the accuracy of the field work, (b) the accuracy of plotting, when the calculations are made from scaled measurements, (c) the method of calculation adopted. It must be realised that the data employed in computation are subject to error and that the accuracy of the final result cannot be increased by needless refinement of figuring.

As there is considerable difficulty in assessing the precision of the data, it is not easy to judge the degree of refinement justifiable in calculation. One must, to a large extent, estimate the probable accuracy of the field and office work, and the final result should be expressed with no more significant figures than can be relied upon.

Note.—The beginner is apt to overlook this point, and, in computing areas for example, would find it helpful to form a mental picture of the extent of

ground corresponding to the last significant figure in his result. Keeping in view the probable accuracy of the field methods and the total area in any case, he will thus acquire some conception of how to express results consistently with the value of the data.

MEASUREMENT OF AREA

By the area of a piece of ground is meant its area in *plan*. The British unit of land measurement is the imperial acre, $1/640$ th of a square mile, subdivided as follows :

$$\begin{aligned} 1 \text{ acre} &= 4 \text{ roods} &&= 160 \text{ poles (or perches),} \\ &= 10 \text{ sq. chains} &&= 100,000 \text{ sq. links,} \\ &= 4,840 \text{ sq. yards} &&= 43,560 \text{ sq. feet.} \end{aligned}$$

Fractional parts of an acre are expressed either in decimals or in roods and poles.

The decimal connection between the acre and the square chain facilitates reduction when the Gunter chain has been used in the field. For example, let the area of a piece of ground be calculated as 349,400 sq. links. By moving the decimal point five places to the left we have at once 3.49400 acres, which, having regard to the circumstances of the measurement, might be stated as 3.494 acres. To convert the fractional part to roods and poles, we have :

$$\begin{array}{r} 3.494 \\ \quad 4 \\ \hline 1.976 \\ \quad 40 \\ \hline 39.040 \end{array}$$

or 3 acres 1 rood 39 poles.

General Methods of Measurement.—Areas may be obtained (*a*) from the dotted plan, (*b*) by direct use of the field notes. The former is the more common and less troublesome method, but the latter is susceptible of greater accuracy since errors introduced in plotting and scaling are eliminated.

Measurement from Plan.—The area is found either (*a*) by dividing the plot into geometrical figures or by ordinates and computing from their scaled dimensions, or (*b*) mechanically by planimeter.

By Division into Geometrical Figures.—(*a*) The most convenient method is to divide up the survey into triangles, either by pencil lines on the plan or on superimposed tracing paper. The base length and perpendicular height of each triangle are scaled and noted, care being taken not to omit any nor to record the same one twice. When one or more of the boundaries of the plot of ground consist of irregular curves, these lines are replaced by straight ones such that the area contained within them is equal to that of the

original figure. This "equalising" of the boundaries is easily accomplished by the use of a piece of black thread or a transparent

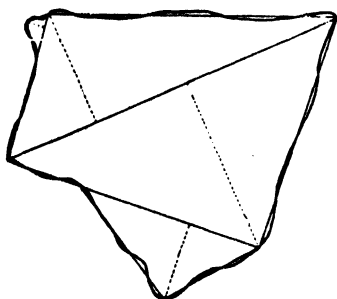


FIG. 247.

set-square, the straights being drawn so that, as nearly as can be judged, the areas contained between them and the irregular boundary are equally disposed on either side of the straight (Fig. 247). No attempt should be made to equalise large irregularities, but small triangles should be introduced in such cases.

(b) A somewhat similar method consists in placing over the drawing a piece of tracing paper ruled into squares, which may conveniently be

an exact fraction of an acre to the scale of the survey (Fig. 248). The number of complete squares within the boundary is counted, but there are in addition a number of partial squares next the boundary, the areas of which have to be evaluated. If a rough estimate only is required, and the squares are small, it may be sufficient to judge these fractional areas by eye in terms of a whole

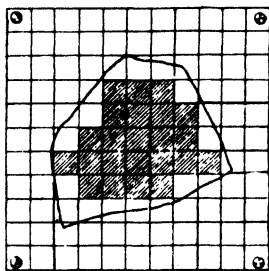


FIG. 248.

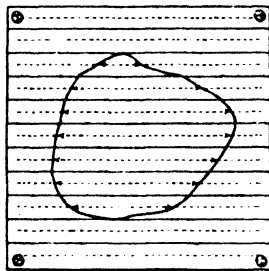


FIG. 249.

square, the resulting errors tending to a certain extent to compensate. A better result is obtained, however, if each partial area is computed from its scaled dimensions, the irregularities being equalised where necessary.

(c) A third method is to divide the area into a series of strips by equidistant parallel straight lines ruled on tracing paper, the constant distance between them being some round number of feet or links. Midway between each pair of lines there is drawn another in a different colour or dotted (Fig. 249). The tracing paper is placed over the drawing, and is moved about until the area to be measured lies between two of the full lines. The figure is thus divided into strips of constant breadth, and, if it is assumed that the strips are either trapezoids or triangles, the area of each is expressed by the length of the mid-ordinate times the breadth.

The lengths of the dotted lines intercepted within the boundary are therefore scaled, and the total sum of their lengths times the constant breadth gives the required area. The assumption that portions of the boundary between adjacent full lines are straight causes some of the partial areas to be over- and others underestimated, the errors being compensating. The smaller the constant breadth of the strips, the more nearly will the correct result be approached.

By Ordinates.—Calculation by ordinates is similar to the last method, and is suitable for the case of a long narrow strip of ground, such as that occupied by a road or railway. A line is described axially through the area, and at equal distances along it perpendicular ordinates are drawn, and their lengths scaled. From these lengths and their common distance apart the area can be calculated by (a) the Trapezoidal Rule, (b) Simpson's Rule.

Trapezoidal Rule.—In this method it is assumed that the short lengths of the boundary between the ordinates are straight lines, so that the area is divided into a series of trapezoids (Fig. 250).

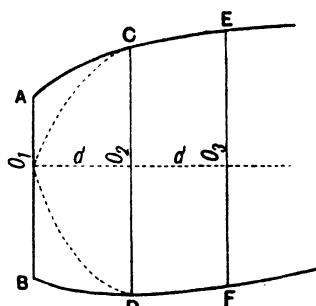


FIG. 250.

If d = the common distance between the ordinates,
 $o_1, o_2, o_3 \dots o_n$ = the lengths of the ordinates ;

$$\text{then area of first trapezoid} = \frac{o_1 + o_2}{2} \times d,$$

$$\text{,, second ,,} = \frac{o_2 + o_3}{2} \times d,$$

$$\text{,, last ,,} = \frac{o_{n-1} + o_n}{2} \times d.$$

Summing up, we have the total area,

$$A = d \left[\frac{o_1}{2} + o_2 + o_3 \dots + o_{n-1} + \frac{o_n}{2} \right].$$

The required area therefore equals the common distance apart of the ordinates multiplied by the sum of half the first and last ordinates plus all the others.

Caution must be exercised in the case where there is an apex at one or both ends of the figure, as shown dotted, resulting in o_1 or o_n being zero. These must not be omitted from the formula, a not uncommon mistake being to disregard them and to treat o_2 and o_{n-1} as the first and last ordinates.

Simpson's Rule.—This rule assumes that the short lengths of boundary between the ordinates are parabolic arcs. With the previous notation, we have (Fig. 250) :

$$\begin{aligned}\text{Area of first two divisions} &= \frac{o_1+o_3}{2} \times 2d + \frac{2}{3} \left[o_2 - \left(\frac{o_1+o_3}{2} \right) \right] \times 2d, \\ &= \frac{d}{3} (o_1 + 4o_2 + o_3),\end{aligned}$$

since they are equivalent to the trapezoid ABEF, obtained by joining AE and BF, and the parabolic segments ACE and BDF.

$$\begin{aligned}\text{Area of next two divisions} &= \frac{o_3+o_5}{2} \times 2d + \frac{2}{3} \left[o_4 - \left(\frac{o_3+o_5}{2} \right) \right] \times 2d, \\ &= \frac{d}{3} (o_3 + 4o_4 + o_5),\end{aligned}$$

and so on to the last two divisions, the area of which is given by

$$\frac{d}{3} (o_{n-2} + 4o_{n-1} + o_n).$$

Summing up, we have the total area,

$$A = \frac{d}{3} [o_1 + 4o_2 + 2o_3 + 4o_4 + \dots + 2o_{n-2} + 4o_{n-1} + o_n].$$

Note.—It is not necessary that the curves should be concave towards the axis, as in Fig. 253. If, in the case of the first two divisions, o_2 is less than $\frac{o_1+o_3}{2}$,

$$\begin{aligned}\text{the area of the first two divisions} &= \frac{o_1+o_3}{2} \times 2d - \frac{2}{3} \left[\left(\frac{o_1+o_3}{2} \right) - o_2 \right] \times 2d, \\ &= \frac{d}{3} (o_1 + 4o_2 + o_3), \text{ as before.}\end{aligned}$$

This method necessitates an even number of divisions of the area, i.e. the total number of ordinates must be odd. If the order of the multipliers, 1, 4, 2, 4, . . . 2, 4, 1, is memorised, confusion will be avoided. The remarks about the case where one or both of the end ordinates are zero again apply.

Simpson's rule is likely to prove more accurate than the trapezoidal formula, but their relative accuracy depends upon the shape of the figure. For the calculation of the area within a regular curve, Simpson's rule is correct not only in the case of $y = a + bx + cx^2$, but also for $y = a + bx + cx^2 + dx^3$. In dealing with irregularly shaped figures, the degree of precision of either method can be increased by increasing the number of ordinates. Several other rules have been formulated, but the above two are sufficient for practical purposes.

By Planimeter.—The planimeter is a mechanical integrator used for the measurement of plotted areas. The various forms which have been devised possess the common feature that a point of the instrument is guided round the boundary of the area, and the resulting displacement of another part of the mechanism is such as to enable the area to be recorded. The most commonly used form is Amsler's polar planimeter, and it only will be considered.

Principle of the Planimeter.—The principle of the instrument may be followed from consideration of the area swept out by a moving line. In Fig. 251, let a straight line of length l , the ends of which move in given loci, be given a small displacement from AB to $A'B'$. The motion is equivalent to a normal translation to $A''B'$ and a rotation about B' through $d\theta$. To the first order of infinitesimals, the area swept out is

$$\begin{aligned} dA &= ldm + \frac{1}{2}l^2d\theta, \\ \text{where } dm &= \text{the distance between the parallels } AB \\ &\quad \text{and } A''B', \\ &= \text{the normal displacement of } B. \end{aligned}$$

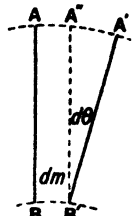


FIG. 251.

Let A describe a closed curve, B being constrained to move along a line and returning to its initial position thereon when A completes a circuit. With the convention that areas swept out to the right are positive and to the left negative, the net area A swept out by AB during a circuit by A is the area round which A has travelled, and

$$\begin{aligned} A &= \int ldm + \frac{1}{2} \int l^2 d\theta, \\ &= lm, \dots\dots\dots (1) \end{aligned}$$

since $\int d\theta = 0$ when AB returns to its initial position.

If B also describes a closed figure of area B , outside or partially outside that traced by A , then, since the line returns to its original position,

$$A - B = lm \dots\dots\dots (2)$$

If B describes a closed curve of area B completely inside that traced by A , then $A - B = lm + \frac{1}{2}l^2 2\pi, \dots\dots\dots (3)$

since AB then makes a complete revolution, so that $\int d\theta = 2\pi$.

The foregoing principle can be applied to the measurement of the area A round which A travels, if the area of the locus of B is known, and if m , the total normal displacement of B , can be determined. It may be more convenient to obtain the total normal displacement n of any other point W on AB . If W is distant a from B , and if dn represents a small normal displacement of W , then

$$dn = dm + ad\theta,$$

a being taken as positive if W is between B and A or on BA produced beyond A , and negative if W is on AB produced beyond B . The results (1), (2), and (3) above now become :

$$A = ln, \dots\dots\dots (4)$$

$$A - B = ln, \dots\dots\dots (5)$$

$$A - B = ln + (\frac{1}{2}l^2 - la)2\pi \dots\dots\dots (6)$$

Amsler's Polar Planimeter.—This instrument (Fig. 252) consists of two bars AB and BC hinged at B. The first carries at A a tracing point, which is guided round the boundary of the area to be measured. C is a stationary point or pole, the bar BC terminating at C with a needle point, which is fixed in the paper and held down by a weight. B is therefore constrained to move along the circumference of a circle of radius CB while the tracing point travels round the area under measurement.

The normal displacement is measured at W by means of a wheel, the plane of which is perpendicular to AB. As AB moves, the wheel partly rotates, and partly slides, over the paper. The amount of rotation measures the total normal displacement: the axial component of the motion causes slip, and does not affect the record.

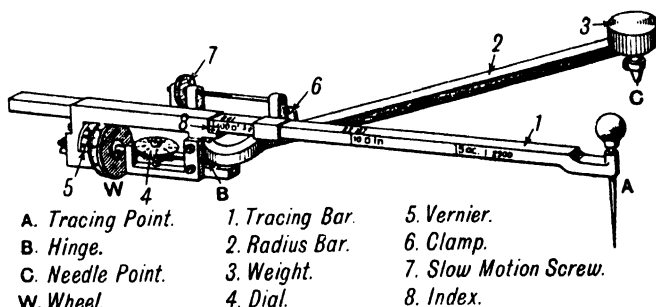


FIG. 252.—AMSLER'S POLAR PLANIMETER.

The wheel is geared to a dial 4, which shows the number of revolutions made by the wheel, ten revolutions of the wheel corresponding to one of the dial. The wheel carries a graduated drum divided into 100 parts, which are subdivided to tenths by a vernier. In the instrument illustrated, the distance between A and B is adjustable, and, on setting the appropriate mark on the tracing arm opposite the index 8 by means of the clamp and tangent screw, readings are obtained in square inches, square centimetres, acres, etc.

Use of the Planimeter.—To measure an area, first adjust the tracing arm so that the result may be given in the desired units. Fix the needle point in the paper outside the area and in a position which will enable the tracing point to reach all parts of the boundary. Mark a point on the boundary from which to start, and place the tracing point there. Read the dial and wheel, or, alternatively, set these to zero. Guide the tracing point in a clockwise direction along the boundary, and, on returning it exactly to the starting-point, again read the dial and wheel. The final reading less the initial reading gives the required area.

In this case, B oscillates along a circular arc, and returns to its

initial position without sweeping out any area, so that, from (4) above,

$$A = ln = l \times 2\pi\rho \times v,$$

where ρ and v are the radius and number of revolutions respectively of the wheel. The graduation of the recording dial is, however, such that the required area is read directly.

If the area is too large to permit of the tracing point traversing the boundary with the pole outside the area, the figure may be divided into parts, which are separately measured. Alternatively, the needle may be fixed inside the boundary. The point B then describes a complete circle within the area traced by A, and, from (6) above,

$$A - B = ln + (\frac{1}{2}l^2 + la)2\pi,$$

the positive sign being used since a is negative.

Denoting the length of the radius arm CB by b , then, since $B = \pi b^2$,

$$\begin{aligned} A &= ln + (\frac{1}{2}l^2 + la + \frac{1}{2}b^2)2\pi, \\ &= ln + \pi r^2, \end{aligned}$$

r being the distance between C and A when the plane of the wheel passes through C (Fig. 253). If A were to travel round the circumference of the circle of radius r , no rotation of the wheel would be caused, and this circle is, in consequence, called the zero circle. Its area forms a constant to be added to the recorded result when the needle is placed within the area to be measured. The values of the constant for various units and scales are engraved on the tracing arm.

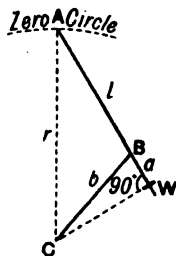


FIG. 253.

Notes.—(1) The accuracy of setting of the tracing arm for any particular unit may be verified by drawing a trial square of known size and measuring its area.

(2) In moving the tracing point round the area under measurement, it should be guided as far as possible by a straight-edge or a French curve. Small errors arising from the tracing point not being maintained throughout on the boundary are compensating, and are rendered negligible by making two or three measurements and adopting the average result.

(3) In the case of areas too large to be measured with one setting of the instrument with the pole outside, it is generally preferred to make the measurement in parts, as a precaution against error in the engraved value of the constant. The constant can, however, be verified once for all by comparison of the result so obtained with that given with the pole within the area.

(4) The plan should be in a horizontal position, not on an inclined drawing board. The surface of the paper on which the wheel rolls must be smooth, but not too highly glazed. Satisfactory results cannot be expected from drawings which have been folded.

Measurement from Field Notes.—The area is divisible into (a) that enclosed by the outside survey lines, (b) the irregular strips between those lines and the boundary. The method of calculating the former depends upon whether the survey has been executed by chain only or by traversing. The areas of the strips are positive or negative, and, since they are divided up by offsets between which the boundary is sensibly straight, are computed as a series of trapezoids.

Area from Chain Survey Notes.—The triangles composing the area within the bounding chain lines are calculated from the lengths of the three sides by the formula,

$$A = \sqrt{s(s-a)(s-b)(s-c)},$$

where a, b, c are the sides, and s is the semi-perimeter.

In computing the areas of the irregular strips, the mean of each successive pair of adjacent offsets is multiplied by the chainage between them, and the results are summed, areas within the extreme survey lines being regarded as negative. The field work should be arranged to obviate difficulty with the corner areas. The best way is to take offsets, such as Aa, Aa' (Fig. 254) at each corner station from both lines meeting there, with such additional measurements as will define the boundary between them. Otherwise, the strips are taken as beginning with the offset from the initial station of each line, and the line is continued to cut the boundary beyond the end station. The separate strips obtained are shown hatched. Small overlaps, shown black, occur at corners where the internal angle exceeds 90° , and a part is omitted at D , where the angle is less than 90° . These

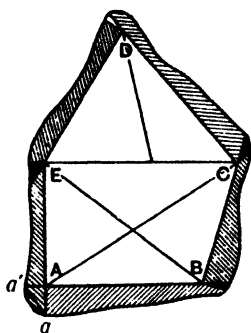


FIG. 254.

will not seriously affect the accuracy of the result, and can be scaled if necessary.

Area from Co-ordinates.—The figure contained by the survey lines of a closed traverse may be so divided up that its area can be computed from the latitudes and departures of the lines. Two methods are used.

1. *By Latitudes and Longitudes.*—The meridian passing through the most westerly station of the survey is to be termed the reference meridian.

The *Longitude* of any survey line is the perpendicular distance of the mid-point of the line from the reference meridian. The longitude of either of the lines meeting at the most westerly station equals half the departure of those lines. A sketch will show that the longitude of any other line may be expressed as the algebraic sum of the longitude of the preceding line plus half the departure of that line plus half its own departure.

Rule: *The area within the lines of a closed traverse survey is the algebraic sum of the products of the latitude of each line by its longitude.*

Since the reference meridian is that of the most westerly station, all longitudes are east or positive quantities. Latitudes are reckoned positive if north, and negative if south, as before.

To test the truth of the rule, let the survey ABCDE (Fig. 255) be considered. Perpendiculars BB', CC' , etc., drawn from each

station to the reference meridian, form a number of figures, one side of each of which is a survey line, another side being a portion of the reference meridian equal to the latitude of that survey line. In general, these figures are trapezoids, but the trapezoids reduce to triangles in the case of the two lines meeting on the reference meridian. The area of each, whether a triangle or trapezoid, is given by the latitude of the line to which it belongs times the longitude L of that line. Now, if from the sum of the two areas CDD'C' and DEE'D', which belong to lines having positive latitudes,

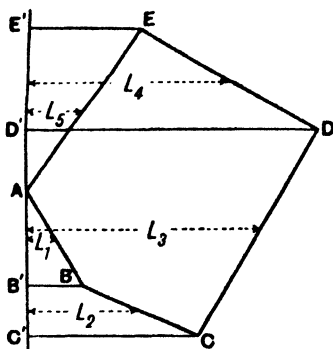


FIG. 255.

we subtract those pertaining to negative latitudes, viz. ABB', BCC'B', and EAE', the remainder is the required area ABCDE.

Since the computation of the longitudes involves halving the departures, it is more convenient and expeditious to omit this step and to work with "double longitudes." The double longitude of either of the lines meeting at the most westerly station equals the departure of the line, and that of any other line is the algebraic sum of the double longitude of the preceding line plus the departure of that line plus its own departure. The algebraic sum of the products of the latitude of each line by its double longitude is then twice the required area.

Before proceeding to the determination of the area, the co-ordinates must be balanced. The calculation is best set out by adding columns to the table of co-ordinates, as in the following numerical example of the survey of Fig. 255. It is desirable, although not essential, that the first line tabulated should be that starting from the most westerly station, and rearrangement of the table may be required to effect this.

Line.	Latitude in Links.	Departure in Links.	Double Longitude.	Latitude \times Double Longitude.	
				+	-
AB	-298	+169	169		50,362
BC	-151	+362	700		105,700
CD	+630	+383	1,445	910,350	
DE	+301	-560	1,268	381,668	
EA	-482	-354	354		170,628

1,292,018 326,690
326,690

Algebraic sum = twice area = 965,328 sq. links.
Area = 482,664 sq. links,
= 4.827 acres.

In this example the positive products happen to exceed the negative products, but it is immaterial whether the algebraic sum is positive or negative. The final division by two is apt to be overlooked, but the seriousness of this error contributes to its detection.

2. *By Departure and Total Latitudes.*—The total latitudes of the various stations, as used in plotting the survey, may be employed for the calculation of the area according to the following :

Rule : *The area within the lines of a closed traverse survey is the algebraic sum of the products of the total latitude of each station by half the algebraic sum of the departure of the two lines meeting at the station.*

Any station may be adopted as the reference point from which to calculate the total latitudes of the others. In the survey ABCDE (Fig. 256), A is used for this purpose, and the total latitudes of the

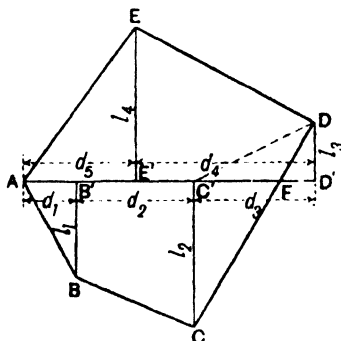


FIG. 256.

stations are shown as the lengths of the perpendiculars dropped from them on the east and west line through A. The traverse is thus divided into triangles and trapezoids, and its area may be expressed as

$$\frac{d_1 l_1}{2} + \frac{d_2 (l_1 + l_2)}{2} + \frac{d_4 (l_3 + l_4)}{2} + \frac{d_5 l_4}{2} + \triangle C'CF - \triangle DFD',$$

but, joining C'D,

$$\begin{aligned} \triangle C'CF - \triangle DFD' &= \triangle C'CD - \triangle DC'D', \\ &= \frac{d_3 l_2}{2} - \frac{d_3 l_3}{2}. \end{aligned}$$

$$\begin{aligned} \therefore \text{Area} &= \frac{d_1 l_1}{2} + \frac{d_2 (l_1 + l_2)}{2} + \frac{d_3 (l_2 - l_3)}{2} + \frac{d_4 (l_3 + l_4)}{2} + \frac{d_5 l_4}{2}, \\ &= \frac{l_1 (d_1 + d_2)}{2} + \frac{l_2 (d_2 + d_3)}{2} + \frac{l_3 (-d_3 + d_4)}{2} + \frac{l_4 (d_4 + d_5)}{2}. \end{aligned}$$

On changing the signs throughout, it will be seen that, with the usual conventions as to signs, the above rule is verified, $l_3 d_3$ being

the only positive product, since l_3 , being north, and d_3 , east, are both positive.

As before, the division by two may be left till the finish if the tabulated products are the total latitude of each station by the algebraic sum of the departures of the lines meeting there.

Working out the same numerical example as before, we have :

Line.	Latitude in Links.	Departure in Links.	Station.	Total Latitude.	Sum of Adjoining Departures.	Products.	
						+	-
AB	-298	+169	A				
BC	-151	+362	B	-298	+531		158,238
CD	+630	+383	C	-449	+745		334,505
DE	+301	-560	D	+181	-177		32,037
EA	-482	-354	E	+482	-914		440,548

Algebraic sum = twice area = 965,328 sq. links.

Area = 482,664 sq. links,
= 4.827 acres.

Since total latitudes are utilised for plotting the survey, this method involves less extra work than the first. A satisfactory check is afforded by making the calculations by both methods.

Note.—The area of the polygon is obtainable independently of the co-ordinates by dividing it into triangles and using the formula,

$$A = \frac{1}{2} ab \sin C,$$

where a and b are adjacent sides, and C is the angle they include. In the case of figures of more than four sides, this method necessitates solution of the interior triangles, and proves very tedious.

Subdivision of an Area into Given Parts from a Point on the Boundary.—Let ABCDEFA (Fig. 257) be a plot of land, and let it be required to cut off a definite area by a line drawn from the point H on the boundary.

Calculate the area of the figure ABCDEFA from the co-ordinates

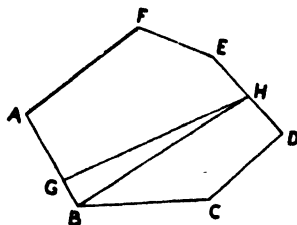


FIG. 257.

and also plot the figure on a fairly large scale. By inspection, or by trial and error on the plotted plan, find the station B so that the area bounded on one side by the line HB is nearer in value to

the given area than that bounded by a line from H to any other station. Compute the length and bearing of HB and the area of the figure HBCDH. Let A be the area required and A' the area of the figure HBCDH. Then, if HG is the line needed for the subdivision, the point G is found from the relation :

$$A - A' = \frac{1}{2} \cdot BG \cdot BH \cdot \sin HBG$$

$$\text{or, } BG = \frac{2(A - A')}{BH \cdot \sin HBG}$$

The length and bearing of the line BH have been computed from the co-ordinates of H and B and, since the bearing of BG is known, the angle HBG is known. Consequently, BG can be computed and the co-ordinates of G found.

Subdivision of an Area into Given Parts by a Line of Given Bearing.

—Let it be required to divide the area ABCDEFGA (Fig. 258) into two parts by a line whose bearing is given.

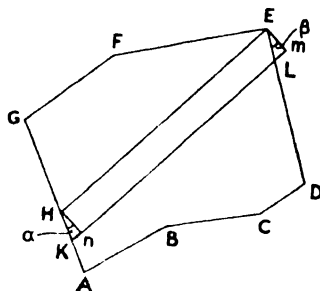


FIG. 258.

Calculate the area of the figure from the co-ordinates and plot it on a fairly large scale. Draw the line EH from one station E, and in the given direction, so that it cuts off an area HEFGH approximately equal to A, the area required. Calculate the bearing and distance of GE. Then, since the bearings of the lines GE, EH, and HG are known, the three angles of the triangle GEH are known and, from these and the computed distance GE, the lengths HE and GH can be calculated. Hence, the co-ordinates of H can be found. Using these co-ordinates, and those of the points E, F, and G, calculate the area of the figure HEFGH. Let A be this area. Then, if LK is the line needed to cut off the area A, we must have :

$$A - A' = \text{Area of figure HKLEH.}$$

From E draw Em perpendicular to EH to meet KL in m , and from H draw Hn perpendicular to KL. Let $Em = Hn = x$ and angle $KHn = \alpha$, and $ELm = \beta$, these angles being known since the bearings of the different lines are known. Then, length $KL = HE - x \cdot \tan \beta + x \cdot \tan \alpha$. Hence,

$$\begin{aligned}
 \text{Area of figure HKLEH} &= \frac{x}{2}(\text{HE} + \text{KL}) \\
 &= \frac{x}{2} \left[2\text{HE} + x(\tan \alpha - \tan \beta) \right] \\
 &= x \cdot \text{HE} + \frac{x^2}{2}(\tan \alpha - \tan \beta).
 \end{aligned}$$

$$\begin{aligned}
 \text{Hence,} \\
 A - A' &= x \cdot \text{HE} + \frac{x^2}{2}(\tan \alpha - \tan \beta).
 \end{aligned}$$

This is a quadratic equation which can be solved for x . Then, having found x , we have :

$$\text{EL} = x \cdot \sec \beta ; \quad \text{HK} = x \cdot \sec \alpha.$$

Hence, the co-ordinates of K and L can be found.

MEASUREMENT OF VOLUME

The reference made at the beginning of the chapter to the sources of error affecting the accuracy of calculated quantities applies with special force to the measurement of volume. The calculations are based upon the results of levelling, and the field work is subject to the usual errors of observation, but an additional and more important source of inaccuracy consists in the deliberate omission to record many of the surface irregularities. Although it would in general be a mere waste of time to measure all the minor undulations, the value of the field work is greatly increased by so setting out the lines of levels that the most faithful record of the surface features may be secured.

The methods of calculation based on the field measurements are not of a precise nature, but involve assumptions as to the geometrical form of the solid which may still further impair the accuracy of the result. It is, however, desirable that the degree of refinement to which both field work and calculations are carried should mutually correspond.

General Methods of Measurement.—The methods of calculation may be classed according as the form of the solid is defined by (a) cross sections, (b) spot levels, (c) contour lines.

Because of its general application, the measurement of earth-work is more particularly referred to below. The unit employed for such quantities is the cubic yard. When the methods are directed to the estimation of the capacity of a reservoir, the results are expressed in millions of gallons.

Measurement from Cross Sections

In this universally applicable method, the total volume is divided into a series of solids by the planes of cross sections. The spacing of

the sections should depend upon the character of the ground and the accuracy required in the measurement. They are generally run at 66 or 100 ft. centres, but sections should also be taken at points of change from excavation to embankment, if these are known, and at places where a marked change of slope occurs either longitudinally or transversely.

Except when the calculations are made directly from the field notes, the sections are plotted, without vertical exaggeration; and, in the case of earthwork measurement, the new surface is represented

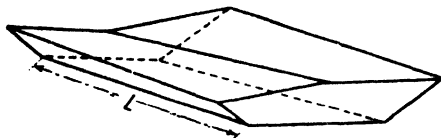


FIG. 259.

on each, its level being obtained from the profile. On drawing the side slopes, the sectional form of the earth to be handled is represented on each section, and, considering two adjacent cross sections, these areas form

the plane ends of a solid of length L equal to the distance between the sections (Fig. 259). The cubic content of each of these solids is to be determined separately, the addition of those in cutting giving the total volume of excavation, and similarly for banking.

In water supply work, measurement of the cubic capacity of a proposed reservoir has to be made for various water levels to determine the surface level and area of the reservoir when impounding a given volume. In this case, therefore, water levels are drawn across the sections, and the volume up to any of them is computed in the same way as for earthwork.

Formulae for Cross Sections.—In the case of the comparatively short cross sections defining the dimensions of the earthwork in road or railway construction, the form of the original ground surface may be determined by only a few field observations. In the process of setting slope pegs (page 463), the particulars obtained are the side widths and side heights, as defined below, the depth of the earthwork on the centre line, with or without that at other points. In cross sections observed by clinometer, the usual data available are the centre depth of earthwork and the slope of the original ground on either side of the centre line.

Formulae for the dimensions of cross sections of cuttings and embankments for the simplest cases are given below, and should be verified by the student as exercises.

- Let b = the constant formation, or "sub-grade," breadth,
- s horizontal to 1 vertical = the side slopes,
- n horizontal to 1 vertical = the transverse slope of the original ground,
- c = the depth of earthwork on the centre line,
- d and d_1 = the side widths, or "half" breadths, i.e. the horizontal

distances from the centre line to the intersections of the side slopes with the original surface,

$$D = d + d_1,$$

h and h_1 = the side heights, i.e. the vertical distances from formation level to the intersections of the side slopes with the original surface,

$$= \frac{d-b/2}{s} \text{ and } \frac{d_1-b/2}{s} \text{ respectively,}$$

w = the formation width of excavation in side-hill sections,

A = the sectional area.

Level Section (Fig. 260).

$$d = d_1 = \frac{b}{2} + sc.$$

$$A = c(b + sc).$$

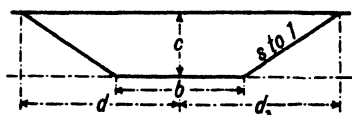


FIG. 260.

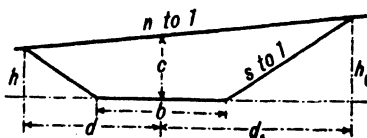


FIG. 261.

Two-Level Section (Fig. 261).

$$d = \left[c + \frac{b}{2s} \right] \left[\frac{ns}{n+s} \right].$$

$$d_1 = \left[c + \frac{b}{2s} \right] \left[\frac{ns}{n-s} \right].$$

$$A = \frac{dd_1}{s} - \frac{b^2}{4s},$$

$$= shh_1 + \frac{b}{2}(h + h_1),$$

the formulæ given for the areas of three-level sections also being available.

Three-Level Section (Fig. 262).

$$d = \left[c + \frac{b}{2s} \right] \left[\frac{ns}{n+s} \right].$$

$$d_1 = \left[c + \frac{b}{2s} \right] \left[\frac{n_1s}{n_1-s} \right].$$

In this case the expression for d or d_1 may apply to both side widths, according as the ground falls or rises from the centre to both sides.

$$A = \frac{D}{2} \left[c + \frac{b}{2s} \right] - \frac{b^2}{4s},$$

$$= \frac{cD}{2} + \frac{b}{4}(h+h_1).$$

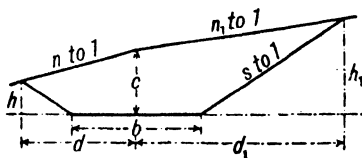


FIG. 262.

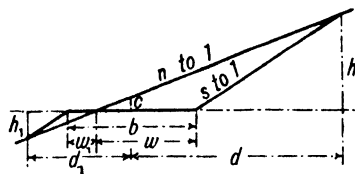


FIG. 263.

Side-hill Two-Level Section (Fig. 263).

The centre line being in excavation, then for the excavation,

$$w = \frac{b}{2} + nc.$$

$$d = \left[c + \frac{b}{2s} \right] \left[\frac{ns}{n-s} \right],$$

$$= \frac{b}{2} + \frac{ws}{n-s}.$$

$$A = \frac{wh}{2} = \frac{w^2}{2(n-s)}.$$

For the embankment, $w_1 = (b-w)$ must be substituted for w , and the first formula for side width becomes

$$d_1 = \left[\frac{b}{2s} - c \right] \left[\frac{ns}{n-s} \right].$$

When the centre line is in embankment, $w = (\frac{b}{2} - nc)$, and the formulæ again apply.

The Prismoid.—To calculate the volumes of the solids between sections, it must be assumed that they have some geometrical form; and, since they most nearly take the form of prismoids, they are considered as such.

The *Prismoid* may be defined as a solid having its end faces in parallel planes and consisting of any two polygons, not necessarily of the same number of sides, the longitudinal faces being plane surfaces extending between the end planes.

The longitudinal or side faces can be regarded as generated by

a straight line extending between the end polygons, and they take the form of triangles, parallelograms, or trapezia having their vertices at those of the end figures.

A prismoid can be divided up into a series of prisms, wedges, and pyramids having a common length equal to the perpendicular distance between the parallel end faces. The performance of such a division may require the temporary addition to the prismoid of one or more pyramids of the same length.

Notes.—(1) The prism forms the special case of the prismoid in which the end polygons are equal, the side faces being parallelograms. If one of the ends degenerates to a line, and the other is a parallelogram, we have the wedge, the side faces being triangles and parallelograms. If one end is a point, the other being any polygon, the solid becomes a pyramid, and the side faces are triangles.

(2) the division of a prismoid into these solids of equal length is sometimes troublesome to follow on a sketch, and it will suffice to notice the simple case of Fig. 264, in which ABCD and EFGH are the parallel end faces, a distance L apart. The bottom longitudinal face BCGF is a rectangle, the sides faces ABFE and CDHG are trapezia, while the upper longitudinal faces ADH and AHE are triangles. By drawing JK on the end face EFGH to make JFGK = ABCD, a prism is formed with these end faces. To divide up the solid lying above the plane ADKJ, let HL be drawn parallel to KJ to meet FE produced at L. HL is in the plane of ADH, and LEH forms the base of a pyramid having its apex at A. The addition of this pyramid to the original prismoid makes the upper face one plane surface ADHL, and the solid having for its ends the area LJKH and the line AD can be split up into a wedge of base JKHM, obtained by drawing JM parallel to KH, and a pyramid JMLA. Each of the constituent figures has the same length L as the prismoid. The sum of their volumes, less that of the added pyramid, is the volume of the prismoid, and the algebraic sums of their areas at each end equal the corresponding end areas of the prismoid.

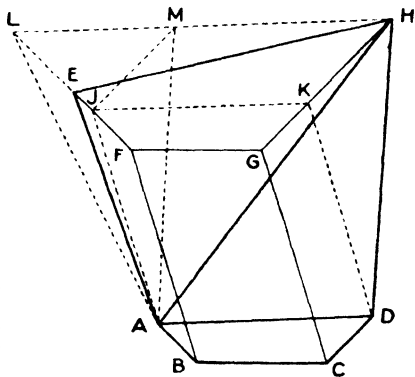


FIG. 264.

The circumstance that the prismoid can be divided up into prisms, wedges, and pyramids of the same length affords a simple method of deducing a formula for its volume.

Let L = the length of the prismoid, perpendicular to the parallel end planes,
 A_1 and A_2 = the areas of those ends,
 M = the mid-area, i.e. the cross sectional area in a plane parallel to the end planes and midway between them,
 V = the volume of the prismoid,

a_1, a_2, m, v = the corresponding quantities for any prism, wedge, or pyramid, a_1 being regarded as the basal area in the latter two forms.

Then, in the case of a prism, $a_1 = a_2 = m$;

„ „ wedge, $a_2 = 0$ (a line), and $m = \frac{1}{2}a_1$;

„ „ pyramid, $a_2 = 0$ (a point), and $m = \frac{1}{4}a_1$.

For each prism, $v = La_1 = \frac{L}{6}(6a_1) = \frac{L}{6}(a_1 + a_2 + 4m)$.

„ wedge, $v = \frac{1}{2}La_1 = \frac{L}{6}(3a_1) = \frac{L}{6}(a_1 + a_2 + 4m)$.

„ pyramid, $v = \frac{1}{3}La_1 = \frac{L}{6}(2a_1) = \frac{L}{6}(a_1 + a_2 + 4m)$.

The volume of each of the constituent parts of a prismoid can thus be expressed in the same terms, and the formula must therefore be applicable to any collection of those components. With the notation applied to the prismoid, we therefore have the *Prismoidal Formula*,

$$V = \frac{L}{6}(A_1 + A_2 + 4M).$$

It must be distinctly understood that M is not the average of the end areas A_1 and A_2 , except in the special case where the solid is composed of prisms and wedges only.

Newton's Proof of the Prismoidal Formula.—A more straightforward proof was enunciated by Newton.

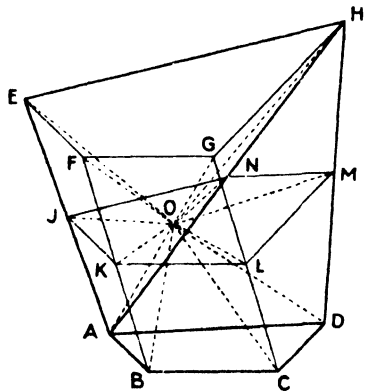


FIG. 265.

In Fig. 265, let JKLMN represent the section midway between the end faces ABCD and EFGH and parallel to them. Take any point O in the plane of the mid-section, and join O to the vertices of both end polygons. The prismoid is thus divided into a number of pyramids, each having its apex at O, and the bases of these pyramids form the end and side faces of the prismoid.

Denoting the end areas by A_1 and A_2 , and the length of the prismoid by L , the volumes of the

pyramids based on the end faces are, respectively, $\frac{1}{3}LA_1$ and $\frac{1}{3}LA_2$.

To express the volume of the pyramids based on the longitudinal faces, consider, say, pyramid OABFE, and let the perpendicular distance of O from JK be h , then

$$\begin{aligned} \text{volume of pyramid OABFE} &= \frac{1}{3}ABFE \times h \\ &= \frac{1}{3}L \times JK \times h = \frac{1}{3}L \times 2 \triangle OJK. \end{aligned}$$

In the same manner, the volume of pyramid OAEH = $\frac{1}{3}L \times 2 \triangle OJN$, and so on for the others, so that

total volume of the lateral pyramids = $\frac{1}{3}L \times 2 \text{ area JKLMN} = \frac{2}{3}LM$.

Summing, therefore, we have the volume of the prismoid,

$$\begin{aligned} V &= \frac{1}{3}LA_1 + \frac{1}{3}LA_2 + \frac{2}{3}LM, \\ &= \frac{L}{6}(A_1 + A_2 + 4M). \end{aligned}$$

Comparison of Earth Solid and Prismoid.—The above formula is strictly correct for any prismoid, but it is desirable to compare the earth solid lying between adjacent cross sections with the prismoid as defined.

(a) The ends of the prismoid are in parallel planes.

This requirement is met in the earth solid provided the centre line is straight. Since the cross sections are taken at right angles to the centre line, an error may be introduced on curves, owing to the non-parallelism of the ends, but this can be allowed for (page 396).

(b) The longitudinal faces of the prismoid are planes.

In the earth solid, the new surfaces (the formation surface and side slopes) are frequently plane, and in the majority of cases where they are not, the departure from a plane is negligible in the distance between cross sections. The original ground surface does not generally fulfil this condition, but will more nearly do so the greater the care taken in spacing the cross sections so that no marked change in the character of the ground occurs between them. It may be shown that the prismoidal formula is strictly applicable to the case where the longitudinal faces of the solid, instead of being plane surfaces, are hyperbolic paraboloids. The original ground surface may closely resemble these ruled surfaces if it exhibits gradual changes of transverse slope between cross sections.

Calculation by Prismoidal Formula.—Although the solid to be measured may differ considerably from a prismoid, the prismoidal formula is generally accepted as correct for earthwork calculation. It is, however, seldom used in a direct manner for ordinary measurements because of the difficulty in estimating the mid-areas. These could, of course, be obtained by running mid-sections, but at the cost of doubling the field work. Their values can be calculated comparatively simply when the sections are defined by two or three levels by taking the linear dimensions of the mid-area as the mean of the corresponding dimensions of the end areas, but with irregular sections the calculations are unwarrantably tedious.

If the cross sections are at equal intervals, the difficulty may be overcome by treating every alternate sectional area as a mid-area.

Thus, if L = the distance between the sections,

$A_1, A_2 \dots A_n$ = their areas,

the volume of the first prismoid of length $2L = \frac{2L}{6}(A_1 + A_3 + 4A_2)$,

„ „ second „ „ „ $= \frac{2L}{6}(A_3 + A_5 + 4A_4)$,

„ „ last „ „ „ $= \frac{2L}{6}(A_{n-2} + A_n + 4A_{n-1})$.

Summing up, we have the total volume,

$$V = \frac{L}{3} \left[A_1 + 4A_2 + 2A_3 + 4A_4 + \dots + 2A_{n-2} + 4A_{n-1} + A_n \right]$$

which is Simpson's rule for volumes, an odd number of sections being required for its application.

The solids of length $2L$ may, however, differ very considerably from true prismoids, and this method does not yield so good results as individual treatment of the solids between adjacent cross sections.

The Method of End Areas.—The difficulty of applying the prismoidal formula to solids of which the mid-area has not been directly measured is avoided in practice by the use of the end areas rule, which states that the volume of a prismoid is given by

$$V = L \left(\frac{A_1 + A_2}{2} \right).$$

This formula, although very convenient, is correct only if the mid-area is the mean of the end areas A_1 and A_2 , i.e. if the prismoid is composed of prisms and wedges only. Since the mid-area of a pyramid is half the average area of the ends, the volume of the general prismoid is overestimated by the formula, but, since the earth solid is not in general exactly represented by a prismoid, the method of end areas may be accepted as sufficiently accurate for most practical purposes, and is that almost exclusively used.

The error made in estimating the volume of a prismoid by this method may be readily ascertained for the simple cases dealt with on pages 389 and 390. For the case in which the ground lines of the sections are level (Fig. 260), the investigation is as follows:

Let L = the distance between adjacent sections,

b = the constant formation breadth,

s hor. to 1 vert. = the side slopes,

c, c' = the centre heights of the sections,

A, A' = their areas.

Since $A = c(b + sc)$, and $A' = c'(b + sc')$,

by end areas, $V = L \left[\frac{bc}{2} + \frac{bc'}{2} + \frac{sc^2}{2} + \frac{sc'^2}{2} \right]$.

Now the mid-area centre height $= \frac{(c + c')}{2}$,

$$\therefore M = \frac{(c+c')}{2} \left[\frac{b+s(c+c')}{2} \right], \text{ so that, by prismoidal formula,}$$

$$V = \frac{L}{6} \left[c(b+sc) + c'(b+sc') + 2(c+c')b + \frac{s(c+c')}{2} \right],$$

$$= L \left[\frac{bc}{2} + \frac{bc'}{2} + \frac{sc^2}{3} + \frac{sc'^2}{3} + \frac{sc c'}{3} \right].$$

On subtracting,

$$\text{Volume by end areas} - \text{Volume by prismoidal formula} = \frac{Ls}{6} (c-c')^2.$$

This quantity is always positive for level sections, and its form shows that a somewhat serious overestimation may result by averaging end areas in cases where adjacent sections differ considerably in area, as, for example, towards the ends of excavations or embankments.

Prismoidal Correction.—If in special cases it is required to obtain a closer approximation than is given by the above method, the precision of the measurement may be increased without undue labour by the application of a correction to the result given by the end area method. The difference between the volume as calculated and that which would be obtained by use of the prismoidal formula is termed the *Prismoidal Correction*, and, if it is known in any case, its application to the approximate result gives the volume as it would be obtained from the prismoidal formula with less trouble than is involved by the direct use of the latter. Since the end area method of calculation overestimates the volume of a prismoid, the correction for the prismoid is always subtractive, but in the case of the actual earth solid the correction, although usually subtractive, may fall to be added to the end area result.

The formula for the correction depends upon the number of levels defining the end areas, and, to deduce it once for all in any case, it is necessary to obtain the expression for the mid-area in order to formulate the volume by the prismoidal method. If the areas are of simple outline, this is accomplished by averaging corresponding linear dimensions, as above. Irregular sectional forms are treated by substituting for them equal areas of simple outline, from the dimensions of which those of a figure corresponding to the mid-area are obtained with more or less accuracy. For the substitution, level, two-level, or three-level equivalent sections are employed according to the degree of accuracy required: further refinement in equalising is of little practical importance.

Formulae for the Prismoidal Correction.—With the previous notation (page 388), unaccented and accented symbols referring to adjacent sections, the following expressions are obtained in the above manner, and should be verified by the student.

Level Sections (Fig. 260).

$$\text{P.C.} = \frac{Ls}{6}(c-c')^2.$$

Two-Level Sections (Fig. 261).

$$\text{P.C.} = \frac{L}{6s}(d-d')(d_1-d_1').$$

Three-Level Sections (Fig. 262).

$$\text{P.C.} = \frac{L}{12}(D-D')(c-c').$$

Side-hill Two-Level Sections (Fig. 263).

The centre line being in excavation,

$$\text{P.C. for the excavation} = \frac{L}{12s}(w-w')(d-d').$$

$$\text{P.C. for the embankment} = \frac{L}{12s}(w_1-w_1')(d_1-d_1').$$

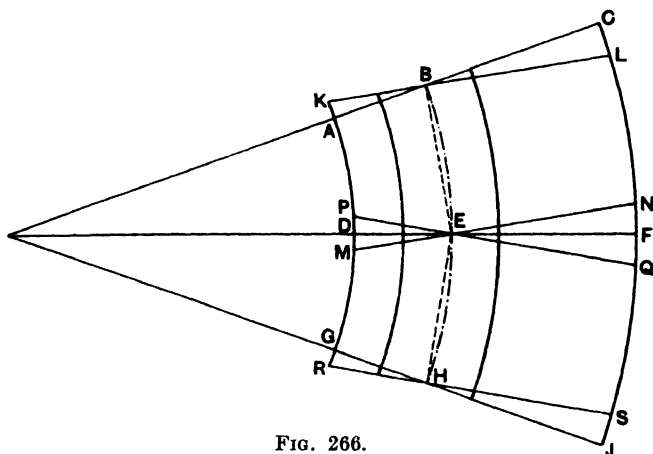


FIG. 266.

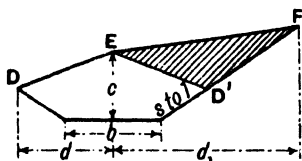


FIG. 267.

Curvature Correction.—Cross sections on curves are run in radial lines, and consequently the earth solids between them do not have parallel end planes. In computing the volumes of those solids, the common practice is to employ the

usual methods, treating the ends as parallel to each other and normal to the chord, and, if circumstances warrant, to apply a correction for curvature.

In Fig. 266, B, E, and H are successive centre line pegs on a

curve, and AC, DF, and GJ are lines of cross sections. The solid from B to E is represented in plan by ADFC, but the volume calculated is that of the solid KMNL, the cross-sectional area normal to the chord being for all practical purposes equivalent to that normal to the tangent. Evidently the calculated volume is too small by the volumes represented by CBL and FEN and too great by those represented by ABK and DEM. When the cross-sectional area is symmetrical about the centre line, these wedge-shaped masses practically balance, and no curvature correction is required. The more unsymmetrical the sectional area, the greater is the value of the correction, which falls to be applied positively or negatively to the calculated volume according as the greater half-breadth is on the convex or the concave side of the curve. It is usual to compute the amount of correction per station, that for station E being (volume NEQ—volume MEP).

In Fig. 267, representing cross section DF, if ED' be drawn symmetrically with ED, the asymmetry of the sectional area is represented by ED'F, and the volume swept out by this area in turning through the angle NEQ also equals the required correction for station E. With the previous notation, area ED'F is expressed by

$\left(\frac{d_1-d}{2}\right) \left(c+\frac{b}{2s}\right)$. The solid swept out by this area, and represented

in plan by ENQ, is, by Pappus' theorem, the product of the area by the length of the path of its centroid. The horizontal distance from the vertical through E to the centroid of ED'F is $\frac{1}{3}(d_1+d)$, and the

travel of the centroid is $\frac{L}{3R}(d_1+d)$, where L is the distance between

the stations and R is the radius of the curve. Otherwise, the solid ENQ may be treated as a truncated triangular prism. The edges normal to the right section EF have lengths at E, D', and F of 0,

$\frac{Ld}{R}$, and $\frac{Ld_1}{R}$, respectively, so that the average length of the prism is

$\frac{L}{3R}(d_1+d)$. By either method the required volume, or curvature

correction for station E, equals

$$\frac{L}{6R}(d_1^2-d^2) \left(c+\frac{b}{2s}\right).$$

This expression obtains for stations between chords of equal length. The correction at a tangent point has half the above value, and that at a station between chords of different lengths is determined by proportion. The formula, deduced for a three-level section, is equally applicable to two-level ground, and may safely be used for irregular sections if the half breadths employed are those of an equivalent three-level section. In the case of a two-level side-hill

section (Fig. 263), in which the width of excavation or embankment at formation level is w , the correction for the excavation or embankment is similarly found to have the value,

$$\begin{aligned} & \frac{Lwh}{6R}(d+b-w), \\ &= \frac{Lw}{6Rs}\left(d-\frac{2}{2}\right)\left(d+b-w\right). \end{aligned}$$

Measurement from Spot Levels

This method of measurement is sometimes applied to large excavations. The field work consists in dividing up the site of the work into a number of equal triangles, squares, or rectangles and observing the original surface level at each corner of those figures. After completion of the earthwork, or during its progress, the lines dividing up the area are again set out, and the levelling is repeated on the new surface, the differences of level thus determined representing the depths of earthwork at a number of points of known position. In estimating the volume of proposed earthwork, the levels of the new surface are obtained from the drawings. The differences of level are regarded as the lengths of the sides of a number of vertical truncated prisms, *i.e.* prisms in which the basal planes are not parallel, the horizontal area of each of which is known. The volume of each prism is obtained as the product of the area of the right section by the average length of the vertical edges.

Division of Ground.—The size of the unit areas into which the site should be divided will depend upon the degree of accuracy required in the measurement and upon the character of the ground.

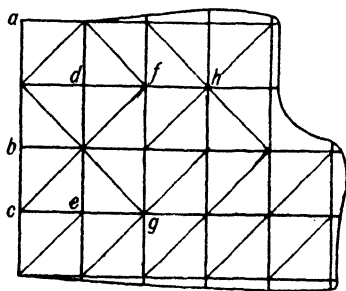


FIG. 268.

The aim should be to have the partial areas of such dimensions that the assumption that the ground surface within each is a plane is not greatly in error, and it is therefore desirable to reduce their size on irregular ground.

Good results are obtained by dividing up the site into a series of squares or rectangles and regarding each as divided by a diagonal into two triangles, the diagonal which more nearly lies along the surface of the ground being selected and noted (Fig. 268). The surface within each rectangular area is then treated as consisting of two triangular planes, and the volume is computed from triangular prisms. This system involves the same amount of levelling as the

method of rectangles, but the freedom of choice in each rectangle as to the pair of planes which more nearly coincides with the ground surface necessarily tends to greater accuracy.

It most frequently happens that the boundary of the work is irregular in plan, in which case there are a number of unequal partial areas adjoining it. These figures approximate in shape to triangles or trapezoids, and may with sufficient accuracy be treated as such. Their areas are obtained from measurements in the field or by scaling from the plan, and, since they are unequal, the volume of each prism must be separately computed.

Method of Calculation.—In the calculation of the volume contained within the limits of the dividing lines, although each triangular prism may be computed separately, the fact that all have the same horizontal area suggests a method of reducing the arithmetical work. It will be seen from Fig. 268 that the corner *a* belongs to one triangle only, and the difference of level between the new and the original surface at *a* is therefore employed only once in the calculations. That at *b* is used twice, that at *c* three times, and so on to that at *h*, which belongs to eight prisms, eight being the maximum number of times a particular height can be employed.

If then *A* = the area of one triangle in square feet,
 h_1 = the total sum, in feet, of all heights used once,
 h_2 = " " " " twice, etc.,
 the total volume, in cubic yards, within the extreme dividing lines is given by

$$V = \frac{A}{27} \left[\frac{h_1 + 2h_2 + 3h_3 + 4h_4 + 5h_5 + 6h_6 + 7h_7 + 8h_8}{3} \right],$$

to which falls to be added that contained within the surrounding irregular strips.

Note.—If certain parts of the ground are of a much rougher character than the rest, a few extra levels may be observed at those places, instead of reducing the size of the triangles throughout. The additional levels are located by measurement of tie lines from the intersections of the reference lines. The original prisms are thus split up into a number of smaller, and generally unequal, triangular prisms of known horizontal area, the volumes of which are computed separately.

The division of a solid into a number of truncated prisms, not necessarily of constant area, affords a convenient method of obtaining the volume in cases like the following.

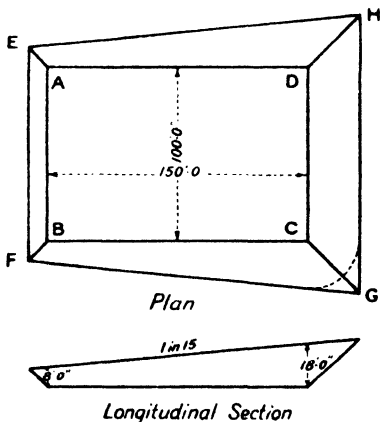


FIG. 269.

Example.—Calculate the volume of the excavation shown in Fig. 289, the side slopes being $1\frac{1}{2}$ horizontal to 1 vertical, and the original ground surface sloping at 1 in 15 in the direction of the centre line of the excavation.

The solid in question, although bounded by planes, is not a prismoid, since no two faces are parallel. The volume will be calculated as that of the vertical truncated prisms appearing in plan as ABCD, ABFE, DCGH, ADHE, and BCGF, of which the lengths of the vertical edges are known.

The horizontal breadths of the slopes on the left and right are computed as 10.9 ft. and 30 ft. respectively, so that we have :

Prism ABCD.

$$\text{Area} = 150 \times 100 = 15,000 \text{ sq. ft.}$$

$$\text{Average height} = \frac{1}{4}(8 + 8 + 18 + 18) = 13 \text{ ft.}$$

$$\text{Volume} = 15,000 \times 13 =$$

$$\begin{array}{r} \text{cub. ft.} \\ 195,000 \end{array}$$

Prism ABFE.

$$\text{Area} = 110.9 \times 10.9 = 1,209 \text{ sq. ft.}$$

$$\text{Average height} = \frac{1}{4}(0 + 0 + 8 + 8) = 4 \text{ ft.}$$

$$\text{Volume} = 1,209 \times 4 =$$

$$4,836$$

Prism DCGH.

$$\text{Area} = 130 \times 30 = 3,900 \text{ sq. ft.}$$

$$\text{Average height} = \frac{1}{4}(0 + 0 + 18 + 18) = 9 \text{ ft.}$$

$$\text{Volume} = 3,900 \times 9 =$$

$$35,100$$

Prisms ADHE and BCGF.

$$\text{Area} = 2 \left[(150 + 40.9) \times \frac{40.9}{2} - \frac{10.9^2}{2} - \frac{30^2}{2} \right] = 6,789 \text{ sq. ft.}$$

$$\text{Average height} = \frac{1}{4}(0 + 0 + 8 + 18) = 6.5 \text{ ft.}$$

$$\text{Volume} = 6,789 \times 6.5 =$$

$$44,129$$

$$\begin{array}{r} \text{Total Volume} = \\ 279,065 \\ = 10,336 \text{ cub. yd.} \end{array}$$

Notes.—(1) The solid may be divided in other ways, and the reader will find it instructive to verify the above result by a different treatment.

(2) It frequently happens, especially in embankments, that the side slope planes are not continued to intersections at the corners, but, as indicated by the dotted curve near C, the earth slope is rotated about the corner to generate a quarter cone there. Unless the original ground surface is horizontal, the bases of these partial cones are not normal to the axis, and the exact calculation of their volumes is unwarrantably troublesome. It is usually sufficient to estimate them as one-third the area in plan of the base times the axial height at the corner.

Measurement from Contour Lines

Rough estimates of volume may be made by treatment of the contour lines of the solid to be measured. In principle, this is no doubt the ideal method, and, if a highly accurate contoured plan were available with a contour interval sufficiently small that full particulars of the irregularities of the solid could be obtained, it would lead to a more precise result than the methods previously considered. Practically, however, this degree of accuracy is not realised because of the trouble of locating contours with an interval small enough to record the minor features of the ground. In dealing with contour lines, one must assume that the surface of the ground slopes uniformly from one contour to the next, and in most cases

this assumption will be incorrect, the resulting error depending upon the vertical interval used as well as upon the character of the ground. To make an estimate of any practical value, the contours should not have a greater vertical interval than 5 ft. on ground of average character, but, if the surface is very irregular, an interval as small as 2 ft. would be required. The method therefore cannot compare with cross sectioning in point of convenience, and is not much used in practice except in the determination of the capacity of reservoirs or the measurement of subaqueous excavation, but even in these cases the method of cross sections is often preferred.

First Case.—The problem is a simple one in reservoir work and in earthwork when the made surface is level, for the new surface is then parallel to the equidistant level surfaces which define the contour lines.

To consider this case, let it be desired to determine the capacity of the reservoir shown in Fig. 270 when the water level is 165.0. The outline of the water surface is first obtained by interpolating

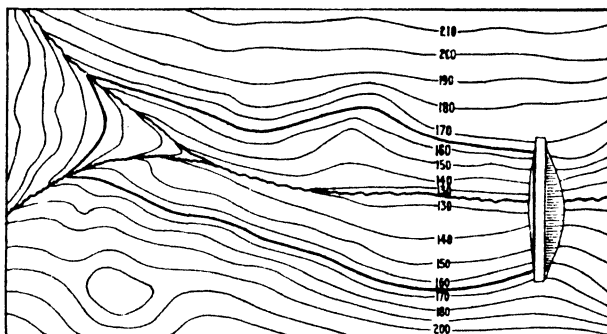


FIG. 270.

the 165 contour. The volume is measured by regarding it as being divided up into a number of horizontal slices by the contour planes. The depths of these are known, and their end areas are obtained from the plan by any of the usual methods, the most expeditious and accurate being by planimeter. The area measured in each case is, of course, the whole area lying within a contour line and not that of the strip between two adjacent contour lines.

The nature of the data is precisely the same as in cross section work, the contour interval corresponding to the distance between cross sections, and the volume may be calculated either by the prismoidal formula, or by the end area method. In using the prismoidal formula, every second area may be treated as a mid area, or the mid-areas may be measured from contour lines interpolated midway between each original pair of contours. The latter method is

recommended when the slopes are flat, so that there is a considerable difference between the areas within successive contour lines. For most purposes, however, the method of end areas yields sufficiently good results.

In connection with the location, design, and operation of reservoirs, it is not sufficient to know the storage capacity for one particular surface level. The volumes in the reservoir for various water levels, such as the contour elevations, should be determined. If these computed volumes are plotted against the corresponding surface levels, the resulting curve is available for reading off the volume up to any water level or for finding the level corresponding to any given volume.

Second Case.—The more general case in earthwork is that in which the ground is not brought to a level surface.

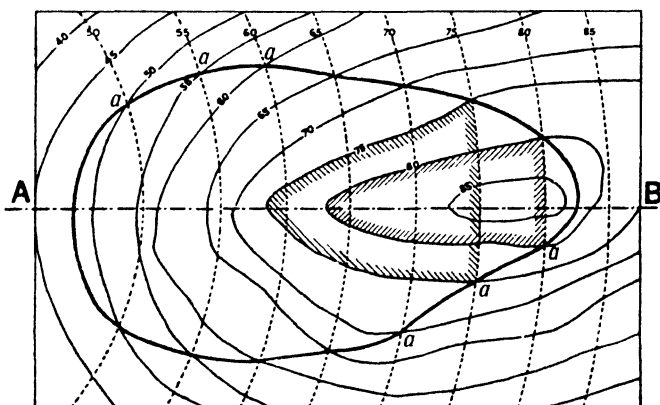
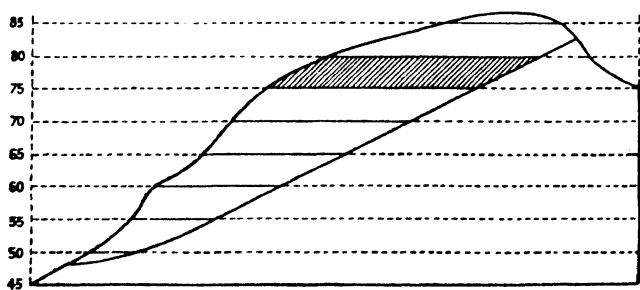


FIG. 271.



Section AB

FIG. 272.

In Fig. 271, let the full lines denote the contours of the original surface, and the dotted lines those of the proposed new surface.

By joining up the intersections *a* of original and new contours of equal value, the line in which the new surface cuts the original is obtained: within this line excavation is necessary, the surrounding parts shown being in embankment. The methods of measurement may be considered with reference to the excavation, the same methods being applicable to embankments.

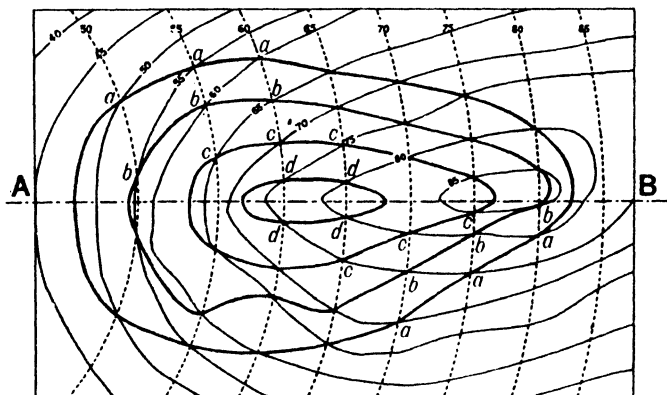
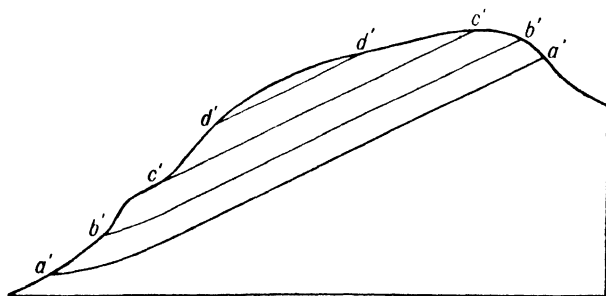


FIG. 273.



Section AB

FIG. 274.

First Method.—It will be seen from the section (Fig. 272) that the contour planes divide the solid to be measured into a series of horizontal layers. The end areas of these can be obtained by planimeter from the plan, those for the strip between the 75 and 80 contours, which is hatched in section, being shown shaded in plan. It frequently happens that an original contour line closes on itself without intersecting the equivalent contour of the new surface, as is the case with the 85 contour in the diagram, and the area enclosed by it is that to be measured. If necessary, a sketch

section will usually clear away any doubt as to the areas required. The required volume is then obtained by applying the method of end areas to each layer.

Second Method.—Instead of dividing the solid by the contour surfaces, it may be divided into layers parallel to the new surface (Figs. 273 and 274).

Just as *aaa* is the line along which the depth of excavation is zero, the depth will be exactly 5 ft. along the line *bbb* got by joining up the intersections of the original contours with new ones of 5 ft. lower elevation. The total areas contained within *aaa* and *bbb* are the horizontal projections of the sloping surfaces which appear in section as the lines *a'a'* and *b'b'*, and the mean of those two areas times the *vertical* distance between them gives approximately the volume of the lowest strip. By joining the intersections of the original contours with the new ones 10 ft. lower, the line *ccc* of 10 ft. excavation is obtained, *ddd* being similarly derived as the line along which the cut is 15 ft., and the areas within these lines are used in the same manner. By inspection of the plan, the greatest depth of excavation appears to be about 17 ft., so that the highest strip has a vertical depth of 2 ft., the same method being applied to it as to the others, since, in this case, it more nearly resembles a wedge than a pyramid.

This method usually involves less measurement of areas than the first, but has the disadvantage that the areas to be measured have to be specially traced out, and this may necessitate the interpolation of intermediate contours.

THE MASS DIAGRAM

In the planning and execution of earthwork certain problems arise which are most simply studied by reference to what is termed a mass diagram.

Labour in earthwork may be analysed under four heads: (a) Loosening; (b) Loading; (c) Hauling; (d) Depositing. For a given class of plant, the unit cost of items (a), (b), and (d) depends almost entirely upon the character of the material, but that of (c) is a function both of the weight of the material and of the varying distance from the working face of the excavation to the tip end. Haulage is the most variable item in the cost of earthwork, and in cases where more than one scheme of distribution from excavation to embankment is possible, it is desirable to be able to compare different projects as regards economy of haulage. This may be accomplished with ample accuracy by means of the mass or haul diagram.

Definitions.—*Haul Distance* (*d*) is the distance at any time from the working face of an excavation to the tip end of the embankment formed from it.

Average Haul Distance (D) is the distance from the centre of gravity of a cutting to that of the tipped material.

Haul is the sum of the products of each load by its haul distance = $\Sigma vd = VD$, where V is the total volume of an excavation.

Change of Volume.—Since excavating involves the loosening and breaking up of the material, the volume available for the formation of an embankment is greater than that measured *in situ* in the unworked excavation. On a rough average, this increase may be estimated at 20 per cent for earth. When banked, however, the material commences to shrink, and after the lapse of a year or two will be found to occupy less space than it did originally in the excavation. The amount of shrinkage depends on several factors, but for the present purpose may be taken as 10 per cent for all earths. On the other hand, excavated rock is permanently swelled by about 50 per cent. Change of volume does not influence ordinary measurements, which should always be based upon original volumes of excavation.

Construction of the Mass Diagram.—The mass diagram is a curve plotted on a distance base, the ordinate at any point of which represents the algebraic sum up to that point of the volumes

Distance. Ft.	Volume. Cub. Yd.		Total Volume. Cub. Yd.
	Cut +	Bank. —	
0			0
100	490		+ 490
200	927		+ 1,417
300	982		+ 2,399
380	279		+ 2,678
400		31	+ 2,647
500		226	+ 2,421
600		654	+ 1,767
700		1,160	+ 607
800		933	— 326
831		92	— 418
900	220		— 198
1,000	428		+ 230

of cuttings and banks from the start of the earthwork or from any arbitrary point. In obtaining the algebraic sum, the usual convention is to consider cuttings plus and embankments minus. The mass diagram is therefore simply an integral or sum curve of the volumes of the several cuttings and banks. When excavation and banking occur on the same section, as in side-hill work, their difference only is used in the summation, the sign being that of the greater volume.

The construction and properties of the diagram are best followed from a simple example. In Fig. 275 let l represent a longitudinal section of, say, a railway siding. The quantities required for plotting the mass curve are tabulated on page 405.

In the example it is assumed that the nature of the material is unknown, and no allowance is made for change of volume. In plotting, the curve should be placed directly above or below the longitudinal section, the same horizontal scale being used for each. Positive total volumes are plotted above, and negative quantities below, the base line, and the ends of the ordinates are joined by a smooth curve, the resulting diagram being as in 2, Fig. 275.

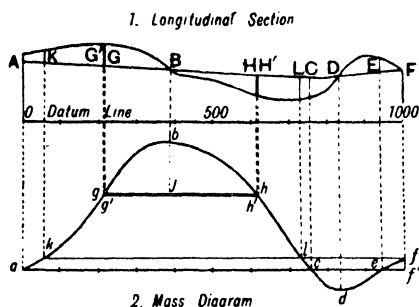


FIG. 275.

✓ **Characteristics of the Mass Diagram.**—With the sign convention adopted :

(1) Upward slope of the curve in the direction of the algebraic summation indicates excavation. Downward slope indicates embankment.

(2) A maximum point occurs at the end of an excavation ; a minimum point, at the end of an embankment.

(3) The vertical distance between a maximum point and the next forward minimum point represents the whole volume of an embankment ; that between a minimum and the next forward maximum point, the whole volume of a cutting.

(4) The vertical distance between two points on the curve which have no maximum or minimum point between them represents the volume of earthwork between their chainages.

(5) Between any two points in which the curve cuts the base line the volume of excavation equals that of embankment, since the algebraic sum of the quantities between such points is zero. The points a and c , for example, show, on being projected to A and C , that the earthwork is balanced between A and C , i.e. the material excavated from AB would suffice to form the embankment up to the point C . There is also balance from E to C .

(6) Any horizontal line intersecting the mass curve similarly serves to exhibit lengths over which cutting and banking are equalised. Thus, gh is a balancing line, the cut from G to B just filling from B to H , the volume moved being represented by bj .

(7) When the loop of the mass curve cut off by a balancing line lies above that line, the excavated material must be hauled forward, i.e. in the direction of summation of the volumes. When the loop lies below, the direction of haul is backward.

(8) The length of balancing line intercepted by a loop of the curve represents the maximum haul distance in that section. Thus, taking the base line as the balancing line, the greatest haul distance involved in disposing of excavation AB is $ac = AC$, so that no material should be hauled past C . In general, the haul distance increases from zero at B to this maximum, and its value at any stage of the work is given by the length of a horizontal line intercepted within the loop. Thus, gh is the haul distance when the face of the excavation is at G .

(9) The area bounded by a loop of the mass curve and a balancing line measures the haul in that section. To take the case of area $abca$, since haul = $\sum vl$, consider a small volume or load, v , initially situated at GG' , and whose final position in embankment is HH' , obtained from the horizontals gh and $g'h'$. From (4) above, the vertical distance between gh and $g'h' = v$, and therefore area $ghh'g' =$ the haul of v . The summation of all such small areas = the area of the loop = the haul involved in transferring cut AB to bank BC . Similarly, area $cde =$ the haul from E to C . Regard must be paid to the scale. If the horizontal scale is 1 in. = x ft., and the vertical scale is 1 in. = y cub. yd., an area of n square inches represents a haul of nxy cub. yd. ft.

(10) The haul over any length is a minimum when the balancing line is so situated that the sum of all areas cut off by it, without regard to sign, is a minimum.

✓ **Use of the Mass Diagram.**—The exact interpretation of a mass diagram is entirely dependent upon the balancing line, each position of which exhibits a possible method of distributing the excavated material, and the selection of the most economical scheme is made by comparing those shown by various balancing lines. Par. (10) above is an important guide, but it most frequently happens that the condition of minimum haul necessitates the wasting or spoiling of material at one place and borrowing at another, the advisability or possibility of which depends upon circumstances. The haul involved in proposed wasting and borrowing is difficult of estimation, but it may be comparatively small if wasting is effected by widening an embankment, and borrowing by widening a cutting. The limit of profitable haul distance, beyond which it is economical to waste and borrow, is, of course, reached when the cost of excavating and hauling one cubic yard equals the cost of excavating and

hauling to waste one cubic yard plus that of excavating and hauling one cubic yard from the borrow pit.

Considering first the previous example of Fig. 275, it will be seen that it is impossible to secure balance over the whole length, as there is an excess ff' of excavation. Trying the base line as a balancing line, there is shown balance from A to E and wasting of the material between E and F. But the haul ($abc+cde$), and the maximum haul distance, ac , can evidently be reduced by raising the balancing line, so that, if the haul required in wasting be assumed constant, the balancing line klf shows a preferable scheme, the material from A to K being wasted.

Fig. 276 illustrates a second example. In this case, excavation and embankment are equal over the length shown, and borrowing and wasting would be unnecessary provided the excavated material were distributed by the method indicated by the base line as the

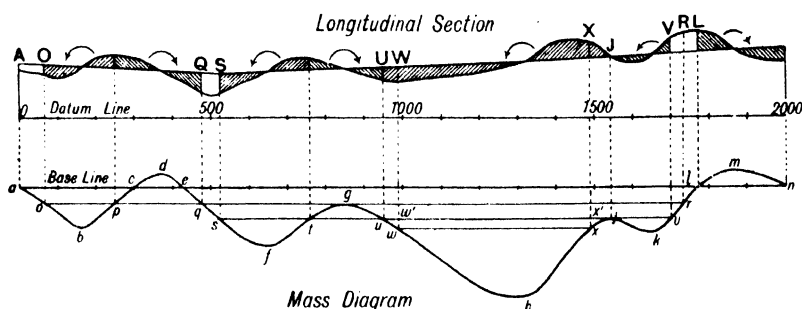


FIG. 276.

balancing line. But, on account of the large area below the base line, this arrangement involves considerable haul, with a maximum haul distance from l to e of about 1,350 ft., and it may be more economical to reduce the haul by wasting and borrowing. A possible method is that shown by balancing lines $opqgr$ and ln , in which there is borrowing from A to O and spoiling of the same volume from R to L. The maximum haul distance is reduced to that from r to g , or about 880 ft. Another possible arrangement is given by balancing lines opq , $stujv$, and ln , with a maximum haul distance ju of about 590 ft. This scheme necessitates borrowing from A to O and Q to S and wasting from V to L, and is shown in longitudinal section. By increasing the amount of borrowing and wasting, haul and maximum haul distance may be still further reduced. In particular, if the haul distance is not to exceed 500 ft., the line wx , scaling that length, will be used as a balancing line. In selecting the most economical of the various methods of disposal, one must be guided by the relationship between the cost of haul and that of borrowing and wasting in so far as it can be estimated for the particular case.

Allowance for Change of Volume.—In comparing schemes for the disposal of proposed excavations, the available information regarding the nature of the material is not usually sufficiently reliable to warrant making refined allowances for the change of bulk caused by excavating, and the common practice in such cases is to neglect these and proceed as above. If, however, sufficient data have been obtained, allowance should be made for the fact that rock increases considerably in bulk on being excavated. Soft earths also swell when loosened, but, chiefly owing to loss of material, they ultimately shrink to a smaller volume than they occupied in their original position in the excavation. The initial swell of earth is largely compensated for by the necessity for making embankments higher than they are intended to be to allow for subsidence, and the ultimate change of volume is in the direction of shrinkage.

Allowance for change of volume is made, before summing the volumes, in either of two ways: (a) by multiplying each computed volume of excavation by a factor which will convert it to the volume of embankment ultimately formed from the material; (b) by multiplying each volume of embankment by a factor to give the volume of excavation from which it can be made. If we suppose that, on a rough average, 100 cub. yd. of solid rock will suffice for forming 150 cub. yd. of embankment, and that 100 cub. yd. of earth in an excavation will ultimately form 90 cub. yd. of embankment, the respective factors would be 1.5 and .9 in the first method, and .67 and 1.1 in the second. Such average values are usually sufficient, since difficulties are encountered in estimating the best allowances for change of volume in a particular case. Shrinkage factors for soft earth depend upon the material, and are greater for low than for high embankments. In addition, mixtures of different earths, and of earth and rock, may have to be dealt with, and, until the mass diagram is prepared and studied, it is not known from which cuttings the material for a given embankment will be derived.

Overhaul.—The terms of contracts for earthwork may stipulate either that the price per cubic yard of excavation is inclusive of the cost of haul regardless of the haul distance involved or that the price includes the cost of haul within a specified distance only. In the latter case, extra payment is made for the haulage of each cubic yard which has to be moved a distance exceeding this specified distance, termed the free haul distance. The excess of haul distance above this amount is called overhaul distance, and the sum of the products of volumes by their respective overhaul distances is termed overhaul, and is the quantity for which extra payment is made. The unit of measurement of overhaul distance is commonly 100 ft., and that of overhaul is therefore cub. yd. \times 100 ft.

The mass diagram affords a convenient aid to estimating overhaul. In the case of Fig. 276, let the scheme with balancing lines *opq*,

stuv, and *ln* be that adopted, and let the free haul distance be 500 ft. Overhaul is required on the section JU. The horizontal line *wx* of length 500 ft. having been drawn on the mass curve and projected to *w'x'* upon the balancing line, the area *w'whxx'* represents free haul. The overhaul required is given by the product of the volume between U and W or X and J multiplied by the excess beyond 500 ft. of the shift of its centre of gravity in being transferred from cutting to bank. The positions of the centroids may be computed or estimated, but, since the total haul on the section JU is represented by the area *uhj*, and the free haul by *w'whxx'*, the difference, or overhaul, is given by the sum of the two areas *uwv'* and *x'xj*. These may be measured by planimeter, and are expressed in terms of cub. yds. $\times 100$ ft.

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EXAMPLES

1. A piece of ground has three straight sides, AB, BC, and CD, and the fourth, AD, is irregular. The dimensions in feet are AB = 422, BC = 640, CD = 456, AD = 798, and AC = 842. Offsets outwards from AD to the irregular boundary have the values 0, 12, 4, 19, 0 at chainages of 0, 150, 330, 434, and 798 ft. from A. Calculate from these figures the area within the boundary expressing the result in acres, roods, and poles.

2. The cross section of a stream 30 ft. wide is measured by means of soundings taken 5 ft. apart. The depths recorded are

0, 1.5, 2.0, 3.5, 2.3, 1.0, 0.

The mean velocity is observed to be 3.4 ft. per second. Compute the discharge of the stream.

3. If areas are measured by planimeter as square inches of map surface, what are the factors for converting the results to acres in the case of the "6-inch" and "25-inch" Ordnance maps?

4. A survey was plotted to the 1/2,500 scale. A certain area on a photo-print reproduction was measured by planimeter, and was found to contain 9.36 square inches. The print shows a shrinkage of 1 per cent both up and down and along the sheet. Obtain the area measured, expressing the result in acres, roods, and poles.

5. In taking a cross section of a stream, a leveller first observed the staff held upon the 10-ft. mark on a gauge post, and obtained a reading of 0.32 ft. The staff was then held on points 20 ft. apart on the cross section, the readings, taken to the nearest tenth of a foot, being

0.3, 4.0, 6.8, 7.4, 9.0, 9.5, 10.2, 10.3, 8.0, 6.4, 4.2, and 0.5.

Obtain the area of flow when the surface of the water is at 5 ft. on the gauge, assuming that the bed of the stream slopes uniformly between the points observed.

6. Calculate the side widths and cross-sectional area of a railway embankment with a formation width of 30 ft., side slopes $1\frac{1}{2}$ to 1, centre height 12 ft., and slope of original ground surface at right angles to the centre line of the embankment of 1 in 10.

7. The data of a closed traverse survey are as follows:

Line.	Latitude in links.	Departure in links.
AB	- 300	+ 450
BC	+ 640	+ 110
CD	+ 100	- 380
DA	- 440	- 180

Calculate by any method the area contained within the survey lines, expressing the result in acres, roods, and poles. (R.T.C., 1913.)

8. The adjusted latitudes and departures of a closed traverse are:

Line.	Latitude in feet.	Departure in feet.
AB	526 N.	113 W.
BC	384 N.	316 E.
CD	227 S.	540 E.
DE	613 S.	104 W.
EF	364 S.	782 W.
FA	294 N.	143 E.

Calculate the area contained within the part ABCDEA of the figure, and express the result in acres, roods, and poles.

9. A corner of a piece of ground is bounded by lines AB, BC, and CD, of which the lengths and bearings are as follows :

Line.	Length in feet.	Bearing.
AB	563	251° 42'
BC	212	193° 16'
CD	404	102° 23'

From a point X on AB it is required to lay off a straight line to a point Y on CD, such that XY will be parallel to BC, and XBCY will have an area of one acre. Compute the distances BX and CY.

10. A city building site, having an area of 5,749 sq. yd., is bounded by the following lines, of which the latitudes and departures, expressed in feet, are :

Line.	Latitude.	Departure.
AB	+ 244.0	0
BC	- 32.8	+ 130.6
CD	- 17.4	+ 128.3
DE	- 271.5	- 144.7
EA	+ 77.7	- 114.2

It is desired to divide the ground by a straight line into two equal areas, each having the same frontage on the line AB. F being the mid-point of AB, obtain the position of the dividing line FG by computing where G falls on DE.

11. A railway embankment is 30 ft. wide with side slopes of $1\frac{1}{2}$ to 1. Assuming the ground to be level in a direction transverse to the centre line, calculate by the end area method the volume contained in a length of 6 chains, the centre heights at one chain intervals being, in feet,

2, 4, $4\frac{1}{2}$, 6, $5\frac{1}{2}$, 4, $\frac{1}{2}$.

12. Calculate the volume of the above embankment by the prismoidal method.

13. Two irregular cross sections of a railway cutting 100 ft. apart have areas of 450 sq. ft. and 194 sq. ft. respectively. The formation breadth is 30 ft., and the side slopes are $1\frac{1}{2}$ to 1. By substituting equivalent level top sections, compute the mid-area and the volume of excavation between them.

14. A pier 300 ft. long is to be built seawards with a top breadth of 40 ft. and side batters of 1 in 8. The top level is to be 14 ft. above low water of spring tides, and the foundation levels at intervals of 50 ft. are 2, 4, 8, 10, 15, 22, and 27 ft. below the same datum.

Estimate by the use of the prismoidal formula the total volume of material required. (R.T.C., 1921.)

15. Calculate the value of the prismoidal correction for the first 300 ft. of a railway cutting of which the formation breadth is 30 ft. and the side slopes are $1\frac{1}{2}$ horizontal to 1 vertical. The original ground surface is level transversely to the centre line of the railway, and the centre depths of the earthwork are 0, 10, 14, and 20 ft. at 100 ft. centres. (T.C.D., 1927.)

16. The slope of a certain piece of ground (which may be regarded as a plane surface) is 1 in 4. On the surface of this ground a line AB, 375 ft. long, is laid out at a gradient of 1 in 9.

Find the slope of the ground in a direction at right angles to AB as seen in plan.

If AB be the centre line of the formation for a path 10 ft. wide, horizontal transversely, and with side slopes of 2 horizontal to 1 vertical, calculate the volume of the earth to be moved in making the path. There is to be neither cutting nor filling along the centre line. (Univ. of Lond., 1916.)

17. A road having a formation breadth of 30 ft. and side slopes of $1\frac{1}{2}$ to 1 is to be constructed on side-lying ground. At adjacent sections, 100 ft. apart, the depths of excavation at the centre line of the road are 2 ft. and 1 ft. respectively and the original ground surface between the sections has a constant inclination of 4 to 1 in the direction at right angles to the centre line. Calculate by the method of end areas the volume of excavation and the excess volume of excavation over embankment between the sections. (T.C.D., 1931.)

18. An embankment of 30 ft. formation width with side slopes of 2 to 1 is to be formed on a curve of 10 chains radius. If the original ground surface slopes at 5 to 1 downwards towards the concave side, calculate the curvature correction per 100 ft. station when the centre height of the bank is 8 ft.

19. A rectangle ABCD, 100 ft. by 80 ft., forms the plan of part of an excavation. AD and BC are the longer sides, and E is the point of intersection of the diagonals. From the following original surface levels and final excavated levels, calculate the volume of excavation within ABCD.

Point.	Original Level.	Final Level.
A	85.30	76.00
B	93.05	76.00
C	92.15	77.20
D	84.70	77.20
E	93.40	76.60

20. The areas within the contour lines at the site of a reservoir and along the face of the proposed dam are as follows :

Contour.	Area in sq. feet.
500	7,054,000
495	6,427,000
490	5,083,000
485	4,612,000
480	2,958,000
475	1,573,000
470	836,000
465	115,000
460	3,000

Taking 460 as the bottom level of the reservoir, find

(a) the volume of water in the reservoir when the surface is at elevation 500, using the method of end areas.

(b) the volume under the same conditions, as given by the prismoidal formula, taking every second area as a mid-area.

(c) the surface level when the reservoir contains 300 million gallons.

$$1 \text{ cub. ft.} = 6.24 \text{ gallons.}$$

CHAPTER X

SETTING OUT WORKS

THIS part of the duties of an engineer on location or construction involves the placing of pegs or marks to define the lines and levels of the work, so that construction may proceed with reference to them. Setting out is, in a sense, the reverse of surveying in that data are transferred from the drawings to the ground.

In the case of works which may be completely defined by a series of straight lines, setting out is a simple operation, and requires little explanation (page 417). The greater part of this chapter is therefore devoted to a description of the setting out of curves.

Location of Works.—Before proceeding to the detailed design of an engineering scheme, the relative merits of different sites must be carefully weighed, and the most economical location selected. For this purpose, in unmapped countries the territory embracing all probable sites must be investigated by surveying. In the case of large projects, time is saved by making successive surveys of increasing refinement, which are used for the gradual elimination of all locations except that finally adopted.

In general, the first survey consists of a rapid examination of a wide area with the object of ascertaining which portions of it merit further investigation. These are surveyed in greater detail, their relative advantages are studied, and finally the work is located in the best position on one of them. The character and amount of the field work involved depend upon the nature of the proposed public works. In any large scheme a considerable amount of detailed investigation is required before the engineer can finally decide upon the site which, from considerations of first cost and running expenses, will prove the most successful.

Without entering into a discussion of the engineering factors which govern the location, the case of a railway may be taken to indicate the field operations undertaken by a locating party.

Reconnaissance Survey.—The first, or reconnaissance, survey is extended over the area embracing all likely routes between the proposed terminals with the object of ascertaining the belt or belts of country which provide practicable routes. The field work has, therefore, the character of an exploratory survey, and is conducted either by rough traversing or by rapid triangulation, as described in Vol. II, Chap. VII. Traversing by prismatic compass

is the most favoured method, and elevations are determined by barometric levelling with the aneroid (Vol. II, Chap. VII).

Although performed by rapid methods, the reconnaissance survey is important as forming the basis of the subsequent work. It is essential that no important topographical or geological features should be overlooked, and the engineer in charge should take full notes of all factors likely to influence the location. A reconnaissance survey on the above lines is not required if topographical maps of the country are available, but, even so, it is usually desirable to supplement the information they afford by a rapid examination on the ground.

If no maps are available, the results of the survey are used to prepare a rough map showing the main topographical features which influence the location. The possible routes are sketched down with due regard to the elevations of these controlling points.

Preliminary Survey.—As a result of the reconnaissance, it may be discovered that a particular belt of country is better than any other as a route for the railway, and it remains to survey this strip in detail, so that the line may be located upon it in the most favourable position. Otherwise the reconnaissance may show that two or more belts of country appear equally suitable, and similar surveys are conducted over each to enable their relative merits to be studied and the line to be finally placed on one of them.

The surveys required at this stage of the investigation are called preliminary surveys. The method which has been most commonly used consists in laying out on the ground a trial line with suitable gradients in what appears on thorough examination of the ground to be the most economical position. The line need consist only of a series of straights, no attempt being made to run in curves unless the angle of deflection between adjacent straights is considerable, in which case one or more short straights may be laid out in the approximate position of the curve. The lines as laid out are surveyed by theodolite, plane table, or compass, as the conditions warrant, and chaining is performed continuously along them, marks being left at intervals of 100 ft. or 200 ft. An essential feature of the survey is the running of a longitudinal section along the traverse lines. The profile constructed from it is used for the laying down of suitable gradients, and, although only approximate because of the neglect of curves, the result is sufficiently accurate for the purposes of the preliminary estimate.

To enable the relationship of the selected line to existing features to be exhibited in plan, a field party is detailed to "take topography." The positions and levels of any artificial features, such as roads and buildings, near the line are obtained, since they influence the question of possible deviations from the trial line. There is considerable variation in field practice regarding the extent to which topographical features are recorded. In some cases, no

topography is taken other than particulars regarding the streams crossed : in others, a strip of country on either side of the projected line is completely contoured on the basis of a 5 ft. interval. If only one belt of country is being examined, a complete contour survey serves for the final location, but when two or more preliminary surveys are being run it is uneconomical to take much topography. Little or no record of the features of the adjacent ground is required in places where there is no doubt as to the best position of the line or where the ground is so flat that a deviation of the trial line will not appreciably affect the estimate of cost. In doubtful situations, on the other hand, a few clinometric cross sections may be run out from the traverse. On plotting these on the plan, and interpolating the contours, the most economical position of the line may be selected over the difficult part.

Location Survey.—The lines projected by the preliminary surveys having been carefully studied and compared with regard to first cost and operating expenses, selection is made of the scheme to be adopted. The remaining field work is directed in the first place to making a more detailed investigation of the ground along the preliminary line selected, with a view to improving the location with reference to the minor topographical features. Finally, after the best centre line is obtained, it must be staked out.

The final location may be performed entirely in the field, the engineer using the gradients shown on the preliminary profile as a guide in seeking for improvements in the alignment. The detail work is then performed by a similar party to that employed on the preliminary survey, and operations are carried out on the same lines as before except that curves are selected and run in. In the hands of a highly skilled locating engineer this method is sufficient, but the more usual and, in general, safer alternative is to make a contour survey of the chosen strip of country and to use the resulting map for the selection of the best centre line, thus making what is known as a paper location. The traverse and levels of the preliminary survey furnish the horizontal and vertical control respectively for the contour survey. The contour interval is usually 5 ft., and the contours are located by clinometric cross sections, by hand or ordinary levelling, or by tacheometry. The width of country to be contoured depends upon the transverse slope of the ground, and is much smaller on steep than on flat ground, since in the latter case the gradient or the depth of earthwork may not be seriously affected by a considerable lateral movement of the centre line. It frequently happens in the course of the survey that local deviations from the projected line appear desirable, and the survey is either widened or looped to include the topography of the alternative route. In making the paper location on the resulting map, the depths of earthwork along any trial line are readily obtainable by comparison of the proposed formation levels with the contour levels. The process is one of successive trial until the line is obtained

in which the gradients and curves are so arranged, with regard to the class of railway proposed, that the first cost plus capitalised working expenses is a minimum.

Location in Mapped Country.—The foregoing operations are greatly simplified when good detailed maps of the country are available. The preliminary trial lines may then be laid down directly on the maps and adjusted at doubtful points in the light of additional topographic data obtained at such places by cross sections or tacheometry. These preliminary lines are transferred to the ground by means of scaled measurements, and the longitudinal sections are run.

When the best scheme has been selected, lithographed drawings of the plan and profile are prepared for deposit in suing for legal power to proceed with the work. These drawings must comply strictly with the statutory requirements as to scale and the amount of information to be exhibited.

Final Location.—When authority has been obtained to proceed with the construction, the line as projected on the paper location is set out on the ground. It is, however, still subject to such minor alterations as may be found desirable, but such deviations from the approved centre line must not be of greater extent than is allowed by the law of the country concerned. In this country, lateral deviations must not exceed 100 yards on either side of the centre line in country districts and 10 yards in towns and villages, and limits are also set as to sharpening curves and steepening gradients.*

Setting Out in General.—Although in most cases the work is very simple in principle, difficulties are commonly encountered in practice, and indirect methods are frequently necessary, especially during the progress of construction. As each case must be dealt with according to circumstances, it is difficult to formulate hard and fast rules, but one or two considerations are of general application.

The need for complete checking of the work is self-evident, as an undetected mistake might have serious consequences. When the work is not such as will enable the closing error to be ascertained, there is a greater sense of certainty if important pegs are located by two different methods based upon independent sets of calculations than if the same method is repeated.

Instruments should be tested at frequent intervals, and should be so used that errors of non-adjustment are reduced to a minimum. A steel band should be used for linear measurements.

* See "The Railways Clauses Act, 1845," "Standing Orders of the House of Lords," "Standing Orders of the House of Commons," and "Rules and Regulations Issued under the Ministry of Transport Act, 1919," or *The Preparation of Parliamentary Plans*, by R. T. Fuller.

In structural work, a setting out plan should be prepared showing those lines of the work which have to be set out. As it may be impossible to locate these directly owing to obstructions, considerable use may have to be made of parallel lines, and it is therefore desirable at the outset to provide a convenient framework of reserve lines clear of the ground to be occupied by plant. All intersections are marked by pegs, the exact point being indicated by the head of a tack driven into the peg, and full information regarding the linear and angular dimensions to these points should be entered on the plan.

In abstracting the required dimensions from the drawings, the surveyor must be on his guard against possible errors in the figuring of the drawings. Scaling should be resorted to as little as possible, and then only for minor features: in the case of skew structures, the skew distances must be calculated. Thus, for setting out a skew bridge the skew span and lengths of the abutments are computed from the given square span and width and the angle of skew. Some of the dimensions necessary to fix the positions of the wing walls may have to be scaled. In the process of setting out such a structure, it is first necessary to locate a centre line and drive a peg thereon to mark the centre point of the bridge. On planting the theodolite over this peg, the angle of skew is set out, and points are established on the faces of the abutments. Pegs are located on the lines of the abutment faces prolonged clear of the work, and their distances from the corners of the abutments are noted. It is unnecessary in the first instance to provide pegs to mark the lines of foundations, back of wall, etc., as such lines can be obtained from that of the face. The end of each wing wall is set out by means of a peg on the abutment face line and an offset distance, but one or more pegs should be driven on the face line of the wing wall clear of the excavation. In the case of battered walls, only the foot of the wall is set out.

CURVES

The ranging of curves is required in the location of various kinds of public works, but it forms so important a part of the setting out of the centre line of a railway that the following discussion refers more particularly to railway work. The methods, however, are similarly applied to other cases.

Except for special purposes (page 442) railway curves are circular, and may be classed as simple, compound, and reverse.

A *Simple Curve* consists of a single arc connecting two tangents (*a*, Fig. 277).

A *Compound Curve* is composed of two arcs of different radii, curving in the same direction (*b*, Fig. 277). The centres of the two arcs are therefore situated on the same side of the curve which itself

lies completely on one side of the common tangent, i.e. the tangent to both arcs at their point of junction.

A *Reverse Curve* consists of two arcs, of the same or different radii, curving in opposite directions (c, Fig. 277). In this case the centres are on opposite sides of the curve, and the curve crosses the common tangent.

Note.—It is to be observed that if the arcs in b and c, instead of running tangentially into each other, have a straight, however short, introduced between them, they become simple curves.

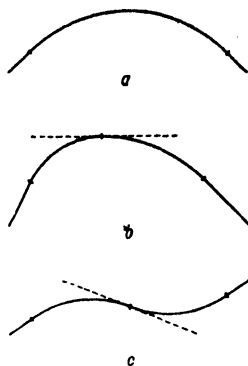


FIG. 277.

Designation of Curves.—A curve may be designated either in terms of its radius or with reference to the angle subtended at the centre by an arc or a chord of a particular length. The former system is adopted in this country, and the unit may be either the foot or the chain, the latter being the more common. The central angle, or degree, system is adopted throughout America. The standard arc or chord length is 100 ft., the angle which it subtends at the centre being called the degree of the curve. Up to fairly recent years the degree (D) of the curve was ordinarily defined in terms of the chord (D_c) but nowadays it is more usual to define it in terms of the arc (D_a). Although perhaps not so convenient for plotting, the degree system possesses considerable advantages in setting out.

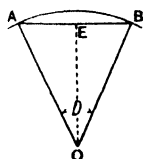


FIG. 278.

Let R = the radius in feet.
and D = the degree of a curve.

In Fig. 278 let AB be an arc of length 100 ft. Then, since D_a degrees = $D_a \times \pi/180$ radians,

$$\frac{3.14159 D_a}{180} R = 100,$$

$$\text{or} \quad R = \frac{5729.58}{D_a} \approx \frac{5730}{D_a}$$

If the degree of the curve is defined in terms of the chord, the chord AB in Fig. 278 is 100 ft. Let E be the middle point of this chord. Join OE and, since OE is perpendicular to AB ,

$$\sin \frac{1}{2} D_c = \frac{AE}{OA} = \frac{50}{R}$$

Hence

$$R = \frac{50}{\sin \frac{1}{2} D_c}$$

If $D_c = 1^\circ$, $R = 5729.65$, and assuming that $\frac{1}{2} D_c$ is a small angle

whose sine can be taken to be equal to its value in radians, we have for all small values of D_c ,

$$R = \frac{5729.65}{D_c} \approx \frac{5730}{D_c}$$

Thus, in the degree-arc system, the radius is exactly inversely proportional to the degree of the curve while, in the degree-chord system, the radius is inversely proportional to $\sin \frac{1}{2}D_c$ and so only approximately inversely proportional to D_c .

Elements of a Curve.—Let AGHKB (Fig. 279) represent a simple circular curve of radius R , connecting the two straights EA and BF, A and B being the tangent points. Let the straights be produced to meet at C.

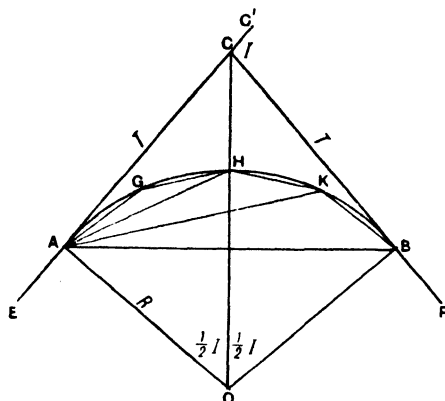


FIG. 279.

1. In the quadrilateral OACB, the angles OAC and OBC are right angles. —

$\therefore C'CB = AOB = I$, the *Intersection Angle*, or *Angle of Deflection*, this quantity representing the amount of deviation given by the curve.

2. The equal distances AC and CB are called the *Tangent Lengths*, T . Join CO.

In the triangle ACO, $\frac{AC}{AO} = \tan AOC = \tan \frac{1}{2}I$,

$$\therefore T = R \tan \frac{1}{2}I. \quad \text{—}$$

3. Let AG, GH, HK, and KB be equal chords. Join G, H, K, and B to A.

The angles AOG, GOH, etc., subtended at the centre by these chords or arcs, are equal, and are each twice the angles GAH, etc., subtended at the circumference. The angle CAG between the tangent and the first chord is also half the central angle subtended by that chord, and therefore equals the angles GAH, etc.

The angles CAG, CAH, etc., contained between the tangent and the rays from A to the various points on the curve, are called the *Deflection Angles* of those points. The most common method of locating points on a curve is based on the use of these angles, the values of which have therefore to be calculated. Since the deflection angle to any point is the sum of the circumferential angles for the chords or arcs lying between the origin of the curve and that point, it is sufficient to examine the calculation of these circumferential angles.

Formulae for Circumferential Angles (Fig. 280).

Let C = chord length in feet,
 A = corresponding arc length in feet,
 R = radius of curve in feet,
 D = degree of curve,
 δ = circumferential angle.

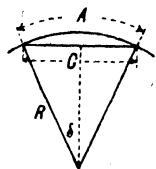


FIG. 280.

From the Figure it is easy to derive the following relationships for δ in terms of (1) A and R , (2) C and R and C and D_c , (3) A and D_c and A and D_a :—

$$(1) \quad \delta = \frac{A}{2R} \text{ radians} = \frac{180 \times A}{2\pi R} = \frac{28.648A}{R} \text{ degrees} = \frac{1718.9A}{R} \text{ minutes}$$

$$(2) \quad \delta = \sin^{-1} \frac{C}{2R} = \sin^{-1} \frac{\frac{1}{2} C \sin \frac{1}{2} D_c}{50}$$

Whence treating δ and D_c as small angles approximately equal to their sines,

$$(3) \quad \delta = 0.573A \sin \frac{1}{2} D_c \text{ degrees} = \frac{AD_a}{200} \text{ degrees with } D_a \text{ in degrees}$$

For short chords on flat curves, expansion of $\sin \frac{A}{2R}$ in terms of $\frac{A}{2R}$ in $C = 2R \sin \frac{A}{2R}$ gives the following useful series :—

$$C = A - \frac{1}{3!} \frac{A^3}{(2R)^2} + \frac{1}{5!} \frac{A^5}{(2R)^4} - \frac{1}{7!} \frac{A^7}{(2R)^6}$$

and, by reversion of series (Vol. 2, Chap. 1), or by expanding $\sin^{-1} \frac{C}{2R}$ in $A - C = 2R \sin^{-1} \frac{C}{2R} - C$, we get

$$A = C + \frac{1}{3R^2} \left(\frac{C}{2} \right)^3 + \frac{3}{4.5R^4} \left(\frac{C}{2} \right)^5 + \frac{3.5}{4.6.7.R^6} \left(\frac{C}{2} \right)^7 +$$

* Note here that, if C is 100 ft. δ is exactly $\frac{1}{2} D_c$.

Since the triangle ACB (Fig. 279) is isosceles, the total deflection angle CAB between the tangent and the chord joining the tangent points = $\frac{1}{2} I$.

4. The length L of the curve AB = the central angle AOB in radians $\times R$.

$$= \frac{\pi IR}{180} = .01745 IR, \text{ where } I \text{ is in degrees.}$$

If n = the number, integral or fractional, of arcs of length A forming the curve, then $n = \frac{\frac{1}{2} I}{\delta}$,

$$\text{and } L = nA = \frac{\frac{1}{2} IA}{\delta}$$

These relations are precise, but if the ratio $\frac{R}{A}$ is great, A and C become nearly equal, and

$$L = nC = \frac{\frac{1}{2}IC}{\delta} \text{ approximately.}$$

SETTING OUT SIMPLE CURVES BY THEODOLITE

Location of Tangents.—In pegging out a line composed of straights and curves, the straights, or tangents, must be set out before the curves connecting them can be located. It is assumed that the surveyor is provided with a copy of the working plan, upon which the line is shown in relation to the controlling traverse of the preliminary survey and to existing features. By scaling distances from the traverse lines or from buildings, fences, etc., several points on each tangent can be obtained on the ground by tape measurements. Such points being temporarily marked, the tangents may then be set out by theodolite, so that, by trial and error, they run as nearly as possible through the marks.

Peg Interval.—It is necessary for purposes of reference, and for conveniently obtaining distances along the line, that the pegs should be at equal intervals from the start of the railway to the end. There must be no break in the regularity of their spacing in passing from a tangent to a curve or *vice versa*. The setting out must therefore start at the beginning of the railway and be continued forwards. The interval between the pegs may be either 66 ft. or 100 ft. The former has the advantage that every tenth peg marks a furlong, and in this country is that generally adopted. In America, and in most British Colonies, the 100-ft. interval is generally used, while in countries in which the metric system is used a distance of 20 metres is most commonly employed. On sharp curves additional pegs may be driven at 33-ft. or 50-ft. centres.

Location of Tangent Points.—For a given pair of straights, there is only one point at which a curve of given radius or degree may leave the first straight tangentially in order to sweep tangentially into the second. The points of commencement and termination of the curve must therefore be determined with greater precision than would be possible by merely scaling their positions from the plan.

1. Having located the two tangents and defined them by ranging poles, peg out the first tangent EA (Fig. 281) up to about the estimated position of A, the theodolite being placed on EA and aligned on one of the poles. By means of the instrument, produce the straight to align two pegs a and b a few feet apart, one being

placed on each side of C, the position of which is estimated by the chainman from the line of the poles on BF.

2. Transfer the instrument to some convenient point on the second straight, and produce the latter to meet a string stretched between *a* and *b*. The point of intersection C of the two tangents thus obtained is marked by a peg.

3. Set up the theodolite over C, and measure the angle ECF. By subtracting the result from 180° , the value of the intersection angle *I* is obtained. Calculate the tangent lengths from $T = R \tan \frac{1}{2}I$.

4. From C measure back the lengths CA and CB = *T*, the tangent points A and B being aligned from the instrument at C. Mark A and B in a distinctive manner, either by painted pegs or by three ordinary pegs, the centre one of which defines the point.

5. Transfer the instrument to A, and set it over the tangent point peg. Measure the angle CAB, which should equal $\frac{1}{2}I$. This provides a convenient check on the equality of the tangent lengths, which may, however, both be in error by the same amount through a mistake in the measurement of *I* or in the calculation of *T*.

6. The chaining of the first straight may now be completed, the chainage of the point A being noted.

Location of Points on the Curve by Deflection Angles.—In setting out pegs on the curve, the most generally adopted method involves the use of one theodolite only. Consideration will be given later to the manner of employing two instruments (page 432).

The interval between the chainage pegs on the curve should strictly be measured as the arc intercepted between them, and consequently, in employing chords to locate the pegs, the length of chord between two adjacent pegs would apparently have to be calculated. In the great majority of cases in practice, however, the difference in length between the chord and the arc is quite negligible, and it is sufficient to use the peg interval as the chord length. In describing the field work, it will be supposed that this is done.

It will seldom happen that the tangent point A falls exactly at a peg interval from the last peg station on the first straight. In consequence, since the chainage must be continuous, the chord AG to the first point G on the curve will be shorter than the regular length *C*. Such a chord is termed a *Sub-chord*. There will also in general be a sub-chord at the end of the curve. Let their lengths be denoted by *c* and *c'*.

1. From the chainage of A obtain the length of the first sub-chord. Thus, if E (Fig. 281) is the last chainage peg on the straight, $c = C - EA$.

2. Calculate the circumferential angle δ for a chord *C* by any of the above formulæ, whether involving *A* or *C*, since these are being regarded as equal. The circumferential angle CAG for the

sub-chord may be obtained in the same manner, but with sufficient accuracy may be taken as $\frac{c}{C} \delta$.

3. Draw up a table of the deflection angles to the various points.

$$\text{1st deflection angle } d_1 = \frac{c}{C} \delta,$$

$$\text{2nd } ,, ,, d_2 = \frac{c}{C} \delta + \delta,$$

$$\text{3rd } ,, ,, d_3 = \frac{c}{C} \delta + 2\delta,$$

and so on, up to a maximum of $\frac{1}{2}I$.

Instead of multiplying δ by the factors 2, 3, etc., it is preferable to obtain each deflection angle from the previous one by the addition of δ . In this way a mistake made at one point will be continued, and may be detected in the following manner. Calculate, from the value of the angle I , the length L of the curve. Then

$$\text{Chainage of A} + L = \text{Chainage of B.}$$

The amount by which the chainage to B exceeds an exact number of peg intervals is the length of the end sub-chord c' , and the calculation of the deflection angles is checked if the circumferential angle corresponding to c' equals ($\frac{1}{2}I$ —the deflection angle to the end of the last whole chord).

The tabulated values of d_1, d_2, d_3 , etc. are the angles to be set on the circle of the theodolite in locating the curve, provided the direction of curvature from the first straight to the second is towards the right, as in Fig. 281, since this direction is the same as that of the graduation of the circle of the instrument. If, however, the curvature is towards the left, and the circle is not graduated both ways, each of the above angles must be subtracted from 360° , and the resulting values tabulated.

4. With the instrument at A, set the vernier to zero, and sight on C. Keeping the horizontal circle clamped, set off the first deflection angle d_1 on the vernier, so that the line of sight is now directed along AG. The point G having to be a distance c from A, the rear chainman holds the chain with the reading c at the peg A, while the leader, holding the end of the chain against a pole, is directed by the surveyor into the line of sight. When the chain is taut, and the pole is at the same time in the line of sight, the position of G is obtained, and a peg is driven at the point.

5. Set the vernier to read the second deflection angle d_2 , so that the line of sight is along AH. If the chords are 66 ft. or 100 ft. long, the rear end of the chain is held at G, and the forward chainman swings about G until his end of the chain is in the line AH, when H is marked. This procedure is repeated up to L, the end of the last whole chord, the position of each peg being determined by a ray from A and a measurement from the previously driven peg.

6. Measure the length LB of the terminal sub-chord. If the result agrees with the calculated length c' , the chaining is checked. To test the accuracy of the whole work, set the vernier to read $\frac{1}{2} I$. If there has been no accidental change in the orientation of the instrument, the line of sight should again pass through B. Now relocate B by this ray and by measuring c' from L. If the new position of B does not coincide with the tangent peg, the distance between the two points is the actual error of tangency, the allowable amount of which will depend entirely upon the circumstances of the case. If this is exceeded, the whole work would require to be checked *ab initio*. In situations where a few inches are of little importance, it is usually permissible to adjust the last few pegs to secure tangency: in city or tunnel work the required degree of precision is, of course, greater.

The diagram shows a series of connected arcs representing a chain line. The starting point is A, and the ending point is B. A straight line segment AB is drawn as a reference. Above this, a series of arcs connect points A, G, H, K, L, and B. Tangent lines are drawn from each interior point (G, H, K, L) to the previous point on the chain (A, G, H, K respectively). At the end of the chain, point L is shown. Two rays originate from L: one ray passes through point B, and another ray extends further to point C. Point C is located above the main chain of arcs. A vertical line segment connects C to a point labeled 'o' on the arc between G and H. The entire figure is labeled FIG. 281.

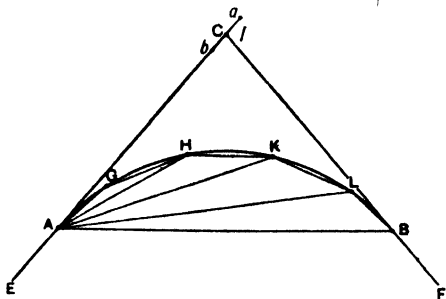


FIG. 281.

7. The first chainage peg on the second straight will be the amount ($C-c'$) distant from B, and the chaining of this line may now be proceeded with on the transfer of the instrument to B or to another convenient point on BF.

Notes.—(1) Assuming the error in the length of the chain is negligible, the most common source of error in the above work is in setting off the wrong value for a deflection angle, the effect of this mistake being continued along the remainder of the curve. The surveyor should therefore always compare each reading, as set, with the tabulated value of the angle. Quite a satisfactory method of detecting mistakes on other than rough ground is for the chainmen, and the surveyor himself where possible, to observe at frequent intervals whether the pegs which have been driven appear to lie on a smooth curve. In this way it is possible to discover a comparatively small local error.

(2) Much time may be saved on a long curve if at each point to be fixed the leading chainman puts himself roughly in position before being guided by the instrument man. He should do so by referring to the position of the previous chord produced. After pegging point H, for example, and pulling forward the chain, he should hold it approximately in the line GH, and then swing his handle through the constant deflection distance to reach K. The surveyor should tell him this distance, which = $\frac{C^2}{R}$ provided the chord produced is not a sub-chord (page 435).

Alternative Method of Setting Off Deflection Angles.—If there were no sub-chord at the commencement of the curve, the values of the deflection angles would be δ , 2δ , 3δ , etc., and a table of these could be prepared beforehand and used repeatedly in setting out curves of the particular radius to which it applies. In cases where there is an initial sub-chord, the manipulation of the instrument

at A may, however, be arranged so that advantage can be taken of such a table.

The chainage of A and the length and deflection angle of the sub-chord having been obtained in the usual manner, set off this angle *behind* the zero of the circle, so that the reading is $(360^\circ - \text{the computed angle})$ for a right-hand curve. With the vernier clamped to this reading, sight on C. Turn the vernier to read zero: the line of sight is now along AG (Fig. 281), and the point G may be obtained. By then successively setting off $\delta, 2\delta, 3\delta$, etc., the tabulated angles, the remaining pegs are located. In setting out a left-hand curve on this system, the tabulated angles can be used without subtraction from 360° provided the vernier reading for the tangent backsight is $(n\delta + \text{the deflection angle for the initial sub-chord})$, where n is the number of whole chords in the curve.

Example of Calculations.—Suppose that in ranging a right-hand curve of 32 chains radius by chords of one chain, with an instrument capable of reading to $30''$, the intersection angle I is found to be $14^\circ 38'$. Let it be required to calculate the quantities necessary for the setting out.

$$\begin{aligned} T &= R \tan \frac{1}{2} I, \\ &= 32 \times \tan 7^\circ 19', \\ &= 4.11 \text{ chains.} \end{aligned}$$

On measuring back this distance from C, let the chainage of A be found to be 265.73 chains (commonly written 265 + 73). The initial sub-chord c has therefore a length of 27 links.

The circumferential angle for a chain chord, in minutes,

$$= \frac{1718.9 A}{R}, \text{ and with the approximation that } A = 1 \text{ chain,}$$

$$= \frac{1718.9}{R \text{ in chains}} = 53'.71,$$

$$\text{and for the sub-chord} = \frac{27}{100} \times 53'.71 = 14'.50.$$

Table of deflection angles to nearest $30''$.

d_1	=	$14' 30''$
$d_2 = 14'.50 + 53'.71$	=	$68'.21 = 1^\circ 8' 0''$
$d_3 = 68'.21 + 53'.71$	=	$121'.92 = 2^\circ 2' 0''$
$d_4 = 121'.92 + 53'.71$	=	$175'.63 = 2^\circ 55' 30''$
$d_5 = 175'.63 + 53'.71$	=	$229'.34 = 3^\circ 49' 30''$
$d_6 = 229'.34 + 53'.71$	=	$283'.05 = 4^\circ 43' 0''$
$d_7 = 283'.05 + 53'.71$	=	$336'.76 = 5^\circ 37' 0''$
$d_8 = 336'.76 + 53'.71$	=	$390'.47 = 6^\circ 30' 30''$
$\frac{1}{2}I$	=	$7^\circ 19' 0''$

To check the calculation of the deflection angles, we have:

$$L = \frac{\pi IR}{180^\circ} = \frac{3.1416 \times 14^\circ 38' \times 32}{180^\circ},$$

$$= 8.17 \text{ chains.}$$

$$\text{Chainage to } B = 265.73 + 8.17$$

$$= 273.90 \text{ chains.}$$

\therefore Length of final sub-chord $c' = 90$ links.

The circumferential angle subtended by 90 links = $\frac{90}{100} \times 53'.71 = 48'.34$,
 but $(\frac{1}{2}I - d_s) = 7^\circ 19' 0'' - 6^\circ 30' 30'' = 48' 30''$.

The calculations are therefore checked, the final difference, due to the assumption that C and A are equal, being negligible.

Note.—If, instead of preparing a list of deflection angles as above, the engineer uses one of the published tables of deflection angles, he calculates the circumferential angles for the sub-chords, viz. $14' 30''$ and $48' 30''$, as before. He sets the vernier to read $359^\circ 45' 30''$ for the backsight AC, and moving the vernier to 360° directs his line of sight towards the first peg on the curve. Thereafter, he sets off the seven successive tabulated angles, $53' 30''$, $1^\circ 47' 30''$, etc., up to $6^\circ 16' 0''$. The reading for the sight on B should be $6^\circ 16' 0'' + 48' 30'' = 7^\circ 4' 30''$.

If the curve were left-handed, the vernier setting for the sight AC would be $6^\circ 16' 0'' + 14' 30'' = 6^\circ 30' 30''$. The successive settings are then the tabulated angles, $6^\circ 16' 0''$, $5^\circ 22' 30''$, . . . $53' 30''$, and 360° , which is the setting for the peg before the tangent point B. For the sight on B, the vernier should read $360^\circ - 48' 30'' = 359^\circ 11' 30''$.

Accuracy of Fix of Pegs on Curve.—Since the position of each peg on the curve other than the first is determined as the point of intersection of the instrument line of sight by an arc centered at the previously driven peg and of radius equal to the chord length, the probable accuracy of the location of the peg will depend upon the character of this intersection. There will not as a rule be any doubt as to which of the two possible intersections is that required, but, if the angle between the chord and the line of sight is considerable, these two intersections approach each other, and the fix becomes unsatisfactory, in that carelessness in chaining produces a relatively great displacement in the position of the point sought.

In Fig. 282, L is the true position of the point to be located, and K is the last driven peg. Let e be the error of measurement of the length $KL = C$, so that the length of the chord as measured is $KL_1 = (C - e)$, and L_1 is the point actually set out. If the angle KLA be denoted by α , the displacement LL_1 in the direction of the line of sight is approximately given by $e \sec \alpha$. The instrument being over the first tangent point, $\alpha = (d_L - \delta)$, and, to secure a satisfactory fix, it is desirable that d_L should not exceed about 30° . If the curve is such that I is greater than 60° , so that some of the deflection angles may exceed 30° , the instrument should be shifted forward according to the method of the following Case 1.

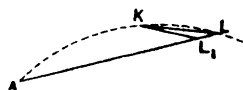


FIG. 282.

Difficulties in Ranging Simple Curves

Case 1.—When the Complete Curve cannot be Set Out from the Starting Point.—It has hitherto been assumed that the curve is sufficiently short and the ground so flat and free from obstructions that all the required pegs on the curve can be set out from the one position of the instrument at A . It is, however, very commonly impossible to do so.

In Fig. 283, the pegs G, H, and J have been located by the deflection angles d_1 , d_2 , and d_3 from A, but let it be supposed that, on setting off d_4 , the line of sight AK is found to be obstructed.

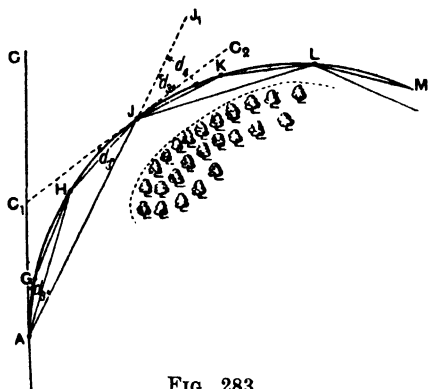


FIG. 283.

1. Transfer the instrument from A, and centre it over J.

2. Set and clamp the vernier to the angle it read at A when sighting C, *i.e.* either zero or $(360^\circ - \text{the deflection angle for the first sub-chord})$ according to the method used. Sight back on A.

3. Transit the telescope. Set the vernier to d_4 , the tabulated deflection angle for the point K, and the line of

sight is now directed along JK, for, if C_1JC_2 represents the tangent at J,

$$C_1JA = J_1JC_2 = d_3, \\ \therefore J_1JK = d_3 + \delta = d_4.$$

4. Continue the setting out from J in the usual manner.

It may not be possible to complete the curve from the station J, in which case the further procedure is as follows.

1. Set up over L, the last point located from J.

2. Sight back on the last point occupied by the instrument or on any peg on the curve, with the vernier reset to read the deflection angle for that peg. Thus, if J is sighted, the vernier must first have been brought to read d_3 .

3. Transit the telescope. Set the vernier to d_6 , the tabulated angle for the next peg M. The line of sight is now along LM, and the setting out may be continued.

Notes.—(1) Points which are to serve as instrument stations, or to which backsights are to be taken, should be marked with particular care, as otherwise it will be found troublesome to check in at the end of the curve.

(2) The above method is arranged to possess the advantage that the checked tabular values, d_1 , d_2 , etc., are employed throughout. On completing the work by sighting B from the last instrument station, the vernier should therefore read $\frac{1}{2}I$. An alternative, but less convenient, method consists in setting off the angles δ , 2δ , etc., anew from the tangent at each instrument station.

Case 2.—When an Obstacle Intervenes on the Curve.—If an obstruction, whether it can be seen over or not, is such that it cannot be chained across, it will be necessary to omit the location of the line across it until the obstacle is removed during construction. To obtain the positions of the pegs beyond the obstacle the usual procedure must be modified as follows.

1. Having located in the ordinary way the points G and H

(Fig. 284) up to the obstruction, find, by setting off the successive tabulated deflection angles, a clear line of sight to a point on the curve. Let AL be this line, the deflection angle being d_s . It is supposed that the point K, although clear of the obstacle, cannot be seen from A.

2. Calculate the length of the long chord AL from the formula,

$$\frac{\frac{1}{2} \text{ chord}}{\text{radius}} = \sin \frac{1}{2} \text{ central angle subtended by chord,}$$

$$\text{i.e. } AL = 2R \sin d_s.$$

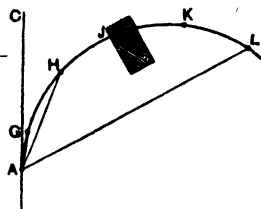


FIG. 284.

3. Measure out this distance from A, aligning the chaining from the instrument at A, and peg the point L:

4. Continue the curve from L in the usual manner.

Notes.—(1) If necessary, pegs such as K can be located by offsets from the long chord by the methods of page 435, or, if the instrument is transferred to L, they may be set out by deflection angles from L. Otherwise they may be left over until the obstruction has been removed.

(2) It may happen that no clear line AL can be obtained on account of obstacles of the type of Case 1. In such circumstances, having calculated the length of the curve, and so determined the length of the final sub-chord, the curve may be set out from the end B in the reverse direction up to K.

Case 3.—When the Point of Intersection of the Tangents is Inaccessible.—This difficulty is of frequent occurrence. Since the intersection point C is employed both in the measurement of I and as the starting-point from which the lengths T are measured back, the field work must be arranged to supply the twofold deficiency when C is inaccessible.

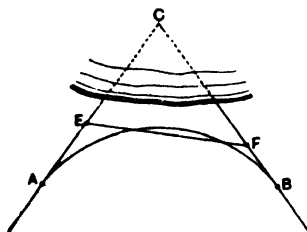


FIG. 285.

1. Select any convenient intervisible points E and F (Fig. 285) on the straights.

2. Measure angles AEF and EFB; then $I = 360^\circ - AEF - EFB$. Calculate the tangent lengths from $T = R \tan \frac{1}{2} I$.

3. Measure EF, and solve triangle CEF for CE and CF.

4. To locate A and B, measure from E the distance EA = $(T \sim CE)$, and from F the distance FB = $(T \sim CF)$.

5. Complete the pegging out of the curve in the usual way.

Note.—It frequently happens that no clear line EF can conveniently be obtained, in which case it is necessary to run a traverse between E and F, the required angles AEF and EFB and the distance EF being obtained by calculation.

Case 4.—When the First Tangent Point is Inaccessible.—The field work must first be directed to the determination of the chainage of the inaccessible tangent point, since the length of the initial

sub-chord and the positions of the pegs on the curve cannot otherwise be known.

1. On chaining back the tangent length from C (Fig. 286) and finding that A is inaccessible, note the measurement from C to a point F clear of the obstacle; then $AF = (T - CF)$.

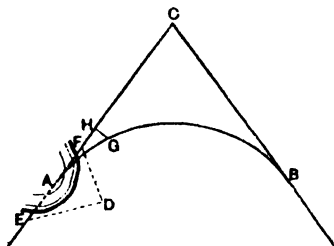


FIG. 286.

2. By any method of measuring past an obstacle, *e.g.* by solution of a triangle such as FDE, determine the distance from F to some convenient point E on the straight and at the other side of the obstacle.

3. Obtain the chainage to E; then the chainage of A = the chainage of E + EF - AF.

4. Compute the length of the curve, and so obtain the chainage of B.

5. Set off the curve in the reverse direction from B. The result may be checked by measuring the length of the offset from the last peg located to the tangent, its required length being $\frac{AH^2}{2R}$ approximately.

Note.—If it is found inconvenient to set out from B, the method of Case 6 may be employed.

Case 5.—When the Second Tangent Point is Inaccessible.—In this case the continuation of the chainage along the second straight forms the difficulty. Let A (Fig. 286) be the second tangent point.

1. On chaining the second tangent length, establish a point F at a known distance from C.

2. Obtain, as in the last case, the distances $FA = (T - CF)$, and FE to a convenient point E on the straight.

3. The chainage of A having been computed from that of the starting-point and the length of the curve, the chainage of E = the chainage of A + FE - FA.

4. From the point E locate the first accessible chainage peg on the second straight.

Case 6.—When both Tangent Points are Inaccessible.—1. Having obtained the length AF and the chainage of A by the method of Case 4, compute, from $FD = R - \sqrt{R^2 - AF^2} = R - \sqrt{(R - AF)(R + AF)}$ the value of the perpendicular offset from F to the curve (Fig. 287).

2. With the theodolite at F, set out the point D very carefully. D will be employed as an instrument station from which the curve may be continued.

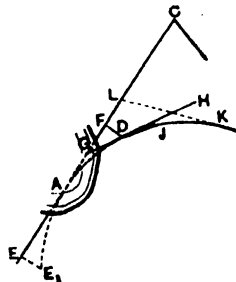


FIG. 287.

3. Calculate the length of arc AD from the circumstance that it subtends at the centre an angle $\sin^{-1} \frac{AF}{R}$; then the chainage to

D = the chainage of A + arc AD.

4. Draw up a table of deflection angles referred to the tangent DH.

5. Set the instrument over D. To lay the line of sight along DJ towards the first peg J, distant c from D, a backsight may be taken on F, and the vernier turned through $FDJ = 90^\circ + \sin^{-1} \frac{AF}{R}$

$+ \frac{c}{C} \delta$. But, unless DF has been made reasonably long, it is much

better to secure a long backsight by regarding the curve as extended beyond A to E_1 , the point E_1 being established by an offset EE_1 . On sighting E_1 from D, and turning through angle $E_1DJ =$

$180^\circ - \frac{1}{2} \left(\sin^{-1} \frac{AE}{R} + \sin^{-1} \frac{AF}{R} \right) - \frac{c}{C} \delta$, the line of sight will be directed

towards J.

6. Continue the location until the obstacle at B is reached. Check the work by measuring an offset from the curve to the second tangent, and use the method of Case 5 in order to continue the chainage.

Notes.—(1) The whole of the field work involved in this method requires considerable care in order that the closure may be effected. The instrument stations should be carefully aligned and marked by a tack on the peg. Checks on the progress of the work on a long curve may be made at intermediate instrument stations, such as K, by noting where the tangent KL cuts the first tangent. The distance CL, for instance, should be found to equal

$$\left[T - R \tan \frac{1}{2} \left(\frac{180^\circ \times \text{arc AK}}{\pi R} \right) \right].$$

(2) Having determined the chainage of A, the following alternative method of setting out the curve is preferable when the intersection point C is readily accessible. Set up the theodolite at C, and bisect angle ACB. Compute the apex distance, or distance from C to the mid-point of the curve, and, by measuring out this distance along the bisector, locate M, the mid-point of the curve. The chainage of M = the chainage of A + $\frac{1}{2}L$. Transfer the instrument to M, backsight on C, and set out the curve in both directions from M.

(3) A third method consists in locating an instrument station on the curve by measuring out from a point, such as E, on the straight. Assuming that D is a point of known chainage to which a measurement can be made from E, the distance ED and the angle CED are obtained by solution of triangle EAD since, the chainage of A having been ascertained, EA, AD, and angle EAD can be evaluated. Alternatively, triangle EGD may be solved. The point D having been pegged, the setting out can be continued from D after backsighting on E.

The foregoing difficulties may occur simultaneously, particularly in the course of setting out in cities. Thus, in Fig. 288, which represents the centre line of part of an underground railway, the setting out is based upon the points D and E on one tangent and

F and G on the other. Very careful traversing must be executed between DE and FG to determine the intersection angle and the positions of A and B. To obtain any point on the curve, such as the centre of a shaft H, the co-ordinates of the point are computed

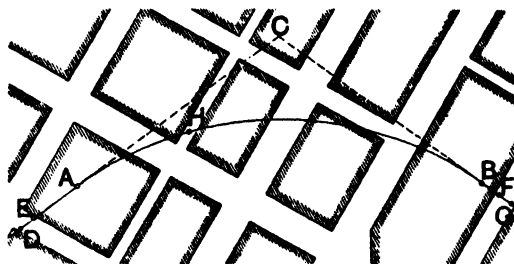


FIG. 288.

from, say, E as origin and DE as meridian. A traverse is conducted from E towards H, the length of the last line being computed so that it ends on the required point. The bearing of the tangent at H is also calculated, and is set out from the traverse to form a base for the location of the curve underground.

Horse Shoe Curves.—If, as in Fig. 289, the angle I subtended at the centre of a curve exceeds 180° , the point of intersection C of the tangents EA and BF lies on the same side of the curve as the centre O . The angles at C are as marked, and evidently from triangle AOC

$$T = R \tan (180^\circ - \frac{1}{2}I).$$

The deflection angles may be laid off from AD , the production of the first straight, and they are tabulated in the usual way up to the maximum angle, $DAB = \frac{1}{2}I$.

In country necessitating the introduction of such curves it will usually be impracticable to obtain the value of I at the intersection point C , and the method of running a series of traverse courses, or, if possible, a single transversal between points on

the two straights, is generally required. If EF be the traverse closing line or the transversal, then $I = AEF + EFB$.

Setting out Simple Curves by Two Theodolites.—The positions of pegs on a curve may be determined, without the necessity of chaining the chord lengths, by locating them as the points of intersection of the lines of sight from two theodolites placed one at each end of the

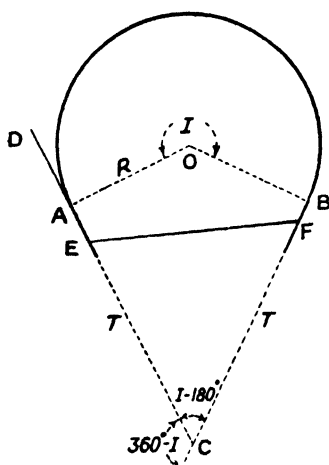


FIG. 289.

curve. Since two instrument men are required, the method is not in general use, but might be warranted in cases where the curve lies on ground of such character that there would be difficulty in performing the chaining with the required degree of accuracy.

1. Having determined the positions of A and B, the chainage of A, and the length c of the initial sub-chord, calculate the length of the curve and from it the chainage of B and the length c' of the terminal sub-chord.

2. Draw up tables of deflection angles from A and B respectively. Thus, for the curve of Fig. 290, we should have

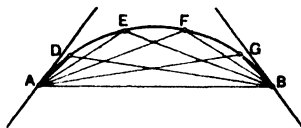


FIG. 290.

Point	Angle from A	Angle from B
A	—	$\frac{c'}{C}\delta + 3\delta + \frac{c}{C}\delta = \frac{1}{2}I$
D	$\frac{c}{C}\delta$	$\frac{c'}{C}\delta + 3\delta$
E	$\frac{c}{C}\delta + \delta$	$\frac{c'}{C}\delta + 2\delta$
F	$\frac{c}{C}\delta + 2\delta$	$\frac{c'}{C}\delta + \delta$
G	$\frac{c}{C}\delta + 3\delta$	$\frac{c'}{C}\delta$
B	$\frac{c}{C}\delta + 3\delta + \frac{c'}{C}\delta = \frac{1}{2}I$	—

The fact that the sum of the deflection angles to each point on the curve equals $\frac{1}{2}I$ provides a check in addition to the usual one.

3. With the instruments at A and B respectively, set off these angles consecutively, the chainman being guided to each point until the pole held by him is simultaneously in the two lines of sight.

Note.—Should it be found necessary to shift one or both instruments to sub-stations on the curve, the tabulated angles will be employed throughout if the orientation of the instrument at each setting is performed as in Case 1, page 427.

SETTING OUT SIMPLE CURVES BY CHAIN AND TAPE ONLY

The location of a simple curve may be performed without the use of an angular instrument in cases where :

(a) A high degree of accuracy is not demanded, as in roads, avenues, etc.

(b) The curve is short, as in certain railway and tramway curves, corners of buildings, curved wing walls, etc.

Location of Tangent Points.—In short curves of small radius, the positions of the tangent points can generally be taken from a large scale plan with sufficient accuracy to permit of the fitting in of the curve between them on the ground. Otherwise, the following method may be used.

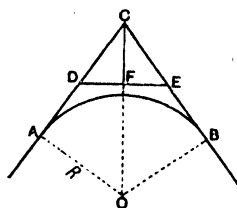


FIG. 291.

1. Produce the two straights by eye to meet at C (Fig. 291).

2. Select any pair of intervisible points D and E, one on each tangent and equidistant from C, making CD and CE as long as is convenient.

3. Measure DE, mark its mid-point F, and measure CF.

4. Since triangle CDF and COA are similar,

$$\frac{CA}{OA} = \frac{CF}{DF},$$

$$\text{so that } T = \frac{CF}{DF} \cdot R.$$

Notes.—(1) If CD and CE cannot conveniently be made equal, select any points D and E in suitable positions on the respective tangents. Measure the three sides of triangle CDE, and solve for angle DCE = $180^\circ - I$. Calculate T from $T = R \tan \frac{1}{2}I$, and measure out the differences DA and EB.

(2) If C is inaccessible, angles ADE and DEB may be evaluated by solution of triangles such as A'DF and FEB' from the measured lengths of their sides, A' and B' being convenient points on the tangents. Thereafter the procedure is as in Case 3, page 429.

Location of Points on the Curve by Deflection Distances.—This is the most useful method for a long curve.

Case (a). When there is no initial sub-chord, or the chainage is not required to be continuous.

1. With the tangent point A as centre (Fig. 292), swing the chord length AD = C on the chain until the perpendicular offset ED from the tangent = $\frac{C^2}{2R}$. D is the

position of the first peg on the curve.

2. By means of poles at A and D produce AD its own length to F. Mark F with a pole or arrow, and, with centre D, swing the chord length to the position DG,

so that $FG = \frac{C^2}{R}$. Peg the point G.

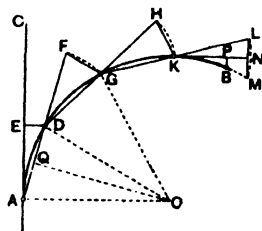


FIG. 292.

3. Produce DG to H, and repeat.

4. To check the work at the second tangent point B, KB being found to be a sub-chord of length c' , set out the points L and M as for a whole chord. Bisect LM at N, then KN is the tangent to the curve at K. From KN set off the perpendicular offset $PB = \frac{c'^2}{2R}$, and the error of tangency will be determined.

Notes.—(1) The above expressions for the offsets from the tangents and from the produced chords are precise. If Q is the mid-point of AD, and A, Q, D, and G are joined to the centre O, the triangles AED and OQA are similar.

$$\therefore \frac{ED}{C} = \frac{\frac{1}{2}C}{R},$$

$$\text{or } ED = \frac{C^2}{2R}.$$

Again triangles DFG and ODG are similar, since both are isosceles and $\angle FDG = 180^\circ - 2\angle DOG = \angle DOG$,

$$\therefore \frac{FG}{C} = \frac{C}{R},$$

$$\text{or } FG = \frac{C^2}{R}.$$

(2) Unless the radius is very small, it is in general sufficiently accurate and more convenient to make $AE = C$, so that the offset $ED = \frac{C^2}{2R}$ is not perpendicular to AE.

In similar circumstances, LN may be set out perpendicular to KL and equal to $\frac{C^2}{2R}$ without the necessity of locating M.

Case (b). When there is an initial sub-chord, c.

A slight modification is necessary in this case on account of the circumstance that the second peg G could not be located in the usual manner from the production of the short chord AD (Fig. 293).

1. Obtain the position of D by swinging the length c about A until $ED = \frac{c^2}{2R}$.

2. Consider the curve extended as shown dotted, and locate the whole chord DD' by turning it about D until the perpendicular offset E'D' has the length given by the approximate formula $\frac{(C-c)^2}{2R}$.

3. Produce D'D a whole chord length to F, and swing through $FG = \frac{C^2}{R}$ as before.

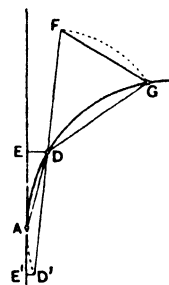


FIG. 293.

Notes.—(1) If the radius is more than, say, three times the chord length, it is usually quite allowable to locate D' by measuring back $AE' = (C-c)$ and erecting the offset from E'.

(2) If the sub-chord AD is not too short for reasonably accurate production, an alternative method of locating G is to produce AD to F', making DF' a whole chord length C. With centre D, the chord length is swung to DG, the distance F'G being approximately $\frac{C(C+c)}{2R}$.

Location of Points on the Curve by Tangent and Chord Offsets.—These methods are useful for short curves. In such cases it is usually unnecessary that the pegs should be equally spaced, and

the field work requires no explanation. Formulæ from which to calculate the lengths of the offsets are as follows :

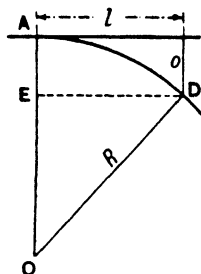


FIG. 294.

(a) Offset o at a distance l on a tangent from the tangent point (Fig. 294).

From triangle EDO,

$$R^2 = (R - o)^2 + l^2,$$

$$\text{whence } o = R - \sqrt{R^2 - l^2}$$

$$= \frac{l^2}{2R} \text{ approximately.}$$

(b) Offset o_1 from the mid-point of a chord of length C (Fig. 295).

In the same manner,

$$o_1 = R - \sqrt{R^2 - \frac{C^2}{4}},$$

$$= \frac{C^2}{8R} \text{ approximately.}$$

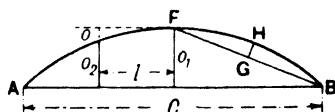


FIG. 295.

(c) Offset o_2 from the chord AB at a distance l from its mid-point.

Since a tangent to the curve at its mid-point F is parallel to AB, o_2 is less than o_1 by the offset o above formulated, so that

$$o_2 = o_1 - R + \sqrt{R^2 - l^2},$$

$$= o_1 - \frac{l^2}{2R} \text{ approximately.}$$

(d) Having obtained F by the offset o_1 , an alternative method of locating other points on the curve consists in bisecting chord FB and erecting an offset GH = $\frac{FB^2}{8R}$ from its mid-point. By dealing similarly with the chords FH and HB, etc., any number of points may be determined.

Note.—The approximate expressions above should not be used unless l or C is small in relation to the radius.

COMPOUND CURVES

Elements of a Compound Curve.—In Fig. 296, ADB represents a compound curve, the two branches of which join at D and have centres O_1 and O_2 and radii R_1 and R_2 respectively, R_2 denoting the greater whether it comes first or second in the direction of chainage. The end straight, on being produced, meet at C, giving an intersection angle I . The tangent lengths AC and BC are necessarily unequal, and will be distinguished as T_1 and T_2 , of

which T_2 is the greater and is adjacent to the arc of greater radius. The common tangent at D meets these end tangents at C_1 and C_2 , and makes with them the angles I_1 and I_2 equal to the respective central angles subtended by the arcs. For the two simple curves, we have

$$C_1A = C_1D = R_1 \tan \frac{1}{2}I_1 = t_1,$$

$$C_2B = C_2D = R_2 \tan \frac{1}{2}I_2 = t_2.$$

The following properties of the compound curve are evident from the figure.

(1) The centres O_1 and O_2 are in the same straight line with D, since the radii O_1D and O_2D are both perpendicular to the common tangent.

(2) $I = (I_1 + I_2)$, since I is an exterior angle of the triangle CC_1C_2 .

$$(3) CC_1 = C_1C_2 \frac{\sin I_2}{\sin I},$$

$$\therefore T_1 = t_1 + (t_1 + t_2) \frac{\sin I_2}{\sin I}, \dots\dots\dots (1)$$

$$\text{and similarly } T_2 = t_2 + (t_1 + t_2) \frac{\sin I_1}{\sin I} \dots\dots\dots (2)$$

(4) T_1 must be greater than $R_1 \tan \frac{1}{2}I$, or R_1 must be smaller than $T_1 \cot \frac{1}{2}I$: for, if $T_1 = R_1 \tan \frac{1}{2}I$, the curve would be simple, and if T_1 were less than $R_1 \tan \frac{1}{2}I$, tangency at the second straight could be secured only by making the radius of the second branch less than R_1 , which has been premised to be the smaller radius. Similarly, T_2 must be smaller than $R_2 \tan \frac{1}{2}I$, and R_2 greater than $T_2 \cot \frac{1}{2}I$.

(5) If the arc AD is extended to E, so that the tangent at E is parallel to CB, then D, E, and B are in the same straight line. For O_1E and O_2B are parallel, and if DE and DB be joined, the triangles DO_1E and DO_2B are similar, since both are isosceles and their angles at O_1 and O_2 are equal. Therefore angles O_1DE and O_2DB are equal, and DEB must be one straight line. Similarly, the corresponding point on the arc BD produced is in the same straight line as D and A.

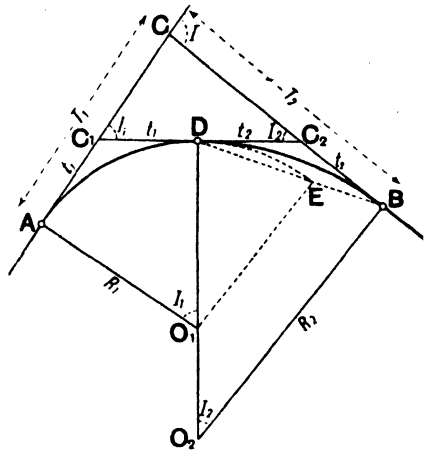


FIG. 296.

Relationships between the Parts of a Compound Curve.—The seven quantities involved in a compound curve of two branches, viz.

$I, I_1, I_2, R_1, R_2, T_1$, and T_2 , are interrelated in a manner deducible from the above expressions (1) and (2) as follows.

$$T_1 = t_1 + (t_1 + t_2) \frac{\sin I_2}{\sin I}, \dots\dots\dots (1)$$

$$= R_1 \tan \frac{1}{2} I_1 + (R_1 \tan \frac{1}{2} I_1 + R_2 \tan \frac{1}{2} I_2) \frac{\sin I_2}{\sin I},$$

$$\text{but } \tan \frac{1}{2} I_{1 \text{ or } 2} = \frac{1 - \cos I_{1 \text{ or } 2}}{\sin I_{1 \text{ or } 2}},$$

$$\therefore T_1 = R_1 \frac{1 - \cos I_1}{\sin I_1} + \left(R_1 \frac{1 - \cos I_1}{\sin I_1} + R_2 \frac{1 - \cos I_2}{\sin I_2} \right) \frac{\sin I_2}{\sin I},$$

$$\text{or } T_1 \sin I = R_1 \left(\frac{1 - \cos I_1}{\sin I_1} \right) (\sin I + \sin I_2) + R_2 (1 - \cos I_2).$$

Substituting $\sin(I - I_1)$ for $\sin I_2$, this reduces to

$$\begin{aligned} T_1 \sin I &= R_1 (\sin I \sin I_1 + \cos I \cos I_1 - \cos I) + R_2 (1 - \cos I_2), \\ &= R_1 (\cos(I - I_1) - \cos I) + R_2 (1 - \cos I_2), \\ &= R_1 (\cos I_2 - \cos I) + R_2 (1 - \cos I_2), \end{aligned}$$

which may preferably be expressed as

$$= R_1 \{ (1 - \cos I) - (1 - \cos I_2) \} + R_2 (1 - \cos I_2),$$

$$\text{or } T_1 \sin I = R_1 \text{ versin } I + (R_2 - R_1) \text{ versin } I_2 \dots\dots\dots (3)$$

$$\text{Similarly, } T_2 \sin I = R_2 \text{ versin } I - (R_2 - R_1) \text{ versin } I_1 \dots\dots\dots (4)$$

Expressions (3) and (4) in conjunction with the relationship $I = (I_1 + I_2)$ provide three simultaneous equations, the solution of which will determine three of the seven parts of the curve if the remaining four form the data. In practice, it will almost always be possible to locate the end straights first, so that I will be known. If, in addition, both radii are given, there remains one quantity to be fixed, and this may be either a tangent length, T_1 or T_2 , or one of the angles I_1 or I_2 . If, as is generally the case, the curve is already plotted, it will usually be preferable to scale a tangent length, but the length of one of the arcs may be fixed instead, in which case I_1 or I_2 is known. By utilising the above three simultaneous equations, the manner of setting out the curve is made independent of the choice of data, but for definiteness in describing the field work it will be assumed that R_1, R_2 , and T_1 are known, and that I is measured in the field.

Setting Out a Compound Curve.—Data : R_1, R_2, T_1 , and I .

1. Locate the end straights, and produce them to their intersection C (Fig. 296). Measure I .

2. Calculate T_2, I_1 , and I_2 .

3. Chain T_1 and T_2 from C, and peg A and B. Determine the chainage of A.

4. Calculate the lengths of the arcs AD and DB and thence the chainages of D and B. Prepare a table of deflection angles for the

are AD : the deflection angle to D should work out as $\frac{1}{2}I_1$. Similarly, tabulate deflection angles up to $\frac{1}{2}I_2$ for the arc DB referred to the tangent at D.

5. Set out the arc AD from A.

6. Transfer the instrument to D, and set the vernier to $\frac{1}{2}I_1$ behind zero. Sight A, transit, and set the vernier to the first tabulated deflection angle for arc DB. The line of collimation is now directed towards the first peg on DB. Continue setting out to B.

Notes.—(1) Having included the value of T_1 in the data, A must be located by measurement from C as above and not merely by its chainage as scaled from the plan. If A is fixed by the latter method, its distance from C must be measured to give the value of T_1 to be used in the calculations.

(2) A check on the positions of A and B is obtained, if B is visible from A, by observation of the angle CAB, the value of which can be calculated by solution of the triangle CAB, but it frequently happens that the sight AB cannot be obtained. If B is visible from D, the check may consist in establishing the direction of the common tangent at D from a backsight on A, or other point on the arc AD, and observing whether $C_2DB = \frac{1}{2}I_2$ or, in other words, verifying that $ADB = (180^\circ - \frac{1}{2}I)$.

Example of Calculations for a Compound Curve.—Let $R_1 = 20$ chn., $R_2 = 40$ chn., $T_1 = 17.54$ chn., and the measured value of $I = 63^\circ 29'$, the arc of smaller radius coming first in the direction of chainage.

I_1 is first obtained from

$$T_1 \sin I = R_1 \text{ versin } I + (R_2 - R_1) \text{ versin } I_2, \dots\dots\dots(3)$$

whence $I_2 = 39^\circ 45'$;
and $I_1 = (I - I_2) = 23^\circ 44'$.

T_2 can now be derived from

$$T_2 \sin I = R_2 \text{ versin } I - (R_2 - R_1) \text{ versin } I_1, \dots\dots\dots(4)$$

whence $T_2 = 22.85$ chn.

On measuring back T_1 , let the chainage of A be found to be 252.46 chn.

$$\text{Arc AD} = \frac{\pi I_1 R_1}{180^\circ} = 8.28 \text{ chn.},$$

$$\therefore \text{chainage of D} = 252.46 + 8.28 = 260.74 \text{ chn.}$$

$$\text{Arc BD} = \frac{\pi I_2 R_2}{180^\circ} = 27.75 \text{ chn.},$$

$$\therefore \text{chainage of B} = 260.74 + 27.75 = 288.49 \text{ chn.}$$

The tabulation of the deflection angles may then proceed as for simple curves.

Difficulties in Ranging Compound Curves.—The procedure in the case of obstacles or inaccessibility of C, A, or B is as for simple curves. When D is inaccessible, the second arc is best set out from B, but, if B is also inaccessible, the location may be made by means of observations from A or any other instrument station on the first arc.

Thus, in Fig. 297, let it be required to locate H, one of the chainage pegs on the second branch, from the station G on the first arc. In triangle GDH, GD and DH can be obtained from the known lengths of arcs GD and DH, and $GDH = (180^\circ - C_1DG - C_2DH)$. But $C_1DG = (\frac{1}{2}I_1 - d_c)$, and $C_2DH =$ the tabulated deflection angle

of H from D, so that the triangle can be solved for GH, DGH, and

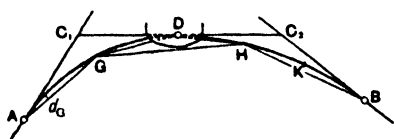


FIG. 297.

DHG. Now $AGH = (180^\circ - \frac{1}{2}I_1 - DGH)$. GH may therefore be set out and H pegged. On transferring the instrument to H, the work may be checked by observing whether $GHB = (180^\circ - \frac{1}{2}I_2 - DHG)$ in the case where this method is

employed with B accessible. To enable the tabulated deflection angles to be utilised in setting out are HB, the backsight on G should be made with the vernier reading $(360^\circ - GHD)$. On turning the vernier to read d_K , the deflection angle for the next peg K, and transiting, the line of sight will be directed for the pegging of K.

Alternative Method of Setting Out a Compound Curve.—An alternative method of locating a compound curve of which the radii are known is designed to avoid the necessity of extending the end straight to their intersection C, and is very useful where C is distant or inaccessible or when the measurement of T_1 and T_2 would be troublesome. It involves the setting out of the common tangent by means of scaled dimensions from the plan in the same manner as for the end straight. The points of intersection, C_1, C_2 (Fig. 298), of the common and end tangents are pegged, and I_1 and I_2 are measured. In order that a compound curve of given radii may fit the three tangents, the necessary condition is that the length of the common tangent should $= (t_1 + t_2) = (R_1 \tan \frac{1}{2}I_1 + R_2 \tan \frac{1}{2}I_2)$.

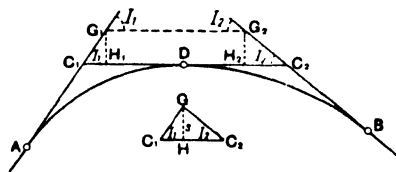


FIG. 298.

The process of setting out is as follows.

1. Locate the end straight and the common tangent. Measure I_1 and I_2 and the length L of the common tangent as laid out.
2. Calculate t_1 and t_2 . If $L = (t_1 + t_2)$, locate D at t_1 from C_1 , and set out the arcs AD and DB.
3. If, however, L differs appreciably from $(t_1 + t_2)$, I_1 and I_2 being assumed to have been correctly measured, the discrepancy is due to the line not occupying the position required for common tangency. The change required to make $L = (t_1 + t_2)$ is most conveniently effected by a movement of the line parallel to itself, so that the values of I_1 and I_2 are unaltered. The direction of shift will be outward, or towards C, if $L > (t_1 + t_2)$, and inward if $L < (t_1 + t_2)$, and the amount of shift can be calculated as follows.

In Fig. 298, let G_1G_2 be the line set out in place of C_1C_2 , its length L being found $< (t_1 + t_2)$. The perpendiculars G_1H_1 and G_2H_2 represent the shift s . Let the triangles $G_1C_1H_1$ and $G_2C_2H_2$

be supposed moved together until G_1H_1 and G_2H_2 are in coincidence. In the resulting triangle GC_1C_2 ,

$$\begin{aligned} C_1H &= s \cot I_1, \\ \text{and } C_2H &= s \cot I_2; \\ \therefore C_1H + C_2H &= s (\cot I_1 + \cot I_2), \\ &= (t_1 + t_2) - L, \\ \text{or, for any case, } s &= \frac{(t_1 + t_2) - L}{\cot I_1 + \cot I_2}. \end{aligned}$$

Note.—It is usually quite justifiable to make the shift a parallel one, as the direction of the common tangent may be set out as accurately as those of the end straights. Alteration in the direction of the common tangent has the effect of changing the lengths of the component arcs and the position of the whole curve.

REVERSE CURVES

Since sudden reversal of curvature is objectionable at high speeds, the use of reverse curves should, where possible, be avoided on main lines, but they are frequently required in sidings, etc., where the traffic is slow.

The geometrical principles involved are similar to those for compound curves, but the radii of the component arcs of a reverse curve may be equal, and the point of intersection C of the end straights may fall on either side of the common tangent and on either side of the tangent points A and B . As before, R_2 will denote the greater radius, but T_2 , the adjacent tangent length, does not necessarily exceed T_1 , as in the compound curve. The intersection point C may coincide with a tangent point, so that one of the tangent lengths vanishes. Figs. 299 to 303 illustrate various conditions.

(a) $I_1 > I_2$: C on O_1 side of common tangent and before A (Fig. 299).

(b) $I_1 > I_2$: C on O_1 side of common tangent and beyond A (Fig. 300).

(c) $I_1 = I_2$: C at infinity (Fig. 301).

(d) $I_1 < I_2$: C on O_2 side of common tangent and before B (Fig. 302).

(e) $I_1 < I_2$: C on O_2 side of common tangent and beyond B (Fig. 303).

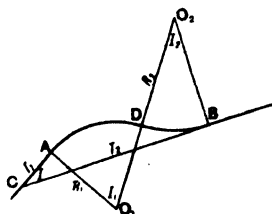


FIG. 299.

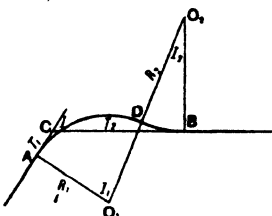


FIG. 300.

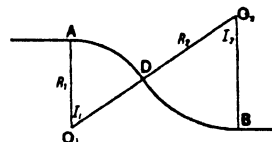


FIG. 301.

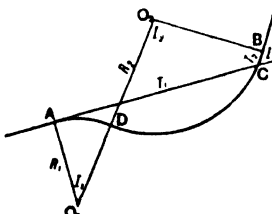


FIG. 302.

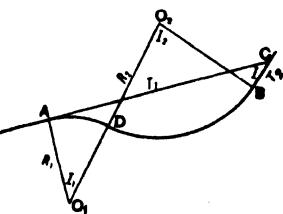


FIG. 303.

As before, the angle I represents the angular deviation produced by the curve, but, in the case of reverse curves,

$$I = I_1 - I_2.$$

The remaining relationships between the seven parts, corresponding to (3) and (4) (page 438), and derived in the same manner, are

$$T_1 \sin I = (R_1 + R_2) \text{ versin } I_2 - R_1 \text{ versin } I,$$

$$T_2 \sin I = (R_1 + R_2) \text{ versin } I_1 - R_2 \text{ versin } I.$$

These formulæ are applied in setting out in the same manner as the similar formulæ for compound curves.

Case of Parallel Tangents.—In case c (Fig. 301) I is zero, and T_1 and T_2 are infinitely great, so that the general formulæ do not serve to enable the curve to be set out. This case is of considerable practical importance, and, since the data almost invariably include the distance between the parallel tangents, formulæ for its solution may be derived as follows.

Let P = the perpendicular distance between the parallel tangents.

In Fig. 304, let EDF be drawn through D parallel to the tangents and meeting O_1A and O_2B in E and F respectively.

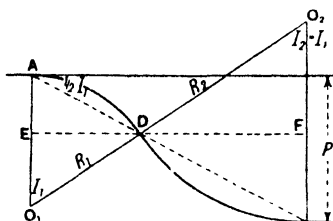


FIG. 304.

Then $P = AE + FB$,

$$= R_1 \text{ versin } I_1 + R_2 \text{ versin } I_2,$$

$$= (R_1 + R_2) \text{ versin } I,$$

$$\text{or versin } I = \frac{P}{(R_1 + R_2)}.$$

If the radii are known, this formula suffices for locating D and enabling the two simple arcs to be set out.

Alternatively, if the radius and central angle of one arc are given, the required radius of the other can be found.

In practice it frequently happens that a reverse curve between parallel tangents is comparatively short, and use may be made of the chord AB in setting out.

It will readily be seen from Fig. 304 that D is always in the line AB. Now $AD = 2R_1 \sin \frac{1}{2}I_1$, and $DB = 2R_2 \sin \frac{1}{2}I_2$,

$$\therefore AB = 2 \sin \frac{1}{2}I_1 (R_1 + R_2) = 2 \cdot \frac{P}{AB} (R_1 + R_2),$$

$$\text{or } AB = \sqrt{2P(R_1 + R_2)}.$$

TRANSITION CURVES ON RAILWAYS

A *Transition* or *Easement* curve is a non-circular arc introduced between a straight and a circular curve, or between the two branches of a compound or a reverse curve. The function of a transition curve in railway alignment is twofold :

1. To eliminate the sudden change of curvature in passing from a straight to a curve or from one arc to another of different radius or direction of curvature.

2. To provide a medium for the gradual introduction, tailing out, or change of the required superelevation of the outer rail.

Change of Curvature.—In passing from a straight into a circular curve, the sudden change of curvature from zero to a finite quantity at the tangent point causes a lurching of the rolling stock due to the abrupt calling into play of centrifugal force. The effect is to throw the train suddenly against the outer rail, and is evidenced by increased wear of that rail in the region of the tangent point. The resulting oscillation not only depends upon the speed, but also increases with increase of change of curvature, and it is therefore more severe at the point of common tangency on a reverse curve and less marked between the two arcs of a compound curve than at the ends of a simple curve of similar radius. By the introduction of a transition curve at those places, the change of curvature may be made continuous and as gradual as desired.

Superelevation.—In railway curves, in order to equalise the wear of the two rails and to eliminate the possibility of the train mounting the outer rail by the action of centrifugal force, that rail is elevated above the inner one by such an amount that the resultant of the weight of the train and the centrifugal force acting upon it is equally distributed to both rails. This is accomplished if the superelevation or cant is such that

$$\frac{\text{cant}}{\text{distance between centres of rails}} = \frac{\text{centrifugal force.}}{\text{weight of train}}$$

The centrifugal force acting upon a body of weight W , travelling with a velocity of v ft. per sec. round a curve of radius R ft., is given by $\frac{Wv^2}{gR}$, where g is 32.2 ft. per sec.², the acceleration due to gravity.

The required amount of cant may therefore be expressed as :

$$\text{cant in inches} = \text{distance between centres of rails in inches} \times \frac{v^2}{gR}$$

A more convenient form of this relation is :

cant in inches =

$$\frac{.8 \times \text{distance between centres of rails in ft.} \times (\text{speed in miles per hr.})^2}{\text{radius in ft.}}$$

With the standard 4 ft. 8½ in. gauge the centres of the rails are about 4 ft. 11 in. apart, and, on substituting this value, the formula becomes :

$$\text{cant in inches} = \frac{3.9 \times (\text{speed in miles per hour})^2}{\text{radius in feet}},$$

$$\text{or} = \frac{.06 \times (\text{speed in miles per hour})^2}{\text{radius in chains}},$$

$$\text{or} = .00068 (\text{speed in miles per hour})^2 \times \text{degree of curve.}$$

In fixing the cant for a particular curve, it is impossible to suit all classes of traffic, and practice varies as to the method of assigning a speed which will determine the value to be adopted. Cant is sometimes made to accord with the probable speed of the fastest trains, regard being paid to circumstances of locality, since speed is reduced on rising gradients and near stations and junctions. Experience shows, however, that too much cant is as objectionable as too little, and it is better to base the cant on some speed less than the maximum, say the average speed of passenger trains. The speed to be allowed for is sometimes fixed from both the maximum and minimum speeds on the curve. Thus, the rule on certain railways is to provide cant suitable for a speed

$$V = \sqrt{\frac{1}{2}(V^2 \text{ max.} + V^2 \text{ min.})}$$

If the cant, as calculated, exceeds a certain limiting value, the speed of fast trains should be reduced on the curve. The most commonly adopted maximum value is 6 in., a greater cant than this being rare.

At the junction of a straight and a circular arc, since the necessary superelevation for the curve cannot be introduced abruptly, three courses are open in the absence of a transition curve :

1. To commence tilting the track on the straight, so that when the tangent point is reached the full superelevation is provided.
2. To introduce the cant along the curve, the rails on the straight being level transversely up to the tangent point.
3. To combine the above methods, so that the cant is introduced partly on the straight and partly on the curve.

These methods are objectionable, since the requirement that superelevation should be proportional to curvature is not fulfilled. The effect of a transverse inclination on the straight is to cause lurching of the rolling stock against the low rail, with excessive wear of that rail. With insufficient cant on the curve, the effect is transferred to the elevated rail. The first and third methods are not uncommon, and if the latter is accompanied by an arbitrary "sweetening" of the junction when laying the rails, its faults are to some extent removed. The objections to the above methods are, however, entirely obviated by the introduction, between the straight and the circular curve, of a transition arc of such varying curvature as will enable the cant to be introduced at any desired rate, the curvature at any point corresponding to the cant required at that point.

Requirements of a Transition Curve connecting a Straight and a Circular Arc.—These are as follows :

1. The transition should be tangential to the straight.
2. Its curvature should be zero at the origin on the straight.
3. The curvature should increase along the transition at the same rate as the cant increases.

4. Its length should be such that at the junction with the circle the full cant has been attained.

5. It should join the circular arc tangentially.

6. Its curvature at this point should be the same as that of the circle.

The conditions as to the tangency and curvature at the origin and the end of the transition are definite, but the rate of change of curvature and the length are dependent on the rate at which the cant is introduced. It is the universal practice to increase the cant at a uniform rate, and, in consequence, the curvature of the transition at any point is proportional to the distance of that point from the origin.

Length of Transition Curves.—The length of transition curve required to enable the cant to be applied at a suitable rate may be determined in three ways.

Let L = length of transition in ft.,

V = speed on which cant is based, in miles per hour,

v = " " " " in ft. per sec.,

C = cant in inches for standard gauge,

R = radius of circular curve in ft.

First Method.—The length may be that required to permit the superelevation to be introduced according to an arbitrary gradient. This is a common system, and the rate of canting varies in practice from 1 in. in 25 ft. to about 1 in. in 100 ft., so that L runs from 300 to about 1,200 times the cant.

On this principle, if the gradient is 1 in. in s ft.,

$$L = \frac{3.9V^2s}{R}$$

Second Method.—The length may be such that trains are canted at an arbitrary time rate. This rate may range from 1 in. to over 2 in. per second.

If r = rate of canting in inches per second,

the time taken in travelling over the transition = $\frac{L}{v} = \frac{C}{r} = \frac{3.9V^2}{Rr}$ sec.,

$$\text{so that } L = \frac{5.72V^3}{Rr}$$

Third Method.—The length may be such as to enable the rate of change of radial acceleration on the transition curve to be limited to the extent that no sensation of discomfort may be experienced by passengers. Mr. Shortt* states that in his experience a rate of change of acceleration of 1 ft. per sec.² per sec. is the maximum that will pass unnoticed.

Let a = allowable rate of change of acceleration in ft. per sec.³,

v, V = max. speed in ft. per sec. and miles per hour, respectively.

* "A Practical Method for the Improvement of Existing Railway-Curves," by W. H. Shortt, *Min. Proc. Inst. C.E.*, Vol. CLXXVI.

The time taken to travel over the transition = $\frac{L}{v}$ sec.

Acceleration attained in that time = $\frac{aL}{v}$ ft. per sec.²,

= $\frac{v^2}{R}$, the radial acceleration on the circular arc ;

whence $L = \frac{v^3}{aR}$;

or, for $a = 1$ ft. per sec.³,

$$L = \frac{3 \cdot 155 V^3}{R}.$$

Either of the methods making $L \propto V^3$ is preferable to the first.

The Ideal Transition Curve or Euler Spiral.—Let

R = the radius of the circular curve (Fig. 305),

r = the radius of the transition at any point P , l ft. along it from the origin T ,

c = the cant at P ,

ϕ = the angle between the straight and the tangent to the transition at P ,

ϕ_1 = the value of ϕ at S , the point of junction with the circle,

L = the total length of the transition.

Since cant will increase uniformly along the transition,

$$\frac{c}{l} = \text{constant},$$

but cant is inversely proportional to radius,

$$\therefore rl = \text{constant} = RL. \quad (1)$$

Now $\frac{1}{r} = \frac{d\phi}{dl}$, the curvature,

$$\therefore \frac{l}{RL} = \frac{d\phi}{dl},$$

$$\text{or } d\phi = \frac{ldl}{RL},$$

and integrating,

$$\phi = \frac{l^2}{2RL} + k.$$

When $l = 0$, $\phi = 0$,

$$\therefore k = 0,$$

so that $\phi = \frac{l^2}{2RL}$, the intrinsic equation of the Euler spiral. (2)

For the purpose of setting out this curve by offsets from the tangent at the origin T , the Cartesian co-ordinates, referred to the tangent as the x -axis, would require to be calculated, and may be obtained as follows.

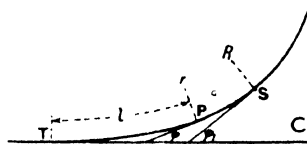


FIG. 305.

$$\frac{dy}{dl} = \sin \phi = \left(\phi - \frac{\phi^3}{3!} + \frac{\phi^5}{5!} - \frac{\phi^7}{7!} + \dots \right),$$

but, from the intrinsic equation, $dl = \frac{RLd\phi}{l}$ and, eliminating l ,

$$dl = \frac{RLd\phi}{\sqrt{2RL\phi}} = \frac{\sqrt{2RL} \cdot d\phi}{2\sqrt{\phi}};$$

$$\therefore dy = \frac{\sqrt{2RL}}{2} \left(\phi^{\frac{1}{2}} - \frac{\phi^{\frac{3}{2}}}{3!} + \frac{\phi^{\frac{5}{2}}}{5!} - \frac{\phi^{\frac{7}{2}}}{7!} + \dots \right) d\phi,$$

by integrating which,

$$y = \sqrt{2RL} \left(\frac{\phi^{\frac{3}{2}}}{3} - \frac{\phi^{\frac{7}{2}}}{7 \cdot 3!} + \frac{\phi^{\frac{9}{2}}}{11 \cdot 5!} - \frac{\phi^{\frac{13}{2}}}{15 \cdot 7!} + \dots \right), \quad (3)$$

the constant of integration being zero, since $y = 0$ when $\phi = 0$.

The terms in the expression that involve the fractional powers may be eliminated by substituting l for $\sqrt{2RL} \phi^{\frac{1}{2}}$ giving

$$y = l \left(\frac{\phi}{3} - \frac{\phi^3}{7 \cdot 3!} + \frac{\phi^5}{11 \cdot 5!} - \frac{\phi^7}{15 \cdot 7!} + \dots \right) \quad (4)$$

By substitution of $\frac{l^2}{2RL}$ for ϕ , the value of the offset at a distance l along the curve from the origin is given by

$$y = \frac{l^3}{3(2RL)} - \frac{l^7}{7 \cdot 3!(2RL)^3} + \frac{l^{11}}{11 \cdot 5!(2RL)^5} - \frac{l^{15}}{15 \cdot 7!(2RL)^7} + \dots \quad (5)$$

$$\text{Similarly, } \frac{dx}{dl} = \cos \phi = \left(1 - \frac{\phi^2}{2!} + \frac{\phi^4}{4!} - \frac{\phi^6}{6!} + \dots \right),$$

$$\text{and } x = \sqrt{2RL} \left(\phi^{\frac{1}{2}} - \frac{\phi^{\frac{5}{2}}}{5 \cdot 2!} + \frac{\phi^{\frac{9}{2}}}{9 \cdot 4!} - \frac{\phi^{\frac{13}{2}}}{13 \cdot 6!} + \dots \right), \quad (6)$$

$$= l \left(1 - \frac{\phi^2}{5 \cdot 2!} + \frac{\phi^4}{9 \cdot 4!} - \frac{\phi^6}{13 \cdot 6!} + \dots \right), \quad (7)$$

$$\text{or } = l - \frac{l^5}{5 \cdot 2!(2RL)^2} + \frac{l^9}{9 \cdot 4!(2RL)^4} - \frac{l^{13}}{13 \cdot 6!(2RL)^6} + \dots \quad (8)$$

The curve therefore does not lend itself to simple expression in Cartesian co-ordinates. The series, however, rapidly converge, since in the practical case ϕ never exceeds a small fraction of a radian. The x co-ordinate is of minor importance, since the offsets y could be employed in conjunction with lengths l along the curve, the setting out being then performed by chords and offsets.

In place of setting out the curve by offsets, use may be made of the method of deflection angles and chords from the starting-point

T. Let α be the deflection angle from the tangent at T to a point l ft. along the curve, then

$$\begin{aligned}\tan \alpha = \frac{y}{x} &= \frac{\frac{\phi^{\frac{3}{2}}}{3} - \frac{\phi^{\frac{7}{2}}}{7.3!} + \frac{\phi^{\frac{11}{2}}}{11.5!} - \dots}{\phi^{\frac{1}{2}} - \frac{\phi^{\frac{5}{2}}}{5.2!} + \frac{\phi^{\frac{9}{2}}}{9.4!} - \dots}, \\ &= \frac{\phi}{3} + \frac{\phi^3}{3.5.7} + \frac{26\phi^5}{3^4.5^2.7.11} - \frac{17\phi^7}{3^3.5^3.7.11.13} - \dots \quad (9) \\ &\approx \frac{\phi}{3} + \frac{\phi^3}{105} + \frac{\phi^5}{5997} - \frac{\phi^7}{198700} \quad (10)\end{aligned}$$

This resembles the expansion,

$$\tan \frac{\phi}{3} = \frac{\phi}{3} + \frac{\phi^3}{81} + \frac{2\phi^5}{3.645} + \dots, \text{ sufficiently closely for the required}$$

values of ϕ to justify the approximation,

$$\alpha \approx \frac{\phi}{3}; \quad (11)$$

or, on substituting $l^2/2RL$ for ϕ ,

$$\alpha \approx \frac{l^2}{6RL} \text{ radians.} \quad (12)$$

By using the expansion

$$\alpha = \tan \alpha - \frac{1}{3} \tan^3 \alpha + \frac{1}{5} \tan^5 \alpha - \frac{1}{7} \tan^7 \alpha + \dots$$

we get from (9)

$$\alpha = \frac{\phi}{3} - \frac{2^3\phi^3}{3^4.5.7} - \frac{2^5\phi^5}{3^5.5^2.7.11} - \frac{23.2^7\phi^7}{3^7.5^3.7^2.11.13} - \dots \quad (13)$$

$$= \frac{\phi}{3} - \frac{8\phi^3}{2,835} - \frac{32\phi^5}{467,775} - \frac{2944\phi^7}{1,915,538,625} - \dots \quad (14)$$

and, also, by reversal of the series (13)

$$\phi = 3\alpha + \frac{2^3}{5.7} \alpha^3 = \frac{43.2^5}{5^2.7^2.11} \alpha^5 + \frac{1621.2^7}{3^4.5^3.7^3.11.13} \alpha^7 + \dots \quad (15)$$

Modifications of the Ideal Transition Curve.—With a view to simplifying the calculation of the quantities required in setting out a transition, various forms of curve, differing only slightly, within the length required, from the ideal curve, have been proposed, and are in use.

The Cubic Spiral.—If ϕ is a relatively small angle, as it usually is in railway work, we can put $\phi = \sin \phi$ so that $\frac{dy}{dl} = \phi$, and, using equation (2₂) the expression for the offset y becomes

$$y = \frac{l^3}{6RL}, \text{ the equation of a cubic spiral.}$$

This curve, which is also directly derivable from the Ideal Transition

curve by omitting all the terms after the first on the right of equation (5), page 447, is well adapted for location by chords and offsets from the tangent. If, however, the method of deflection angles is preferred, the approximation $\alpha = \frac{1}{3}\phi = \frac{l^2}{6RL}$ radians is quite valid

The Cubic Parabola.—This curve, which is sometimes known as Froude's Transition Curve, is obtained as an approximation to the cubic spiral by writing x instead of l in the equation of the latter, so that the equation of the new curve becomes

$$y = \frac{x^3}{6RL}$$

Owing to the approximations made, the curve is inferior to the cubic spiral, and still more so to the Euler spiral, but, because of the ease of setting out by rectangular co-ordinates, it is sometimes used as a transition curve.

Relationship between Transition and Main Curves.—In order to fit in equal transitions between the ends of a circular curve of pre-determined radius, it is usual to move the centre of the circular curve away from the point of intersection of the straight tangents along the line bisecting the angle of intersection of these tangents. This means fitting the actual circular curve between new tangents parallel to the old ones but shifted from them by an amount, s , called "the shift". These new tangents are called the "shift tangents".

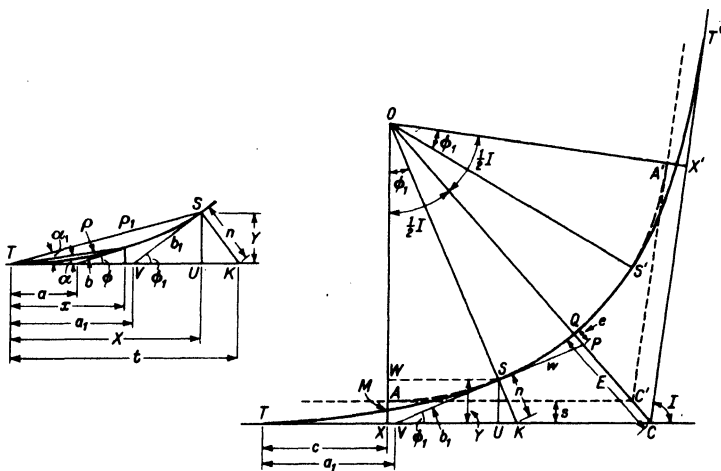


FIG. 306

In Fig. 306 TC and T'C are the main tangents, the angle of intersection of these measured from the forward direction of the tangent TC being I . The arc SS' is the circular arc of radius R and centre O and TS and S'T' the equal transition curves. From O draw OX and OX' perpendicular to the tangents TC and CT' and produce the circular arc SS' to meet OX and OX' at A and A' respectively. Through A and A' draw the lines AC' and A'C' parallel to TC and T'C to meet at C'. These lines are the shift tangents tangential to the circular arc ASS'A' at A and A' and AX = A'X' is the shift s . From S draw SV tangential to SS' at S and meeting TC in V. Then SV is also the tangent to the transition at its end S and the angle SVC = ϕ_1 is the total deflection of the transition curve. QC, where Q on the line OC is the middle point of the arc SS', is the external distance E . TX is the shift point distance c and TV and VS the long and short subtangents, a_1 and b_1 , respectively.

From the Figure,

$$T = TC = TX + XC = c + (R + s) \tan \frac{1}{2} I \quad (1)$$

$$s = AX = WX - WA = WX - (OA - OW) = Y - R \operatorname{versin} \phi_1 \quad (2)$$

$$c = TX = TU - XU = TU - WS = X - R \sin \phi_1 \quad (3)$$

By using the expansions for Y , X , $\operatorname{versin} \phi_1$ and $\sin \phi_1$, we easily obtain

$$s = \frac{1}{2} L \left\{ \frac{\phi_1}{3.2!} - \frac{\phi_1^3}{7.4!} + \frac{\phi_1^5}{11.6!} - \frac{\phi_1^7}{15.8!} + \dots \right\}. \quad (2a)$$

$$c = \frac{1}{2} L \left\{ 1 - \frac{\phi_1^2}{5.3!} + \frac{\phi_1^4}{9.5!} - \frac{\phi_1^6}{13.7!} + \frac{\phi_1^8}{17.9!} - \dots \right\}. \quad (3a)$$

Also, it will be seen that if OS is produced to meet TC at K and SK = n and TK = t , then

$$n = (R + s) \operatorname{cosec} \phi_1 - R = Y \sec \phi_1 = b_1 \tan \phi_1, \quad (4)$$

$$t = c + (R + s) \tan \phi_1 = X + Y \tan \phi_1 = a_1 + b_1 \sec \phi_1. \quad (5)$$

In the case of the cubic spiral and cubic parabola (2a) becomes

$$s_s \approx \frac{L^2}{24R} \quad \text{and} \quad s_p \approx \frac{X^2}{24R}$$

$$\text{Now as} \quad \phi_1 = \frac{AS}{R} = \frac{L}{2R}, \quad \text{we have}$$

$$AS = \frac{1}{2} L \approx \frac{1}{2} X.$$

In practical cases, MS is very approximately equal to AS so that we can put $MS \approx AS$, and M is very approximately the mid-point of the transition curve.

Again, for both cubic spiral and cubic parabola, the offset

$$MX \approx \frac{(\frac{1}{2}L)^3}{6RL} \approx \frac{L^2}{48R} \approx \frac{X^2}{48R},$$

so that the transition curve bisects the shift very approximately.

Setting Out.—The setting out of a railway transition curve, say the cubic spiral, may be performed (a) by deflection angles, (b) by tangent offsets.

By Deflection Angles.—1. Locate the tangent point T by measur-

mg out the tangent length CT, which may with sufficient accuracy be taken $= (R+s) \tan \frac{1}{2}I + \frac{1}{2}L$.^{*} Alternatively, the position of T may be found by first locating X from the measurement CX $= (R+s) \tan \frac{1}{2}I$, setting out the perpendicular offset XM $= \frac{1}{2}s$, and swinging the distance MT $= \frac{1}{2}L$.

2. Obtain the chainage of T and from it the length of the first chord in the transition. It is desirable to make the standard chord length C on the transition $\frac{1}{2}$ or $\frac{1}{3}$ of the usual peg interval, and the deflection angles are obtained from

$$\begin{aligned} a &= \frac{l^2}{6RL} \text{ radians,} \\ &= \frac{n^2 C^2}{6RL}, \text{ where } n = \text{the number, integral or fractional, of} \\ &\quad \text{chord lengths } C \text{ from the origin,} \\ &= \frac{573n^2 C^2}{RL} \text{ minutes.} \end{aligned}$$

Tabulate the deflection angles up to that of S, which $= \frac{1}{3}\phi_1 = \frac{L}{6R}$ radians $= \frac{573L}{R}$ minutes.

3. Obtain the chainage of S by adding L to that of T, and tabulate the deflection angles for the circular curve referred to the tangent at S. The maximum deflection angle is $(\frac{1}{2}I - \phi_1)$, and the length of the circular arc $= \frac{\pi(I - 2\phi_1)R}{180^\circ}$.

4. Set the theodolite over T, sight along TC, and proceed to lay off the successive deflection angles and peg the curve up to S. The position of S may be conveniently checked by measuring the offset $SU = \frac{L^2}{6R}$.

5. Transfer the theodolite to S. Since TSV $= \frac{2}{3}\phi_1$, the direction of SV may be obtained from a backsight on T. For a right-hand curve this angle should be set off behind zero. After sighting T and transiting the telescope, the vernier can be set to the first deflection angle for the circle.

6. Set out the circular arc in the usual way up to the end S¹, and obtain a check by measuring the offset to the second tangent at that point. Another check is to chain the external distance $E = [(R+s) \sec \frac{1}{2}I - R]$ from C the point of intersection of the tangents to Q the middle point of the circular curve.

Notes.—(1) Continuity of the chainage is sometimes disregarded on the transition curve, no initial sub-chord being used. The chainage of S and the position of the first peg on the circular arc are, however, obtained as above. Setting out can be much simplified by the use of Tables such as Table I for the Euler Spiral on pages 638-640.

^{*} The formulae given here are sufficient for railway transition curves where the deflection angle ϕ_1 of the transition is small. See Appendix IV, pages 630-641, for the case where the deflection angle is large, as in road transition curves.

(2) In place of setting out the circular curve from S, the theodolite may be set up at A and the deflection angles laid out with reference to the shift tangent. This method has usually the advantage of affording a longer back-sight. The necessary changes in the computations will be easily deduced.

By Tangent Offsets.—In this method, T is located and its chainage obtained as before. Offsets are computed from $y = \frac{l^3}{6RL} = \frac{(nC)^3}{6RL}$ for the initial sub-chord and the succeeding uniform chords, each peg being located by swinging the chord length from the preceding peg until the required offset is obtained. The chainage of S is determined as before, and the setting out of the circular arc proceeds by deflection angles from S or A.

Note.—In the case of the cubic parabola, convenient abscissæ are measured out along TC.

Transitions between Branches of Compound and Reverse Curves.—The length L of transition required between the branches of a compound curve is that necessary to deal with the difference of cant or with the difference of radial acceleration, according to the principle employed for determining length. The formulæ of pages 445 to 449 are therefore made applicable by putting $\frac{R_1 R_2}{(R_2 - R_1)}$ for R .

In this case the shift at the junction between the branches is the radial length of gap between the two shifted arcs. Its value is obtained by means of the same substitution for R , giving $s \approx \frac{L^2(R_2 - R_1)}{24R_1 R_2}$. As before, the transition bisects, and is bisected by,

the shift. The values of deflection angles are obtained by similarly modifying the previous formulæ, but in this case the length of the transition curve is frequently so short that it is commonly sufficient to peg the mid-point and ends of the transition and, if necessary, locate one additional peg on each half by offsets.

In the case of a transition between the two branches of a reverse curve, the length L is the sum of the lengths l_1 and l_2 necessary for the individual radii R_1 and R_2 . The gap between the two shifted arcs at the original point of reverse is $\frac{L^2(R_1 + R_2)}{24R_1 R_2}$. This original inflection point bisects the transition, but, unless the radii are equal, it does not bisect the gap, the individual shifts of the circular arcs from the common tangent being given by $\frac{l_1^2 + 3l_2^2}{24R_1}$ for the curve of radius, R_1 , and by $\frac{3l_1^2 + l_2^2}{24R_2}$ for that of radius R_2 . The new tangent to the spirals at their common origin passes through the original point of inflection and makes an angle θ with the original tangent at that point, where θ is given by $\tan \theta = \frac{l_1}{2R_2} = \frac{l_2}{2R_1}$. The commencement of the two transition curves is shifted along this line in

the direction of the flatter curve by the amount $\frac{1}{2}(l_1 - l_2)$, and the shifts of the curves measured from the line are given by $\frac{l_1^2}{24R_1}$ and $\frac{l_2^2}{24R_2}$ respectively.*

The most convenient method of setting out the transition is to locate the points at which the transition joins the shifted curves by measuring out $\frac{1}{2}L$ on either side of the original point of reverse. Points on the transition are then obtained by laying off deflection angles or by cubic offsets from the shifted arcs. The point of inflection of the transition coincides with that of the original curve only when the radii are equal.

TRANSITION CURVES ON ROADS

The transition curve was originally designed for application to railway curves, but, owing to the appearance of the high-speed motor car, it has now been applied to roads, although no serious effort to do so appears to have been made in England until 1925-26, when road transition curves were used on the Great North Road, in the county of Rutland.†

There are several fundamental differences in the conditions governing the application of transition curves to railways and to roads. The first is that curves on railways are usually much flatter than curves on roads, so that, for a given deflection between tangents, the length of curve used is much greater for the one than it is for the other. As a consequence, railway curves consist for the most part of circular curves with short transition curves at either end. On the other hand, road curves may consist of quite short circular arcs with comparatively long transitions at each end, and, in certain circumstances, the curve may be transitional throughout—that is, one-half may consist entirely of a transition curve and the other half of a mirror image of the same curve. In this way, the curvature is gradually increased from zero to a certain maximum and then gradually reduced again.

A second, and the most important point, is that, on a railway, any sideways thrust along the plane of the rails due to centrifugal force, which is not balanced by the sideways component of the weight of the vehicle due to cant or super-elevation, is taken up almost entirely by the pressure exerted by the rails on the flanges of the wheels. In the case of roads, any unbalanced side thrust must be

* For derivation of these formulæ see Perrott and Badger, *The Practice of Railway Surveying and Permanent Way Work*.

† See "A Simple Treatment of Superelevation, Transition Curves and Vertical Curves as used on the Great North Road in the County of Rutland, 1925-26," by H. Criswell, M.Inst.C.E. *Public Works, Roads and Transport Congress, Final Report*, 1929.

taken up by the adhesion between the tyres of the vehicle and the surface of the road, and, if the side thrust exceeds a certain value, the vehicle will either sideslip or, in certain cases, overturn. On a flat road, with no cant, the condition for overturning before slipping takes place is that :

$$\mu \geq \frac{\text{half-gauge}}{\text{height of centre of gravity.}}$$

where μ = co-efficient of adhesion between tyre and road surface. In all modern motor cars, however, the centre of gravity is so low that, for all normal road conditions, the car will side-slip before it overturns. Consequently, in curve design, the factor governing the maximum amount of curvature allowable is that at which side-slip will occur.

Maximum Allowable Curvature for Standard Velocity.—Let R be the minimum radius of curvature for the standard velocity V . Then, on a curve where there is no cant, side-slip will occur if the side thrust due to centrifugal force is greater than the adhesion between the tyres and the road surface. That is, if :

$$\frac{Wv^2}{gR} > \mu W.$$

Hence, the smallest radius of curvature allowable is given by :

$$R = \frac{v^2}{\mu g}.$$

If V is v in miles per hour, $g = 32.2$ ft. per sec. per sec., and R is in feet, this gives :

$$V^2 = 14.969\mu R.$$

On roads, under average conditions, the co-efficient of adhesion, μ , may be taken as 0.25. Here, inserting this value,

$$R = \frac{V^2}{3.742} = 0.2672 V^2.$$

If R_{κ} and V_{κ} are the values of R and V in meters and in kilometers per hour respectively, we get :—

$$R_{\kappa} = \frac{V_{\kappa}^2}{31.82} = 0.03143 V_{\kappa}^2.$$

If there is superelevation, or cant, of angle β , the sideways thrust Wj , parallel to the surface of the road is the difference between the components of the centrifugal force and the weight resolved along the direction of the cant, viz.

$$Wj = \frac{Wv^2}{gR} \cos \beta - W \sin \beta,$$

in which j , the unbalanced sideways thrust per unit weight, is called the "Lateral Ratio". As β is a small angle not greater than

$\sin^{-1} 0.1$ (page 462), we have, with β in radians or expressed as a gradient:—

$$j = \frac{v^2}{gR} - \beta,$$

$$= f - \beta,$$

where $f = \frac{Wv^2}{gR}/W = \frac{v^2}{gR} = 0.066805 \frac{v^2}{R}$ is called “the Centrifugal Ratio”. It is now generally assumed that j is the factor which causes discomfort to passengers and its permissible value is usually taken from 0.16 for low speeds to 0.10 for high speeds. When $\beta = 0$, j is equivalent to μ so that this value of j gives a somewhat lesser value of μ than that already given above, and this gives a greater value of R for a given value of V , which makes for rather safer running.

When $j = 0$, $f = \beta$ and the velocity at which this takes place is known as the “hands off” speed. At this speed, which is given by $v_h = \sqrt{gR \tan \beta}$, the car will steer itself round the curve without guidance from the wheel.

Length of Transition Curve Required for Maximum Curvature.—

The length of the transition curve required for maximum curvature is generally determined from a , the rate of change of radial acceleration, so that (page 446) we have

$$a = \frac{v^3}{LR}$$

Different authorities use different values of a but a common rule in this country is to adopt the value of 1 ft. per sec³ recommended by Mr. Shortt for use in railway transition curves. In America it is more usual to adopt a value of 2 ft. per sec³, while, as one among compromises, Tyson has suggested a sliding scale of $a = 1.75 - 0.0075V$, (desirable) or $a = 2.10 - 0.0090V$ (emergency), where a is in ft./sec³ and V is in miles per hour.

Assuming a value of $a = 1$ ft./sec³, and reckoning L and R in feet and V in miles per hour, we have as in page 446—

$$L = \frac{3.155V^3}{R}$$

If L and R are expressed in meters, we have

$$L = 12.73 \sqrt{R_{\text{m}}}.$$

Choice of Transition Curve.—The theoretical condition to be fulfilled by a transition curve is that the radius of curvature should decrease in proportion to the length, or very approximately so. In addition, it is desirable that the rate of decrease of the radius of curvature should, if anything, be less towards the end of the transition than at the beginning.

The theoretical curve in which the radius of curvature is exactly inversely proportional to the length is known as the transition or

Euler spiral or clothoid, which we have already studied in connection with railway transition curves, and whose equation is:

$$\phi = \frac{l^2}{2RL}$$

or,

$$l = m\sqrt{\phi}$$

$$\text{where } m = \sqrt{2RL}$$

The lemniscate, whose properties will be investigated later, is a curve which satisfies the condition that the rate of decrease of the radius of curvature should decrease towards the end of the transition.

A third curve, which approximates to the other two in the early stages, is the cubic parabola.

Of these three curves, the third is not generally used for road work as the curvature increases up to a maximum, and then, as soon as the polar deflection angle exceeds 9° , commences to decrease again. Hence, this curve is not suitable as a transition when the polar deflection angle exceeds 9° .

The spiral, or clothoid, and lemniscate are very similar up to a polar deflection angle of 45° . In both curves, the radius of curvature gradually diminishes, but, whereas in the spiral the radius of curvature goes on diminishing, so that it tends to reach zero as a limit, in the lemniscate it reaches a minimum value when the polar deflection angle is 45° and then gradually increases again. Also, in the case of the lemniscate, the rate of diminution of the radius is more gradual than it is in the case of the spiral, and advocates of the lemniscate claim that this makes for greater safety, as it is equivalent to an easing off of the transition at its junction with a circular curve.

The lemniscate also has the valuable property, from the setting-out point of view, that the exterior deflection angle between the end tangents is always exactly three times the polar deflection angle, no matter how large the latter may be. This is not rigorously true of the spiral, although it is very approximately so when the polar deflection angle is small. However, in practice this is not really a very important point as the correction $\gamma = \frac{1}{3}\phi - \alpha$ to be subtracted from $\frac{1}{3}\phi$ to give α in the case of the Euler spiral can be easily computed or obtained from Tables.

The lemniscate is a symmetrical curve which can be used with large deflection angles, an extreme case being when the exterior deflection angle between the tangents is 270° . In this case, the curve doubles back on itself and crosses the original starting-point at right angles to its original direction, thus forming one loop of the figure 8. Against this is the fact that the length of the curve is awkward to compute, though it may be obtained from Tables. Moreover, the use of the lemniscate involves the determination of the additional parameter C .

Both the spiral and lemniscate have been extensively used in modern road work in this country, the lemniscate often being chosen

in preference to the other when the deflection angle is large. One main reason for this preference seems to be the ease of setting out by polar deflection angles in virtue of the exact relation $\alpha = \frac{1}{3}\phi$. Apart from this, for all cases met with in ordinary road practice the difference in alignment of the curves for the same conditions is very small, even with large deflection angles, and, with good Tables, there is no difficulty in setting out the spiral. It is the curve most generally used abroad and at home the tendency also now is to employ it in preference to the lemniscate.

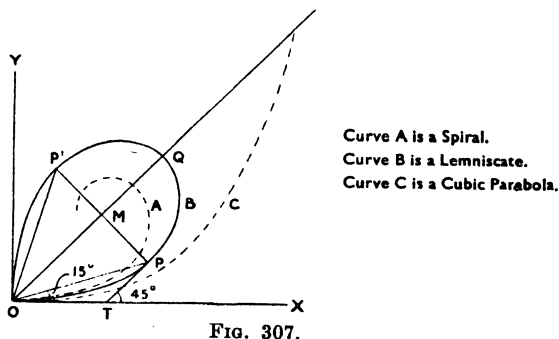
The Ideal Transition Curve or Euler Spiral as a Road Transition Curve.—When the Euler Spiral is used as a road transition curve the deflection angle is often large and for this reason the approximations justifiable in the case of its use as a railway transition curve are no longer valid and we must use the more complete formulae and proceed as described in Appendix IV, pages 630-641.

The Lemniscate Transition Curve.*—The polar equation of the lemniscate is :

$$\rho = C\sqrt{\sin 2\alpha}$$

where ρ denotes the polar ray or radius vector, α the polar deflection angle and C is a constant.

The maximum value of ρ occurs when $\alpha = 45^\circ$, when $\rho = C$; while ρ is 0 when $\alpha = 0^\circ$ and when $\alpha = 90^\circ$. The curve is therefore of the form shown in Fig. 307†, in which, for purposes of rough com-



* The first to draw attention to the possibilities of the lemniscate of Bernoulli as a transition curve appears to have been Paul Adams in France, who, in 1895, suggested its application to railway curves, and, in 1919, C. Galatoire-Malegarie suggested its application to road curves. The theory of the lemniscate as a road transition curve was not, however, put on to a really scientific basis until the publication, in 1931, of Professor F. G. Royal-Dawson's book, *Elements of Curve Design for Road, Railway and Racing Track on Natural Transition Principles*.

† Only one "leaf" of the curve—that in the first quadrant—is shown in Fig. 307. The complete curve consists of four "leaves"—one in each quadrant—each similar to the one shown.

parison of general shapes, the inner dotted curve shows part of the spiral and the dotted outer curve part of the cubic parabola.

The following are the essential formulæ for the lemniscate:—

$$\begin{aligned} \rho &= C\sqrt{\sin 2a} & \rho_1 &= C\sqrt{\sin 2a_1}. \\ a &= \frac{1}{3}\phi, & a_1 &= \frac{1}{3}\phi_1. \\ C &= \sqrt{3\rho r} = \sqrt{3\rho_1 R} \\ &= 3r\sqrt{\sin 2a} = 3R\sqrt{\sin 2a_1}. \\ x &= \rho \cos a, & y &= \rho \sin a. \\ X &= \rho_1 \cos a_1, & Y &= \rho_1 \sin a_1. \\ l &= C\sqrt{2a} \left[1 + \frac{1}{15}a^2 + \frac{1}{90}a^4 + \frac{61}{24,570}a^6 + \frac{1,261}{1,927,800}a^8 \right. \\ &\quad \left. + \frac{79}{415,800}a^{10} + \dots \right], a \text{ being in radians,} \\ &\approx \frac{ra^\circ}{9.55} \text{ for small values of } a \text{ in degrees.} \\ s &= \text{shift} = Y - R \text{ versin } \phi_1. \\ c &= \text{shift point distance} = X - R \sin \phi_1. \\ T &= \text{total tangent length} = (R + s) \tan \frac{1}{2}I + c. \end{aligned}$$

Proofs of the less obvious of these formulæ are given in Appendix IV, pages 634-636.

The following relations for the lemniscate may also be noted :

In Fig. 307, OQ, the polar ray for $a = 45^\circ$ is the major axis. Draw the polar ray OP making angle $a = 15^\circ$ with OT. Then tangent at P makes angle $\phi = 45^\circ$ with OT. Consequently, PT is parallel to OQ. Draw PMP' perpendicular to OQ, meeting the other side of curve in P'. Then PMP' is the minor axis and POP' is an equilateral triangle. Also, $PP'/OQ = 1/\sqrt{2}$ and ratio of length of curve OPQ to OQ is 1.31115 : 1, while radius of curvature at Q is $\frac{1}{3}$ OQ.

As mentioned earlier, the chief difficulty with the lemniscate is the calculation of the length as this is obtained in the form of a series which is awkward to compute. The series given above is due in the first place to the late Capt. G. T. McCaw. A short Table to facilitate the computation of the length of the arc for different values of a is given in Appendix IV, page 641.

Unit-Chord System of Measurement for Transition Curves.—

Prof. Royal-Dawson has pointed out that, as a means of measuring transition curves, the use of radii of curvature or of chords subtending a unit of angle, as in the degree system, has the disadvantage that the formulæ involved depend upon the particular unit of length employed, so that inconvenience is caused when transformations

have to be made from one unit of length to another. Hence, he has proposed a different system in which the unit to be used in measuring transition curves is some quantity which is inherent in the curve itself, and, for this purpose, he takes the polar ray given by a deflection of 16 minutes of arc.* This unit is independent of the unit of length and is applicable to all transition curves. When the polar deflection angle is small it is very approximately proportional, in the case of all transition curves, to the square of the length of the curve. Hence, if the curve is set out in unit-chords, the deflection of the first chord will be 16', of the second $2^2 \times 16'$, of the third $3^2 \times 16'$, and so on for the first few degrees, after which modifications or corrections have to be introduced. Also, if the unit-chord length is inconveniently long for setting-out purposes, and it is desired to use, say, a half unit-chord length instead, the initial deflection angles become 4', 16', 36', 64', etc.

In the case of the spiral, when ϕ and α are small we can write $\phi = 3\alpha$ and hence, from the equation of the curve,

$$3\alpha = \frac{l^2}{2RL}$$

Now for $\alpha = 16'$ the chord length will be practically the same as that of the arc and hence putting $l = 1$, we have

$$RL = \frac{1}{6\alpha},$$

where R and L are measured in unit-chord lengths and α is in radians. But $6\alpha = 96' = 0.027\ 9253$ radians and therefore

$$RL = 35.80982 = 35.81 \text{ approximately.}$$

Again, $rl = RL = 35.81$

and so the radius of curvature in unit-chords at the end of the first chord where $l = 1$ will be $r_1 = 35.81$ unit-chords. At the end of the second chord the radius of curvature $r_2 = 35.81/2 = 17.905$ unit-chords and at the end of n chords it will be $35.81/n$ unit-chords.

Similarly, for the lemniscate we have.

$$\rho = C\sqrt{\sin 2\alpha}$$

When $\alpha = 16'$, $\rho = 1$ and we have

$$C = \frac{1}{\sqrt{\sin 32'}},$$

giving

$$C = 10.36490 \text{ unit-chords.}$$

Also, as

$$C = \sqrt{3\rho r} = \sqrt{3\rho_m R},$$

$$\sqrt{3\rho r} = 10.3649 \text{ unit-chords,}$$

provided ρ and r are both measured in unit-chords.

* See *Elements of Curve Design for Road, Railway and Racing Track*, by F. G. Royal-Dawson, 1932. Also, *Road Curves for Safe Modern Traffic*, 1936, by the same author.

Determination of Length of a Unit-Chord in Terms of a Particular Unit of Length.—The length of the unit-chord in terms of a particular unit of length is found from the formula giving the maximum change of radial acceleration. In the spiral, the rate of change of radial acceleration is constant throughout the curve, and, in the lemniscate, it is a maximum at the beginning of the curve, after which it begins to decrease very slowly to a minimum at the end of the major axis. Hence, in both curves, if D is the length of a unit-chord in feet, a rate of change of 1 ft. per sec.³ will nowhere be exceeded if we put $L=D$, $R=r_1 \times D = 35.81 \times D$ in the formula $L = \frac{3.155 V^3}{R}$, which is based on this rate of change.

$$\text{This gives: } D = \frac{3.155 V^3}{35.81 D}$$

$$\text{or, } 11.4 D^2 = V^3$$

and, from this expression, the value of D in feet can be calculated.

If V is given in kilometers per hour and D is required in meters, then:

$$510 D_m^2 = V_k^3$$

Application of Unit-Chord System to the Lemniscate.*—Substituting the value $C = 10.3649$ unit-chords in the equation of the lemniscate, this equation becomes:

$$\rho = 10.3649 \sqrt{\sin 2a} \text{ unit-chords.}$$

When D has been computed, and it is required to find ρ in feet, the formula is:

$$\rho = 10.3649 \cdot D \sqrt{\sin 2a} \text{ feet,}$$

and, from this, different values of ρ can be calculated for different values of a .

The minimum allowable value of r , in feet, is given by :

$$R = 0.2672 \times V^2,$$

and the maximum value, ρ_m , of ρ can then be found from :

$$\begin{aligned} \sqrt{3\rho_m R} &= 10.3649 D, \\ \text{or, } \rho_m R &= 35.8104 D^2. \end{aligned}$$

If it is considered advisable, or if natural features make it essential, to fix a value of R greater than the minimum theoretical value, the maximum value of ρ can be found by substituting the chosen value of R in this formula.

Having found ρ_m , the maximum permissible value, a_m , of a is

* For method of application to the spiral see the hints given with problem 47 in the examples at the end of this chapter.

found from the equation of the curve. If this value is greater than $\frac{I}{6}$, where I is the exterior deflection angle of the tangents between which it is proposed to run the curve, the latter can be transitional throughout. In this case, the curvature will not reach the maximum allowable for the standard speed, and the maximum value which α will have will then be $\frac{I}{6}$. If α_m is less than $\frac{I}{6}$, the curve cannot be transitional throughout, and, in that event, it is necessary to introduce a circular arc which subtends an angle 2θ at the centre of the circle, where 2θ is given by :

$$2\theta = I - 6\alpha_m.$$

Graphical Methods of Solving Problems in Transition Curve Design.

—Many problems in transition curve design can be solved graphically by means of celluloid lemniscate transition curves which have been designed by Prof. Royal-Dawson and may be obtained from the makers, Messrs. W. F. Stanley & Co. Ltd., who can also supply a sixpenny pamphlet explaining how to use them.

Setting Out the Lemniscate.—When it has been decided whether or not the curve is to be transitional throughout, and the maximum value of α has thus been determined, intermediate values of ρ , for intermediate values of α up to the maximum, can be computed from the equation of the curve.

The next step is to determine the tangent lengths. Referring to Fig. 306, these are given by

$$\begin{aligned} T &= (R + s) \tan \frac{1}{2}I + c, \\ \text{where } s &= Y - R \operatorname{versin} \phi_1, \\ c &= X - R \sin \phi_1, \\ X &= \rho_1 \cos \alpha_1, \quad Y = \rho_1 \sin \alpha_1. \end{aligned}$$

As before, the work can be considerably simplified if suitable Tables are available.

If D , the unit-chord length, is too long for convenient setting out, a working chord length, which is some fraction of D , say $\frac{1}{2}$ or $\frac{1}{4}$, should be chosen. Let this fraction be p , so that the working polar rays become $p \times D$, $2p \times D$, $3p \times D$. . . , and the corresponding polar deflection angles in minutes will be : $p^2 \times 16$, $4p^2 \times 16$, $9p^2 \times 16$. . . , until a polar deflection angle of about 2° is reached. Up to this point the length of the curve will be very approximately equal in length to the polar ray, so that the individual chord lengths between successive points are all approximately equal to $p \times D$.

After the polar deflection angle exceeds 2° successive values of α should be taken such that $\alpha_r = r^2 \times \alpha_1$, where α_1 is the initial polar deflection angle at station 1 and α_r that at station r , but the individual chord lengths should be found by computing the rectangular co-ordinates of successive points on the curve from the relations : $x_r = \rho_r \cos \alpha_r$; $y_r = \rho_r \sin \alpha_r$. The length of the

One commonly used rule is to take the maximum sideways slope equal to 0.4 of the maximum centrifugal ratio, $\frac{V^2}{gR}$, provided the result does not exceed 1 : 10. Having fixed the maximum value at the end of the point of maximum curvature, intermediate amounts of super-elevation are proportional to the length of the curve from its origin.

The datum line to be used in laying out the super-elevation should be the centre line of the road. Thus, with a road 30 ft. wide and a super-elevation of 1 : 10, the outside edge should be raised 1.5 ft. above the centre of the road and the inner edge lowered 1.5 ft. below the centre. The super-elevation is then represented by a uniform gradient at the outside edge of the curve, rising from zero at the origin to the maximum amount at the point of maximum curvature.

Camber should not be combined with super-elevation, although this is sometimes done in practice, and, where camber is used on the straight approaches to the curve, it should be gradually reduced from a point some little distance back from the beginning of the curve until it becomes zero at the zero point of the transition, when it is replaced by the super-elevation.

In order to avoid sudden changes in gradient at the extremities and centre of a curve, it is advisable to insert parabolic vertical curves at these points.

Further information regarding road transition curves, including such matters as cant or super-elevation on roads, widening roads on curves, vertical curves, etc., will be found in Prof. Royal-Dawson's book, *Elements of Curve Design for Road, Railway and Racing Track*, and in its sequel, *Road Curves*. Both volumes contain numerous tables to facilitate the design and setting out of transition curves. Other more recent information will be found in books or papers by J. Barnett (for American practice), H. Creswell, J. J. Leeming and A. G. Tyson listed in the References on pages 470-473.

MISCELLANEOUS OPERATIONS IN SETTING OUT

Setting Slope Stakes.—The operation of slope staking consists in locating and pegging points on the lines in which proposed earth slopes intersect the original ground surface.

In the case whether cross sections have been plotted and the new work is shown thereon, all that is necessary is to scale the horizontal distances representing the side widths or "half-breadths" (page 388) and to set out these measurements on the ground. Slope stakes can, however, be located without the use of plotted cross sections by a trial and error process, which is best followed from an example.

In Fig. 309, let the formation level at a certain point on an embankment, as derived from the profile, be 123.60, the top width being

30 ft., and the side slopes $1\frac{1}{2}$ horizontal to 1 vertical. Let the instrument height of a level commanding the ground be, say,

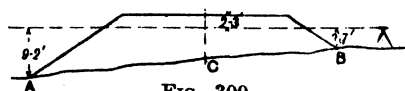


FIG. 309.

121.3. The difference of 2.3 ft. between instrument height and formation level represents a quantity, called the grade staff reading, which in this

case falls to be added to readings of the staff when held on ground points in order to give the depths below formation of those points. If, for example, the centre height of the earthwork is not known, by reason of a local deviation or otherwise, and the staff is held on the ground at the line peg C, giving a staff reading of, say, 5.6 ft., the centre height of the embankment will be 7.9 ft.

To locate the slope peg A, the staffman must estimate where the foot of the slope will come, and he holds the staff there, measuring also its distance from the centre peg C. Let us suppose the staff reading to be 8.3 ft. This represents a depth from formation level of $8.3 + 2.3 = 10.6$ ft., and, if the point selected is really on the toe of the slope, the corresponding side width ought to be $1\frac{1}{2} \times 10.6 + 15 = 30.9$ ft. If the measured distance happens to have this value, the staffman has estimated the position correctly, but it will usually require a second or third trial before the correct point is found. Thus, if the staff reading of 8.3 ft. was obtained at a distance of 27 ft. from C, the surveyor, on finding that the computed side width of 30.9 ft. does not correspond with the measurement, would direct the staff holder to proceed farther from the centre. The next results might be 9.2 ft. on the staff and 32 ft. on the tape, which would be practically correct. In the same manner, the results of the final trial in setting the right-hand slope stake B might be 3.7 ft. on the staff at 24 ft. from C.

This method is very convenient for taking and recording cross sections, and is invariably adopted by American railway engineers. The results in the case of the three level section of our example are conveniently written,

$$\begin{array}{r} -11.5 \\ \hline 32.0 \end{array} \quad -7.9 \quad \begin{array}{r} -6.0 \\ \hline 24.0 \end{array}$$

the upper figures in the case of the side points representing the vertical distances between formation and ground levels, reckoned positive for cuttings and negative for embankments, and the lower figures their distances from the centre. Additional points between the centre peg and the slope stakes may be recorded by observing the staff reading and distance from the centre and applying the grade staff reading positively or negatively to the former. From the side widths and centre and side heights, the area may be computed by the formulæ on pages 389 and 390.

Setting Out Vertical Curves.—At the junction of two railway gradients it is desirable to introduce a vertical curve to round off

the angle and give a gradual change from the one gradient to the other. For convenience in setting out, the parabola is generally employed for this purpose.

The length of curve required to afford a suitable rate of change should evidently depend upon the algebraic difference between the gradients, but the most suitable length for any particular case is largely a matter of opinion. The practice recommended by the American Railway Engineering and Maintenance of Way Association, and which will serve as a good guide, is that for first-class railways the gradient should not change at a greater rate than 0.1 ft. per 100 ft. station on summits and 0.05 ft. per station in sags, while for second-class lines the rate of change should not be more than twice as much. According to this rule, the curve joining a rising gradient of 1 in 100, or 1%, with a falling gradient of 0.5% at a summit on a first-class road would require to have a length of $\frac{1+0.5}{0.1} \times 100$ ft. = 1,500 ft. The length of curve need not,

however, exceed that of the longest train likely to pass over it.

Vertical curves are also required in road work. In this case they are of special importance at summits, and should be of such length as will afford drivers a suitable limit of vision.

Whatever length is adopted, half of it is placed on either side of the apex at which the straight gradients meet. To enable the curve to be set out, its elevation at each chainage peg is obtained from the corresponding elevation of the tangent gradients, given on the working profile, by computing and applying the vertical distance between the tangent and the curve.

In Fig. 310, let two gradients meet at C, and let it be required to join them with a parabolic vertical curve n chains long. Pegs A and B, situated $\frac{1}{2}n$ chains from C, mark the beginning and end of the curve ADB. By taking the average of the known elevations of A and B, obtain that of E, the mid-point of AB, which is situated on the vertical through C. The elevation of C being given on the profile, the dimensions CE is now known, and since the parabola bisects CE at D, the elevation of D is obtained. Now, from the property of the parabola that offsets from a tangent are proportional to the squares of distances along the tangent, the elevations of points on the curve may be computed. Thus, let $\frac{1}{2}n = 5$ chains, the chainage of C being integral as is usual, and let it be supposed that $CD = \frac{1}{2}CE$, has been found to be 0.70 ft., then the offsets in order from A, as shown dotted, are respectively $\frac{1}{16}$, $\frac{1}{4}$, $\frac{9}{16}$, and $\frac{1}{4}$ of 0.70 ft., the corresponding offsets from CB having the same values.

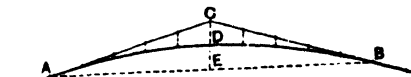


FIG. 310.

Setting Out Tunnels.—The setting out of a tunnel is an operation demanding a high order of precision throughout. The usual type of

engineer's theodolite is not always sufficient, particularly with regard to the quality of the telescope and general stability, and a 6-in. or larger instrument is sometimes necessary.

The simplest case occurs when the two ends of the tunnel are intervisible or visible from an intermediate point, so that its direction can be laid out on the ground without difficulty and continued beyond the entrances. Permanent ranging marks are constructed clear of the work, and from these the line may be projected into the tunnel at each end. If the tunnel is also to be driven from shafts, these must be carefully aligned on the surface, and, when the shaft is sunk, the alignment at the bottom is commonly obtained by suspending two plumb lines from a frame above the shaft. Each consists of copper, brass, or steel wire, and carries a rather heavy weight immersed in a pail of water for steadiness. The plumb lines should be capable of fine adjustment into line, and the upper end of each should therefore be mounted in a manner permitting lateral movement by means of a screw. The distance between the wires forms a short base which is prolonged underground by setting the theodolite in line with them, numerous face right and face left observations being taken to reduce instrumental and observational errors to a minimum. A lamp, screened by having a sheet of tracing cloth pasted over the glass, is placed behind the far wire when the instrument is being aligned, and a similar lamp is used behind any mark being set out. When the shaft is not on the centre line of the tunnel, the plumb lines are usually placed so that the line through them is roughly normal to the centre line. This line is prolonged on the surface to meet the centre line, and the intersection angle is measured, as well as the chainage of the intersection point and its distance from the plumb lines. By setting out the same dimensions underground, the corresponding point on the centre line and the direction of the tunnel are obtained.

Similar care is required in the levelling. The relative levels of formation at the two ends of the tunnel must be carefully determined by running two or more lines of levels between them, the precautions against error following those adopted in precise levelling (Vol. II, Chap. VI). The formation levels at the various shafts are computed, and are established underground by means of vertical steel tape measurements from level marks at the surface.

The more usual case occurs when the ends of the tunnel are not intervisible, and their relative positions must then be carefully determined. This is generally done by triangulation extended from a carefully measured base line (Vol. II, Chap. III), and, in very rough broken country, this may be the more convenient method, great care being taken to see that the signals are truly vertical and properly centered, as failure to exercise the greatest possible vigilance with regard to this point may be a serious source of error in triangulation when the sides are short. In flat country, or in built-up areas, traversing may be the more convenient method,

and there is no reason why it should not be used, even for very long tunnels, provided the taping is very carefully done with a properly standardised steel or invar tape, the legs are sufficiently long, and the angular measurements made with a good micrometer theodolite, reading directly to 10" or less, and, if necessary, each angle measured on several different zeros. The length of the tunnel is ascertained, and intermediate points are located, by co-ordinate calculation from the triangulation or traverse, and the centre line, whether straight or curved, may then be set out on the surface. Instrument stations are established, and by observations from them the plumbing wires at the shafts are aligned as before. In the case of curves, the centre line is best located underground by offsets from a series of chords.

The procedure in setting out tunnels naturally varies according to local conditions. Short of actual experience, an adequate knowledge of the methods employed can be derived only by a careful study of the descriptions given in the publications of engineering societies and in the technical press.

Setting Out Viaduct Piers on a Curve.—In setting out a river viaduct, the location of points on the piers and abutments has generally to be performed several times at various stages of the work. The first operation during construction may consist in the location of staging surrounding each pier, and, according to the class of foundation adopted, cofferdams, cylinders, or caissons will subsequently have to be aligned. On completion of the foundations, the outlines of the piers and abutments are set out upon them, and it may at this stage be practicable to employ instrument stations on the piers. The methods will be sufficiently indicated by describing the location of the centre lines of the piers of a curved viaduct by observations from the river banks only.

Two cases may occur: (1) The tangent points may fall on the banks behind the abutments, so that the whole viaduct is on the curve; (2) The tangent points may be situated in the river, so that only a part of the viaduct is on the curve.

Case 1.—Tangent Points Accessible.—The preliminary data will include: (a) the radius of the curve; (b) the chainage to the face of the first abutment; (c) the angle of skew of that abutment relatively to the tangent to the curve at the point at which the latter cuts the face; (d) the angles between adjacent piers or piers and abutment faces, if they are not parallel; (e) the lengths of the square or the skew spans; (f) the dimensions of the piers and abutments. The situation fixed for the first abutment governs the position of the other lines to be located, and calculation may be made of the further items; (g) the chainage to the centre of each pier and to the second abutment face; (h) the angles of skew made by the centre lines of the piers and by the face of the second abutment with the curve. These quantities may be derived as follows.

In Fig. 311, NPQ and STU represent the parallel centre lines of

two piers or an abutment face and a pier situated on a curve of radius R .

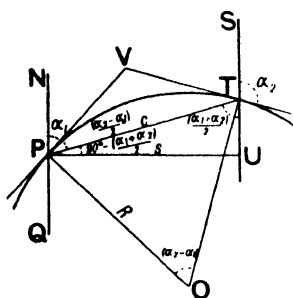


FIG. 311.

Let α_1 and α_2 = the angles of skew made by these lines with the tangents at P and T respectively,

s = the square span PU,

c = the chord length PT = the skew span.

Given R , s , α_1 , and the chainage of P, it is required to calculate α_2 and the chainage of T.

The angle subtended at the centre O = the change in direction of the curve from P to T = $(\alpha_2 - \alpha_1)$,

$$\therefore c = 2R \sin \frac{(\alpha_2 - \alpha_1)}{2}.$$

$$\text{But } PTU = \frac{(\alpha_1 + \alpha_2)}{2},$$

$$\therefore c = s \operatorname{cosec} \frac{(\alpha_1 + \alpha_2)}{2}.$$

Equating these values of c , we have

$$2R \sin \frac{(\alpha_2 - \alpha_1)}{2} \sin \frac{(\alpha_1 + \alpha_2)}{2} = s,$$

$$\text{i.e. } R (\cos \alpha_1 - \cos \alpha_2) = s,$$

whence α_2 .

$$\text{Chainage of T} = \text{chainage of P} + \text{arc PT},$$

$$= \text{chainage of P} + \frac{\pi(\alpha_2 - \alpha_1)R}{180^\circ}.$$

If NPQ is not parallel to STU, there is no square span, and c is substituted for s in the data. To α_2 , as calculated on the assumption of parallelism, it is then necessary to apply the known angle between NPQ and STU.

Setting Out.—1. C being inaccessible, locate the tangent points A and B (Fig. 317) by the method of Case 3, page 429, the length of a transversal EF being obtained by measuring out a base line such as ED, observing two or all angles of the triangle EDF, and solving for EF.

2. A and B, being found accessible, are to be used as instrument stations from which the required points can be located by simultaneous observations. In setting out a point on the centre line, such as G, the deflection angles CAG and CBG are readily computed from the lengths of the arcs AG and GB obtained as above.

Setting Out.—1. Having solved triangle CEF as before, and found that A and B are inaccessible (Fig 314), select two instrument

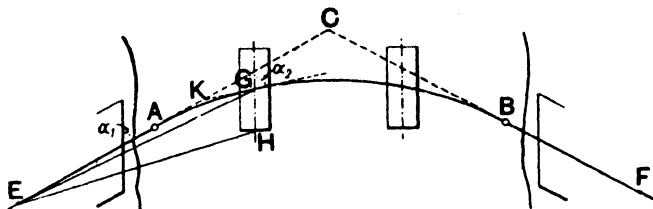


FIG. 314.

stations, such as E and F, from which to set out simultaneous deflection angles. The chainage of E being known, that of F is readily found since EA, arc AB, and BF are known.

2. Calculate the deflection angles: for a centre line point such as G, these angles are CEG and CFG. KG being the tangent to the curve at G, CEG is obtained by solution of triangle EKG, since $EKG = (180^\circ - i)$, where i is the central angle subtended by arc AG, $KG = R \tan \frac{1}{2}i$, and $EK = (EA + R \tan \frac{1}{2}i)$. Similarly for CFG.

3. The value of angle CEH required in locating H is obtained by solution of triangle EGH, in which EG is obtained from triangle EKG, GH is known, and $EGH = (\alpha_2 - KGE)$. Similarly for CFH.

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EXAMPLES

1. The chainages at the beginning and end of a 2° curve are $320+34$ and $339+75$ respectively. Find the intersection or central angle.
2. If in the above case the intersection angle is increased by 5° by turning the second tangent about the vertex, find the new chainages of the tangent points of the 2° curve.
3. To ascertain the radius of an existing curve, two 100 ft. chords, AB and BC, are measured out, and the versed sine of chord AC is found to be 4 ft. 9 in. Find the radius.
4. Show that the tangent offset at the end of the N th chord on a D° curve is approximately given by $\frac{1}{3}N^2D$ ft.
5. A certain railway curve is to have a radius of 20 chains, the tangents intersect at chainage $320+30$, and the angle of deflection is $37^\circ 50'$, find the following quantities : (a) tangent distances, (b) apex distance, or distance of middle point of the curve from the intersection, (c) length of curve, (d) chainage of the beginning, end, and apex of the curve. (Inst. C.E., 1907.)
6. Some particulars of part of a railway survey are given in the annexed table :

Point or Apex.	Apex Angle	Radius or degree of Curve.	Distance from previous Apex.
A_0			
A_1	$163^\circ 18'$	$R = 1200$ ft.	4872.4
A_2	$165^\circ 36'$	4° Curve	6319.5
A_3			3824.7

The line starts at A_0 , and all distances are in feet. The distances are from A_0 to A_1 , and so on.

You are required to find the whole length, along the centre line of the railway, from A_0 to A_3 , and the chainage of each tangent-point. (Univ. of Lond., 1916.)

7. From a tangent AB, having a bearing of $133^\circ 42'$, a left-hand curve of 3,000 ft. radius is to be set out in 100 ft. chords to join a tangent BC, the bearing of which is $126^\circ 18'$. The chainage of the point of commencement of the curve is 19,638 ft. Give a list of the bearings to the pegs from the initial tangent point, expressing them to the nearest $\frac{1}{4}$ min.

8. The bearing of the first tangent to a railway curve of 40 chains radius is $72^\circ 19'$. What length of curve is required if the second tangent is to have a bearing of $88^\circ 55'$? If the chainage of the first tangent point is 236.24 chains, calculate the first three deflection angles to the nearest $20''$, for chords of one chain.

9. A curve of 30 chains radius has been pegged out to connect two railway tangents having the intersection angle $I = 15^\circ 26'$, and the chainage of the initial tangent point has been found to be 384.17 chains. On further examination of the ground it is decided to alter the radius to 45 chains. Calculate the

chainage of the new initial and final tangent points, and the distance between the new and original curves at their mid-points. (T.C.D., 1928.)

10. The lengths and bearings of three successive courses in the final location survey for a railway are

Line.	Lengths in chains.	Bearing.
<i>PQ</i>	63.22	122° 39'
<i>QR</i>	39.87	149° 57'
<i>RS</i>	46.25	131° 21'

It is proposed to connect *PQ* and *QR* by a curve of 80 chains radius, and *QR* and *RS* by one of 60 chains radius. If the chainage of the tangent point on *PQ* is found to be 642.35, find that of the other tangent points.

11. The positions of eight columns are to be set out on a railway station platform, the columns to be 40 ft. apart centre to centre. The centre points of columns 1 to 5 are to lie upon a circular arc of 1,905 ft. radius, and the tangent to this arc through column No. 1 has been established. The remaining columns will lie on a straight line tangential to the arc at column No. 5.

Obtain to the nearest 10' the deflection angles required in locating the columns on the curve from a theodolite placed at the centre point of the first column.

12. Two railway tangents have been located on the ground, and the difference between their bearings has been found to be 21° 48'. They are to be connected by a circular arc, and it is desired that the tangent lengths should not exceed 500 feet in length. Calculate the degree of curve, to the nearest quarter-degree, which will be most suitable, and find the corresponding tangent lengths and the length of the curve.

13. Part of the traverse survey on a railway location is as follows :

Line.	Bearing.	Length in feet.
<i>PQ</i>	93° 14'	1,967
<i>QR</i>	75° 44'	1,608
<i>RS</i>	101° 8'	2,710

Tangents are laid along these lines in the course of the final location, and it is decided to connect *PQ* and *QR* by a $1\frac{1}{4}$ degree curve. What is the maximum radius which can be used for the curve joining *QR* and *RS* so that there may be not less than 100 ft. of straight on *QR* between the end of one curve and the beginning of the other ? (T.C.D., 1931.)

14. From a line *AB*, 820 ft. long, perpendiculars *AD* and *BC* are laid out of lengths 122 ft. and 264 ft. respectively. A pond is to be constructed in a public park, and will have straight sides lying along *AB* and *DC*, the ends being formed of circular arcs to which *AB*, *DC*, and the end perpendiculars are tangential. Calculate the radii of the two ends and the perimeter of the pond.

15. A 10-chain curve *AB* runs through a number of obstacles, for the clearing of which points *C* and *D* are to be established on the curve by running out chords *AC* and *AD* at 10° and 15° to the first tangent. Calculate the lengths of the chords.

16. In laying out a circular railway curve it is found that the tangents intersect at chainage 257+34 and that the deflection angle is 27°. Find the radius of the curve which will pass through a point 40 ft. from the intersection and equally distant from the tangents, and write down the chainage at the beginning of the curve. (Inst. C.E., 1906.)

17. The internal angle *ABC* between two tangents is 120°, and they are to be connected by a circular curve. Owing to the presence of buildings it is found necessary that the curve should pass through a point *D*, the length of

the perpendicular DE on to the tangent AB being 24 ft., the distance BE being 500 ft. Find the radius and tangent distance of a suitable curve. If the chainage at the intersection point is 150 chains 20 links, find the chainage at the end of the curve. (R.T.C., 1913.)

18. In the course of a railway survey the following bearings were observed : AB , $77^\circ 18'$; BC , $92^\circ 54'$; and CD , $105^\circ 12'$. BC is 12.08 chains long. AB and CD are to be connected by a circular arc, right-handed from AB , which is required to pass through a point E on BC situated at 4.90 chains from B . Determine the required radius of curve.

19. A curve AB , of 60 chains radius, is to be set out by chain chords to join two tangents AC and CB , the intersection angle of which is $19^\circ 46'$. The chainage of the initial tangent point A is found to be 521.28 chains. The part of the curve adjacent to A passes through a dense strip of wood, and will not be pegged out in the first instance, but a line AD can be laid off from the tangent point, making angle $CAD = 3^\circ 54'$. Calculate (a) the length of AD so that D may be on the curve, (b) the chainage of D , (c) the length of curve to be set out from D to B . (T.C.D., 1928.)

20. A light railway is to be carried round the shoulder of a hill, and its centre line is to be tangent to each of the three lines AB , BC , and CD as follows :

Line.	Bearing.	Length.
AB	North 30° East	—
BC	East	600 ft.
CD	South	—

Calculate the radius of the curve and the lengths required for setting out the tangent points. (Univ. of Lond., 1913.)

21. A curve of 60 chains radius has been pegged out to connect two railway tangents, the difference between the bearings of which is $18^\circ 0'$. It is found necessary to alter the alignment by introducing a straight length of 6 chains midway between the existing tangents and connecting it to them by two arcs of equal radius. If the original tangent points are to remain in the same position, compute the radius of the required curves and the maximum distance between the original and amended centre lines.

22. AC and BC are two railway straights which are to be connected by a 2° curve. The intersection point C is found to be inaccessible, and a line AB is run out between convenient points on the straights. The length of AB is found to be 900 feet, and the observed values of the angles BAC and ABC are $9^\circ 20'$ and $8^\circ 30'$ respectively. Calculate the length of the curve and the positions of its tangent points relatively to A and B respectively.

23. The intersection point C of two tangents, AC and CB of a railway which is being set out is found to be inaccessible, and a traverse is run from a point D in AC to a point H in CB with the following results :

Line.	Bearing.	Length in feet.	Latitude.	Departure.
DA	$344^\circ 38'$			
DE	$123^\circ 12'$	186	-101.8	+155.6
EF	$159^\circ 30'$	427	-400.0	+149.5
FG	$100^\circ 48'$	144	-27.0	+141.4
GH	$186^\circ 0'$	251	-249.6	-26.2
HB	$143^\circ 34'$			

If the tangents are to be joined by a curve of 2,640 feet radius, calculate the distance of the initial tangent point from D . (T.C.D., 1929.)

24. A 4° curve is to connect two straights, the bearings of which are 20° and 80° respectively. The intersection point is inaccessible, and from a point A of chainage 13,350 ft. on the first straight a traverse is run to D on the second straight as follows :

Line.	Bearing.	Length.
AB	60°	468
BC	110°	360
CD	30°	950

Find the chainage of the first tangent point and the distance from D to the second. (R.T.C., 1920.)

25. C is the inaccessible intersection point of two railway straights AC and CB which are to be joined by a circular curve. Points D and E are selected in AC and CB respectively, and the following bearings are observed : AD, $173^\circ 42'$; DE, $187^\circ 24'$; EB, $198^\circ 18'$. The distance DE is measured and found to be 960 feet. If on examination of the ground it is found desirable to locate the initial tangent point of the curve at D, obtain the necessary radius in feet and the distance from E to the second tangent point.

26. A right-hand curve of 60 chains radius, of which A and B are the tangent points, has been pegged out to connect the tangents AC and CB, the intersection angle of which is $19^\circ 18'$. It is desired to alter the alignment of the tangent CB to DE, D being on CB and 1.57 chains from B towards C, and angle CDE on the side of CB remote from A being $177^\circ 16'$. If the original radius of the curve is preserved, find the distances of the new tangent points from A and D respectively.

27. From a given point A it is required to set out a tangent AD to an existing 40-chain curve BDC. A straight line ABC is run to intersect the curve, making AB = 10.20 chains and BC = 4.32 chains. Calculate the angle BAD and the length of arc BD.

28. ABC is a railway line of which AB is straight and BC is a 20-chain curve towards the left : 2 chains to the right of AB and parallel to it is a straight railway DE. Find at what distance from B a 40-chain curve must leave BC to join DE.

29. AC and CB are two railway tangents giving an intersection angle of $12^\circ 26'$. They are joined by a curve AB of 60 chains radius, the chainage of the initial tangent point A being 346.27. From a point D in AC situated at 3.04 chains from A it is desired to set out a tangent DE to the curve. Calculate (a) the angle CDE, (b) the chainage of E, (c) the distance from B at which DE produced meets BC.

30. It is proposed to lay down a right-hand curve of 792 ft. radius at a dock, and the initial and terminal tangent points, A and B, will occupy the positions given by the following total co-ordinates :

	Total Latitude in feet.	Total Departure in feet.
A	+3,479	-1,096
B	+2,963	-1,208

Calculate the shortest distance between the curve centre line and a point C the total co-ordinates of which are +3,204 ft. and -1,129 ft. in latitude and departure respectively.

31. The initial tangent point of a 40-chain railway curve is situated in water, but its chainage is found to be 1,936.44 chains. It is desired to set out the peg at 1,941 chains by sighting from station 1,934 on the straight. Calculate, to the nearest $20''$, the angle to be set off from the tangent and the distance to be measured out to locate the peg.

32. In setting out a compound curve of radii 20 and 40 chains, the intersection angle of the arc of smaller radius is found to be 90° , and that of the other 60° . The common tangent is then chained and found to have a length of 43 chains. Find by how much and in what direction it will be necessary to shift this tangent parallel to itself, so that the curve may be accurately set out. Explain how this subsequent setting out is performed. (R.T.C., 1913.)

33. A compound curve is to consist of an arc of 30 chains followed by one of 40 chains radius, and is to connect two straights which yield an intersection angle $I = 84^\circ 32'$. At the intersection point the chainage, if continued along the first tangent, would be 77.64, and the starting point of the curve is selected at chainage 45.14. Calculate the chainage at the point of junction of the two branches and at the end of the curve.

Describe briefly the steps involved in setting out the curve. (R.T.C., 1915.)

34. A railway siding is to be curved through a right angle. In order to avoid buildings the curve is to be compound, the radii of the two branches being 8 chains and 12 chains. The distance from the intersection point of the end straights to the tangent point at which the arc of 8 chains radius leaves the straight is to be 10.08 chains. Obtain the second tangent length, or distance from the intersection point to the other end of the curve, and the length of the whole curve. (T.C.D., 1927.)

35. A compound railway curve ABC is to have the radius of the arc AB 60 chains and that of BC 40 chains. The intersection point V of the end straights is located, and the intersection angle is observed to be $35^\circ 6'$. If the arc AB is to have a length of 20 chains, calculate the tangent distances VA and VC. (T.C.D., 1930.)

36. A reverse curve having the two branches of common radius is to connect two parallel straights 30 ft. apart. If the distance between the tangent point of one straight and that on the other is 240 ft., find the necessary radius.

37. The first branch of a reverse curve has a radius of 8 chains. Find the second radius so that the curve may connect parallel tangents 84 links apart, the distance between tangent points to be 5 chains. Calculate also the lengths of the branches.

38. Two parallel railway straights, 27 ft. apart centre to centre, are to be connected by a reverse curve the radius of each branch of which will be 10 chains. You are given the position of one of the tangent points. Describe in detail how you would set out the curve either by offsets from the line joining the tangent points or by deflection angles to points $\frac{1}{2}$ chain apart on the curve. Give all the necessary calculations. (T.C.D., 1926.)

39. A rule which is sometimes used for determining the required superelevation of the outer rail on railway curves on standard gauge is that the superelevation should equal the offset from the centre of a chord of length equal to the distance travelled in one second. Compare the superelevation given by this rule with the theoretical value derived from mechanical principles

40. Calculate (a) the superelevation required on the outer rail of a curve of 30 chains radius for a speed of 45 miles per hour, (b) the length of transition curve necessary so that the rate of change of radial acceleration may be 1 ft. per sec.³, (c) the amount of shift required for this transition.

41. A railway curve of 50 chains radius and about half a mile long is to be set out to connect two tangents. The maximum speed on this part of the railway will be 60 miles per hour, and transition curves are to be introduced at each end of the curve. Find a suitable length for the transition curves, and calculate the necessary "shift" of the circular arc.

Describe how you would set out the first transition and locate the initial pegs on the circular curve. (T.C.D., 1926.)

42. Explain the utility of introducing a vertical curve at the junction of two railway gradients.

The levels of a number of successive points 100 ft. apart, as shown on the longitudinal section, are :

236.42, 235.62, 234.82, 234.02, 235.13, 236.24, 237.35.

Calculate the levels of the pegs which should be set out to define a vertical curve 600 ft. long. The curve may be either a parabolic or a circular arc. (T.C.D., 1927.)

43. The exterior angle between two tangents on a road is 85° and a curve is to be fitted between them to suit a standard velocity of 35 miles an hour. Using the method of unit-chords and assuming that the transition curve is to be a lemniscate, find :—

- (1) Minimum radius permissible, in feet.
- (2) Value of D , in feet.
- (3) Maximum value of ρ , in feet.
- (4) Maximum value of α .
- (5) Will curve be transitional throughout? If not, what angle will the circular arc subtend at the centre of the circle?
- (6) Length of tangent point.
- (7) Maximum superelevation at end of transition.
- (8) Compute the values of the polar ray and the lengths of the chords. Check your work by computing the theoretical length of the transition curve and seeing if this agrees with the sum of the chord lengths.

44. With the same data as in the last question, but assuming that the radius of curvature is not to exceed 1000 ft., find :—

- (1) The maximum values of ρ and α .
- (2) The angle 2θ subtended by the circular arc at its centre.
- (3) The length of the tangent point.
- (4) The maximum superelevation.
- (5) The deflection angles and working chord length assuming that the latter is to be $\frac{1}{4}D$.

45. If the exterior angle between the tangents is 60° and the standard velocity is 45 miles per hour, work out the quantities mentioned in question 43.

46. Prove that, if l is in unit-chords, the equation of the spiral is $l = 8.463\sqrt{\varphi}$.

47. If the deflection angle between the two tangents is 110° and the standard speed is 50 miles per hour, work out the polar deflection angles for a spiral transition curve to suit these conditions. (Hints: Calculate D , and find the minimum permissible value of R in unit-chords from the formula

$R = \frac{0.2672V^3}{D}$. Then find L in unit-chords from $LR = 35.81$, and, from this, compute the maximum value of φ from $L = 8.463\sqrt{\varphi}$. See whether the curve is wholly transitional or not and choose a working chord length. Now compute values of φ for 1, 2, 3, m working chord lengths. Polar deflection angles for the larger angles, for each working chord length, can then be calculated from the series given on page 448, viz. :—

$$\tan \alpha = \frac{\varphi}{3} + \frac{\varphi^3}{105} + \frac{\varphi^5}{5997} - \frac{\varphi^7}{198700} \dots\dots$$

In this problem the usual assumption, that the sum of the chords is equal to the length of the curve, may be made.)

CHAPTER XI

TACHEOMETRICAL SURVEYING

TACHEOMETRY or, as it is otherwise termed, **Tachymetry** or **Telemetry**, is a branch of angular surveying in which both the horizontal and vertical positions of points are determined from the instrumental observations, the labour of chaining being entirely eliminated. The usual instrument employed is called a **Tacheometer**, and is nothing more than a transit theodolite adapted for distance measuring by the provision of special fittings.

Scope.—The fundamental object of tacheometry is the preparation of a contoured plan. For this purpose it possesses the merit that the field work can be executed with considerable rapidity, more especially in rough country where ordinary levelling is tedious and chaining is both slow and inaccurate. This advantage has led to its wide adoption by engineers in location surveys for lines of communication, reservoirs, etc. Tacheometry is, however, sometimes applied in small surveys in which elevations are not determined.

Systems.—The underlying principle common to different systems of tacheometry is that the horizontal distance between an instrument station, A, and a point, B, as well as the elevation of B relatively to the instrument, can be deduced from (1) the angle at A subtended by a known short distance at B, and (2) the vertical angle from A to B. The various tacheometric methods available employ the principle in different ways, and differ from each other in methods of observation and reduction, but may be classified under two heads :

(a) The **Stadia System**, in which the necessary observations to the point are secured with one pointing of the telescope.

(b) The **Tangential System**, in which two pointings are required.

The general conduct of the field work does not differ materially in the two methods, and their principles and the instruments used will first be considered.

THE STADIA SYSTEM

This is the more extensively used system of tacheometry, particularly for detailed work such as is required in location surveys. The principle appears to have been discovered in 1770 by James Watt, who constructed an instrument, and used it in Scotland in 1771.

Principle.—Let it be supposed that the theodolite employed differs from the ordinary transit only in having the diaphragm fitted with two additional horizontal hairs, called stadia* hairs, one on each side of the centre hair. The only other article of equipment required is a graduated staff or stadia rod, which may be an ordinary levelling staff, and which is held on the points to be located with respect to the instrument station.

On viewing the staff through the telescope, the stadia hairs are seen to intercept or subtend a certain length, which will be greater the farther off the staff is held, and from the observed value of the intercept the distance to the staff station is deducible. This constitutes the *Fixed Hair Method*. The procedure may be modified in that, instead of the intercept being the variable, a constant length of staff between two targets may be used, provided the stadia hairs are adjustable so that they can be set to subtend it. Provision is made in this case for the measurement of the variable interval between the hairs, from which quantity the required horizontal distance is computed. This system is distinguished as the *Movable Hair Method*. The former is much the more widely adopted method of stadia surveying, but the latter can be usefully applied to surveys involving sights of greater length than is consistent with accurate reading of a graduated staff.

Fixed Hair Method. Distance and Elevation Formulæ.—Reduction formulæ for the horizontal distance and elevation of a staff station with respect to the instrument will be deduced first for the case where the staff can be read with the telescope horizontal. In the general case, the telescope is inclined, and the data include the vertical angle.

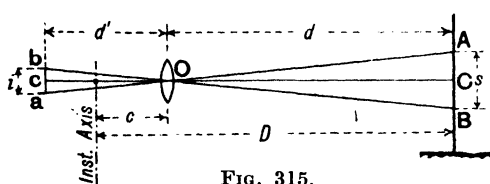


FIG. 315.

(1) *Horizontal Sights.*

In Fig. 315, O is the optical centre of the object glass of an external focussing telescope, a, b, c represent the three horizontal hairs, and A, B, C, the

points on the staff which appear cut by the hairs, so that ab is the length of the image of AB.

Let f = focal length of object glass,

i = stadia hair interval, ab,

s = staff intercept, AB,

d = horizontal distance from O to the staff,

d' = " " " " to the plane of the hairs,

c = " " " " to the vertical axis of the instrument,

D = horizontal distance from the axis to the staff.

* The term *stadia* should strictly be confined to the staff or stadia rod, but is loosely used in referring to the hairs, the entire instrument, and the method generally.

The rays AOa and BOb passing through O are straight lines, so that triangles AOB and aOb are similar, whence

$$\frac{d}{d'} = \frac{s}{i}.$$

But d and d' are conjugate focal distances (page 14),

$$\therefore \frac{1}{f} = \frac{1}{d'} + \frac{1}{d},$$

or $d = \frac{fd'}{d'} + f$, on multiplying throughout by fd' .

Substituting $\frac{s}{i}$ for $\frac{d'}{d}$, we have

$$d = \frac{fs}{i} + f, \text{ and, on adding } c \text{ to each side,}$$

$$D = \frac{fs}{i} + (f+c).$$

The intercept s is observed as the difference of the stadia hair readings, and, to evaluate D , the quantities, $\frac{f}{i}$ and $(f+c)$, must be known for the particular instrument used. The elevation of the staff station is obtained in this case exactly as in ordinary levelling by observation of the reading C of the centre hair.

Note.—The reader may deduce the distance formula by similar treatment with the other easily drawn rays, viz. those passing through either the interior or exterior principal focus of the object glass.

(2) *Inclined sights.*—In this case the staff may be held either vertically or normal to the line of sight, the former method being generally preferred.

(a) *Staff Vertical* (Fig. 316).—Let θ be the angle of elevation or depression of the line of sight from the horizontal. The inclined

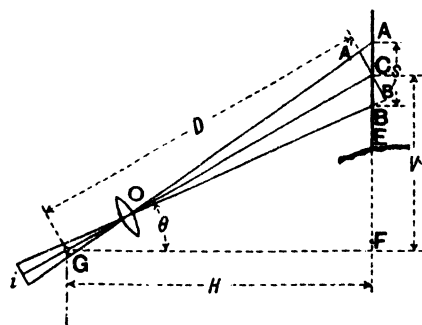


FIG. 316.

distance D from the trunnion axis G to the point C on the staff could be obtained directly from the previous formula, were it not that the observed intercept AB is not, as before, normal to OC. Let an intercept A'B' be drawn through C perpendicular to OC. Since $\angle ACA' = \theta$, and $\angle AA'C$ is practically 90° , A'B' may be expressed as $s \cos \theta$ with negligible error, whence

$$D = \frac{fs}{i} \cos \theta + (f+c),$$

and, since the required horizontal distance $H = D \cos \theta$,

$$H = \frac{fs}{i} \cos^2 \theta + (f+c) \cos \theta.$$

Let the difference of level FC between the telescope axis and the point C on the staff be denoted by V . The value of V can be obtained from $H \tan \theta$ or $D \sin \theta$, so that

$$V = \frac{fs}{i} \cos \theta \sin \theta + (f+c) \sin \theta,$$

$$\text{or} = \frac{fs}{i} \frac{1}{2} \sin 2\theta + (f+c) \sin \theta.$$

Denoting CE, the reading of the centre hair, by h , the difference of level between G and E for an angle of elevation is given by

$$FE = V - h,$$

and, if the elevation of the trunnion axis above datum is expressed by H.I.,

$$\text{Reduced level of E} = \text{H.I.} + V - h.$$

In the case of a depressed sight, $FE = V + h$, so that

$$\text{Reduced level of E} = \text{H.I.} - V - h.$$

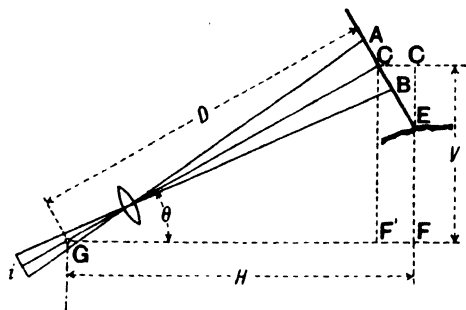


FIG. 317.

(b) *Staff Normal* (Fig. 317).

In this case, $D = \frac{fs}{i} + (f+c)$;

$$\text{but } H = D \cos \theta + CC' = D \cos \theta + h \sin \theta,$$

$$\therefore H = \frac{fs}{i} \cos \theta + (f+c) \cos \theta + h \sin \theta,$$

the $(h \sin \theta)$ term being subtractive when θ is a depression, since the staff then leans away from the instrument.

The vertical component $V = F'C = D \sin \theta$, whence

$$V = \frac{fs}{i} \sin \theta + (f+c) \sin \theta.$$

For an angle of elevation, $FE = V - h \cos \theta$, and

$$\text{Reduced level of E} = \text{H.I.} + V - h \cos \theta$$

For an angle of depression, $FE = V + h \cos \theta$, and
 Reduced level of $E = H.I. - V - h \cos \theta$.

Movable Hair Method. Distance and Elevation Formulæ.—The above formulæ apply in this case, but s is now the constant distance between the staff targets, and the variable i is measured. The hairs are set to intercept s by means of screws with micrometer heads, and their distance apart is read in terms of the pitch of the screws.

Let p = pitch of the screws,
 n = number of pitches between the stadia hairs, or the number of turns of the micrometer heads necessary to carry both stadia hairs from the centre hair until they subtend s .

Then the expression, $D = \frac{fs}{i} + (f+c)$, becomes

$$D = \frac{fs}{np} + (f+c),$$

which involves the two instrumental constants $\frac{f}{p}$ and $(f+c)$. The remaining formulæ are derived from the fundamental expression as before.

The Tacheometer.—Although an ordinary transit fitted with stadia hairs or points can be employed for tacheometry, accuracy and speed are promoted if the instrument is specially adapted for the work. The telescope formerly provided in small theodolites by some makers is not sufficiently powerful for tacheometry. The magnification should be at least 20 diameters, and, to obtain a sufficiently bright image, the effective aperture of the objective should not be less than $1\frac{1}{2}$ in. For the same reason, it is inadvisable to employ an erecting eyepiece on account of the resulting loss of light. It is desirable that the objective should produce a flat and undistorted image, since the stadia hairs are not in the optical axis. Two pairs of stadia hairs are sometimes fitted. The interval between the extreme hairs corresponds to a value of $f/i = 50$ and is twice that of the inner pair, for which $f/i = 100$. To facilitate reading the extreme hairs, the eyepiece may be arranged to move up or down by means of a rack and pinion. The widely spaced hairs are intended for the taking of short sights, but are seldom used.

Since the measurement of vertical angles constitutes an important item of the observations, the instrument should have a sensitive spirit level mounted upon the vernier arm of the vertical circle. The vertical circle is commonly graduated in quadrants with the zeros horizontally opposite each other, but some surveyors prefer a whole circle graduation, as thereby there is no need to distinguish readings as elevations or depressions in booking, and mistakes are avoided (page 67).

The instrument required for observations on the movable hair system differs from the simple tacheometer only in having a special diaphragm. In the commonest form, the fixed horizontal and vertical hairs are stretched across the frame, and each movable hair is mounted upon a slide, which can be raised or lowered by a micrometer screw. To prevent fouling, the hairs lie in slightly different vertical planes. The screws are provided with milled heads and drum scales. The drums are divided into 100 parts, and are read against a fixed index to 0.1 of a division either by estimation or by vernier. Readings are therefore made to 0.001 of the pitch of the screws. A comb scale with teeth of the same pitch as the screw is provided for the purpose of exhibiting the number of complete pitches intercepted between the axial hair and either stadia hair.

A transit theodolite provided with either fixed or movable stadia hairs and having a telescope of the quality specified above may be classed as a tacheometer, but the term is sometimes limited to instruments in which the telescope is fitted with an anallactic lens.

The Anallactic Lens.—The formula, $D = \frac{fs}{i} + (f+c)$, shows the staff intercept to be proportional to $D - (f+c)$, the distance between the staff station and the exterior principal focus of the objective. The latter point therefore forms the apex of a constant visual angle between the sides of which the quantity s is intercepted. If this apex were situated on the vertical axis of the instrument, the term $(f+c)$ would vanish, and D would be proportional to s . This was accomplished by Porro by the introduction in the telescope of an additional convex lens, called an anallactic lens, placed between the eyepiece and object glass, and at a fixed distance from the latter.

The anallactic lens is generally provided in external focussing tacheometer telescopes by English makers. While its use simplifies the reduction of observations, it is open to the objection that it increases the absorption of light in the telescope with consequent reduction in brilliancy of the image. It is not fitted in internal focussing telescopes (see page 486).

Theory of the Anallactic Lens.—In Fig. 318, O represents the optical centre of the object glass, and L that of the anallactic lens, the rays drawn being two of those passing through the principal focus, F_1 , of the latter. The actually formed image of the staff intercept AB is represented by ba , and $b'a'$ is that which would be produced if no anallactic lens were interposed. The effect of the introduction of this lens is best followed by reference to these two images.

Let D , d , f , and c = the same quantities as before (page 480),

d' = distance from optical centre of objective to $b'a'$,

d_1 = distance from optical centre to actual image ba ,

e = distance between optical centres of object glass and anallactic lens,

f_1 = focal length of anallactic lens,

i = length $b'a'$,

I = length ba , the actual stadia hair interval.

The equations governing the size and position of $b'a'$ are, as before,

$$\frac{1}{f} = \frac{1}{d'} + \frac{1}{d} \dots \dots \dots (1)$$

$$\text{and } \frac{d}{d'} = \frac{s}{i} \dots \dots \dots (2)$$

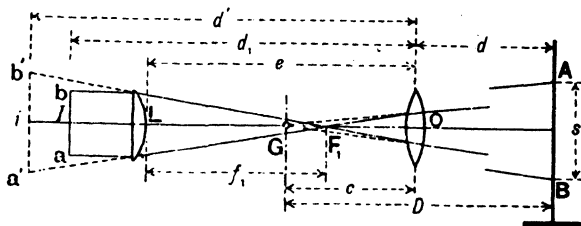


FIG. 318.

Due to refraction through the anallactic lens, the rays which would converge to form $b'a'$ actually form the image ba , so that $b'a'$ and ba are conjugate, and their distances $(d'-e)$ and (d_1-e) from L are connected by

$$\frac{1}{f_1} = \frac{1}{(d_1-e)} - \frac{1}{(d'-e)}, \dots \dots \dots (3)$$

the negative sign being required since $b'a'$ and ba are on the same side of L (page 14).

The lengths of $b'a'$ and ba are proportional to their distances from L , so that

$$\frac{i}{I} = \frac{(d'-e)}{(d_1-e)}, \dots \dots \dots (4)$$

An expression for D can now be obtained by eliminating d' , d_1 , and i from these equations.

From (3), $(d_1-e) = \frac{f_1(d'-e)}{(f_1+d'-e)}$; whence, from (4),

$$i = \frac{I(f_1+d'-e)}{f_1};$$

but, from (1), $d' = \frac{fd}{(d-f)}$,

$$\therefore i = \frac{I\left(f_1 + \frac{fd}{(d-f)} - e\right)}{f_1}$$

A second expression for i follows from (1) and (2), viz.

$$i = \frac{fs}{(d-f)}.$$

Equating these two expressions for i , and solving for d , we have

$$\frac{fs}{(d-f)} = \frac{I\left(f_1 + \frac{fd}{(d-f)} - e\right)}{f_1},$$

$$\text{whence } d = \frac{ff_1s}{I(f+f_1-e)} - \frac{f(e-f_1)}{(f+f_1-e)},$$

$$\text{and } \therefore D = \frac{ff_1s}{I(f+f_1-e)} - \frac{f(e-f_1)}{(f+f_1-e)} + c. \dots (5)$$

Now the condition that D should be proportional to s requires that the second and third terms should vanish, so that

$$\frac{f(e-f_1)}{(f+f_1-e)} = c,$$

which is secured by placing L so that

$$e = f_1 + \frac{fc}{(f+c)}. \dots \dots \dots (6)$$

Under these conditions, the apex of the tacheometric angle is situated at G , the centre of the trunnion axis. The focal length f_1 must be such that e is less than f , and the value of f_1 and I must

be so arranged that the multiplier $\frac{ff_1}{I(f+f_1-e)}$ is a suitable round number, say 100, so that $D = 100s$.

The formulæ for inclined sights on either the fixed or the movable hair system are similarly modified by omitting the term involving the second constant.

The anallactic lens is usually provided with means for adjusting its position in the telescope, so that, if the constant is found to differ from 100, the distance e may be adjusted until the desired value is obtained.

The Internal Focussing Telescope in Tacheometry.—If the instrument is fitted with an internal focussing telescope, the distance formula, $\frac{fs}{i} + (f+c)$, is not applicable, nor can the internal lens be regarded as the exact equivalent of an anallactic lens. The system consisting of the object glass and the internal lens is not of constant focal length, since the interval between these lenses is varied in focussing, whereas an anallactic lens is placed at a constant distance from the object glass. Investigations of the distance formula for

the internal focussing telescope are given by Louis and Caunt* and by Henrici.†

The variation of focal length of the objective system is very small for sights of more than, say, 100 ft., and practically it is sufficient to adopt the same basic reduction formula, $D = Cs + C_1$, as in the case of the externally focussed telescope. The multiplier C is a constant, usually 100, but C_1 , although practically constant for long sights, is not so for all distances, and may be regarded as a correction term which neutralises the effect of the varying focal length. Its value is usually only a few inches, and it is often disregarded.

It is impossible to proportion a focussing lens to serve as an anallactic lens, but several designs have been evolved with the object of reducing to negligible limits the variations in the magnitude and position of the tacheometric angle. In that patented by Messrs. Zeiss in 1910, the internal lens, when focussed for very distant objects, is placed about midway between the plane of the stadia hairs and the image projected by the object glass of the point in which the apex of the tacheometric angle then lies. When focussed for very near objects, the internal lens, if negative, is moved half-way at most towards the reticule. This scheme places the apex of the tacheometric angle in a practically constant position at the instrument axis. Alternatively, if the internal lens is placed about midway between the object glass and the plane of the hairs when the telescope is focussed for very distant objects, the apex is situated in the object glass. The additive term is then the constant c . In both arrangements the tacheometric angle is practically constant for all sights over, say, 20 ft. in length.

Messrs. J. S. Wilson and E. W. Taylor, of Messrs. Cooke, Troughton and Simms, patented in 1925 a design in which the focal length of the internal lens is equal in magnitude, but opposite in sign, to that of the object glass, the distance between the centre of the object glass and the plane of the stadia hairs being 1.11 to 1.12 times the common focal length. If the horizontal axis of the telescope intersects the optical axis midway between the objective and reticule, the design secures that distances derived as a constant times the staff intercept represent distances from the instrument axis within 1 part in 1,000 for sights of from 20 to 600 ft. The telescope is therefore for practical purposes anallactic. Messrs. A. and E. W. Taylor have produced an internally focussed telescope which is perfectly anallactic, but three lenses are introduced between the objective and diaphragm, so that a larger aperture is required to compensate for the loss of illumination.

Instrumental Fittings to Simplify Reductions.—Whether the telescope of a tacheometer is, or is not, anallactic, a good deal of labour is involved in the determination of the horizontal and vertical

* *Tacheometer Tables*. London, 1919.

† "The Use of Telescopes with Internal Focussing for Stadia Surveying." *Trans. Optical Society*, Vol. XXII.

distances. Certain instruments and attachments are available in which fittings are provided for the purpose of facilitating the reduction of observations. These include the Beaman Stadia Arc, and different types of "self-reducing" tacheometers.

The Beaman Stadia Arc.—The reductions would be simplified if the only values of θ used were those for which either $\cos^2 \theta$ or $\frac{1}{2} \sin 2\theta$ is a convenient figure. The former varies too slowly for the small angles usually required, but a list of values of θ for which $\frac{1}{2} \sin 2\theta = .01, .02$, etc., can be prepared as follows:

$\frac{1}{2} \sin 2\theta$	θ to nearest second.			$\frac{1}{2} \sin 2\theta$	θ to nearest second		
	°	'	"		°	'	"
-01	0	34	23	-06	3	26	46
-02	1	8	46	-07	4	1	26
-03	1	43	12	-08	4	36	12
-04	2	17	39	-09	5	11	6
-05	2	52	11	-10	5	46	7
				etc.			etc.

If these particular angles were used, the vertical component V , for a multiplying constant of 100 and no additive constant, would be $s, 2s, 3s$, etc.

The idea is applied in the Beaman Stadia Arc, which has been used to a considerable extent in tacheometers and plane table alidades, and has increased in popularity in recent years. It is fixed to the vertical circle, and consists of a scale engraved with the above angles on either side of zero up to about $26^\circ 33' 54''$, for which $\frac{1}{2} \sin 2\theta = .40$. The scale is figured in terms of $100 \times \frac{1}{2} \sin 2\theta$, and is read against a fixed index mark. To avoid possible confusion between elevations and depressions, the zero is commonly marked 50, so that 50 must be subtracted from every reading. No fitting is required to enable fractional parts of the scale to be read, since for every sight a graduation of the scale is to be brought opposite the index by means of the vertical circle tangent screw.

Let it be supposed that a staff is sighted with an instrument having the tacheometric constant 100 and the stadia arc zero 50, and that the tangent screw is adjusted to bring a graduation of the stadia arc, say 36, exactly opposite the index. Let the observed axial hair reading be 6.10, and the staff intercept 4.95.

The stadia arc reading = $36 - 50 = -14$.

The vertical component = $-14 \times 4.95 = -69.3$ ft., and the staff point is below the trunnion axis by $69.3 + 6.1 = 75.4$ ft.

To facilitate the calculation of horizontal distances, the stadia arc also carries a scale of percentage reductions to be applied to the distance readings, $\frac{fs}{i}$. In the above example, the distance scale will read 2.0, so that

the horizontal distance = $495 - 2 \times 4.95 = 485$ ft.

One example of an instrument fitted with a Beaman Stadia Arc is the Watts Microptic Optical Alidade described on pages 611-613.

Self-Reducing Tacheometers for Use with a Vertical Staff.—The principle underlying these instruments involves an automatic variation in the stadia interval so contrived that multiplying the observed stadia intercept read on the staff by a fixed constant gives the reduced horizontal distance, and in some cases also the vertical component of the inclined line of sight is obtained automatically in the same way.

Among early instruments of this type were the Stanley tacheometer fitted with the Stanley Compensating Diaphragm and the Jeffcott Direct-Reading Tacheometer. In both these instruments the horizontal and stadia hairs in the diaphragm were replaced by three platinum or platinum iridium pointers, of which the two outer ones were movable. In the case of the Jeffcott instrument both horizontal and vertical components of the line of sight could be obtained directly without having to measure the vertical angle. When the telescope was elevated or depressed, the two outer pointers were actuated by a system of levers and cams in such a manner that the staff interval read between the fixed central pointer and the movable right hand pointer when multiplied by 100 (or 200 in some instruments) gave the reduced horizontal distance, and the staff intercept read between the central pointer and the left hand pointer when multiplied by ten gave the vertical component.*

In the modern self-reducing tacheometers, the movable pointers are replaced by special curves engraved on rotating glass circles. In the Kern DKR self-reducing tacheometer, the engraved circle is mounted at right angles to the axis of the telescope with the face on which the curves are engraved almost in contact with the ordinary diaphragm carrying a vertical line. The circle rotates as the telescope is elevated or depressed and the appropriate parts of four curves are seen in the eyepiece against the centre of the staff when the fixed vertical line is directed to it. The staff intercept between the two outer curves multiplied by 100 gives the reduced horizontal distance and the staff intercept between the two inner curves when multiplied by a factor which is indicated by special marks placed at short intervals between the elevation curves (20 for slopes of 0° to 12° , 50 for slopes of 12° to 27° , and 100 for slopes of 27° to 40°), gives the vertical component.

In the Wild RDS Reduction Tacheometer and the Zeiss Reduktions-Tacheometer Dahlta 020 the glass circles carrying the reduction curves are in vertical planes parallel to the axis of the telescope, and an image of the curves is brought to focus on the diaphragm by means of lenses and reflecting prisms. Fig 319 shows the field of

* Somewhat fuller descriptions of the Stanley Compensating Diaphragm and the Jeffcott Direct-Reading Tacheometer were given in the 4th and earlier editions of this book.

view in the telescope of the Wild RDS instrument. The value of the intercept between the lower and upper curves in centimetres gives the horizontal distance in metres and the value in centimetres of the intercept between the lower and middle curve when multiplied by $+0.2$, the factor shown above the middle curve, gives the

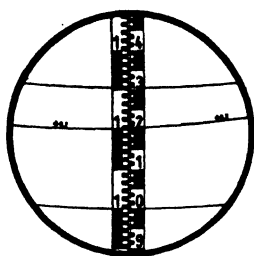


FIG. 319.—STADIA CURVES IN THE WILD RDS REDUCTION TACHEOMETER.

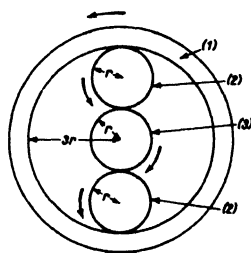


FIG. 320.—OPERATION OF PLATE CARRYING STADIA CURVES IN THE WILD RDS REDUCTION TACHEOMETER.

(By courtesy of Messrs. Henry Wild & Co.)

vertical component in metres. In the diagram, the distance intercept is 30.6 cms. so that the reduced horizontal distance is 30.6 m. and the vertical intercept is 20.4 cms. so that the vertical component is $20.4 \times +0.2 = +4.08$ m.

The operation of the plate carrying the curves in the Wild tachometer is shown in Fig. 320. The internally toothed ring (1) is rigidly fixed to the body of the telescope and the toothed planet gears (2) revolve on axes attached to the main standard, while the sun gear (3), to which is attached the plate carrying the curves, is free to rotate about the transit axis. The radii of the gears (2) and (3) are each one-third of the radius of the ring (1), and, as the telescope rotates, the plate carrying the curves rotates three times as much in the reverse direction, thus allowing the use of long flat curves.

The Kern, Zeiss and Wild self-reducing tachometers are fitted with glass horizontal and vertical circles which are read by optical scale or micrometer. They can therefore all be used as an ordinary theodolite.

The Stadia Rod.—An ordinary levelling staff may be used, but the graduation of some patterns is not sufficiently bold for tachometry, as sights are frequently of much greater length than is usual in ordinary levelling. Fig. 321 illustrates graduations typical of the many patterns which have been proposed. The rod has a length of from 12 to 15 ft., and is either telescopic, hinged, or in separate pieces fitting together as in the Scotch levelling staff. Lightness is a desideratum, and rigidity may be secured by means of a stiffening

piece screwed to the back or by side strips projecting a little beyond the graduated face.

For movable hair observations, diamond-shaped or circular vanes may be attached to the back of an ordinary staff: for moderate lengths of sights, patterns painted on the staff itself are sufficient. A centre vane or mark should always be provided at a known distance from the foot of the staff. It is useful to have two intercept

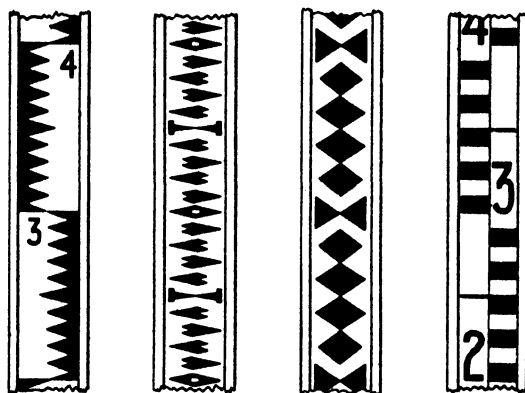


FIG. 321.—STADIA ROD GRADUATIONS.

distances of, say, 5 ft. and 10 ft., distinguished by different patterns of targets, the smaller being used for short sights. In this way the alteration of the hairs necessary to suit widely different lengths of sights is reduced.

If the staff is to be held vertically, it is desirable that some appliance should be provided to enable the holder to see if he is maintaining it in the vertical position. One method is to attach a pendulum plumb-bob to the back, such that when the staff is vertical the point of the plummet lies in the centre of a projecting ring fixed to the staff. A neater method is to fit a small circular spirit level, which may be hinged to fold up (Fig. 322). For normal holding, a staff director is usually attached to the rod. A common form consists of a tube fitted with cross wires which provide a line of sight at right angles to the staff. The latter is inclined until the telescope of the theodolite appears bisected by the wires. The tube may be fitted with lenses forming a small telescope to assist the staffman in setting the rod for long sights.

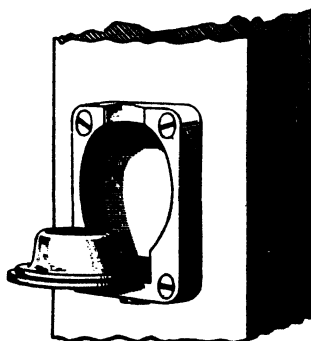


FIG. 322.—STAFF PLUMBING LEVEL.

If, however, in these circumstances he can sight the upper part of the instrument roughly, the error of inclination of the staff will be negligible, so that the use of a telescopic director is a needless refinement. Strictly, the director should be attached to the rod at the reading of the centre hair, but it is sufficient to place it at a convenient height for the staffman.

Instrumental Constants.—Taking first the case of the external focusing telescope without an anallactic lens, the expression, $\frac{fs}{i} + (f+c)$, and its modifications involve the two constants, f/i , and $(f+c)$, the values of which must be known for the instrument used. The actual distance, i , between the stadia hairs is not required, since the ratio f/i can be obtained without direct measurement of i .

The constant f/i is by far the more important of the two, since it is a multiplier. It is usually arranged by the maker to be 100, but its actual value should be determined as carefully as possible. The $(f+c)$ term is not strictly a constant, since the distance c will vary a little for different lengths of sight owing to movement of the objective in focussing. This variation will, however, seldom exceed a small fraction of an inch, since very short sights are comparatively rare, and it is quite allowable to regard c as constant. The value of $(f+c)$ varies from less than 1 ft. to nearly 2 ft., according to the size of the telescope.

To determine the values of the constants, a base line of about the same length as the longest sight to be used in the survey is measured on fairly level ground with a steel tape. Pegs are driven along it at intervals of, say, 200 ft. The instrument is set at one terminal, and readings are taken with the rod held at the various distances. The fact that there are two constants to be determined suggests the use of pairs of simultaneous equations. Thus, if D_1 , D_2 , etc., represent distances from the instrument, and s_1 , s_2 , etc., the corresponding staff intercepts, then

$$D_1 = \frac{fs_1}{i} + (f+c),$$

$$D_2 = \frac{fs_2}{i} + (f+c), \text{ etc.}$$

If horizontal sights are impracticable, the corresponding formula for inclined sights will be applied. The equations are solved in pairs for f/i and $(f+c)$, and the average values are adopted.

A simpler, and rather better, method is to determine $(f+c)$ by direct measurement on the instrument. The length f is measured between the centre of the object glass and the plane of the cross hairs when the telescope is focussed on a remote object: c is best measured with the telescope focussed for an average length of sight. The results of the observations of the several measured lengths are now employed for the determination of an average value of f/i .

The reduction is somewhat simplified if the instrument is placed at a distance ($f+c$) beyond the end of the base line.

The accompanying table shows a typical set of readings with level sights. The constants are worked out for the half-intervals between the axial hair and the stadia hairs, as well as for the whole interval. Although it can be shown that distances derived from half-interval observations are not strictly correct in the case of highly inclined sights, these subsidiary constants are used in the reduction of observations in which the whole stadia interval cannot be used, as (a) when an obstacle intervenes in the line of sight of one of the stadia hairs, (b) when a very long sight is taken so that

R.T.C. 5 in. Transit No. 3.

($f+c$) = 1.4 ft.

Distance in feet. 1.4 +	Readings.			Constants.		
	Lower.	Centre.	Upper.	Interval. Whole	Lower Interval.	Upper Interval.
200	3.30	4.32	5.32	99.01	196.08	200.00
400	6.67	8.71	10.70	99.26	196.08	201.01
600	3.46	6.53	9.51	99.17	195.44	201.34
800	4.79	8.86	12.85	99.26	196.56	200.50
1,000	2.99	8.10	13.08	99.11	195.69	200.80
			Average	99.2	196.0	200.7

both stadia hairs do not cut the staff. If the stadia hairs are equidistant from the middle hair, these constants will each have a value of twice that for the whole interval, but, since the intervals may not be exactly equal, the constants should be evaluated. If they differ sensibly, care must be exercised in observing with a half-interval to avoid confusing between the two constants in the direct and reversed positions of the telescope.

An important source of error in the determination of the value of f/i , and one which affects all stadia work, is that caused by the difference in the effect of atmospheric refraction on the two sight lines through the stadia hairs. Near the surface of the ground the density of the strata of air may vary rapidly at different distances from the ground, the nature and extent of the variation depending upon the relative temperature of the atmosphere and the ground. The effect is most marked during the midday hours in summer. The density then decreases downwards, and a ray of light situated at less than about 4 ft. from the ground is convex towards the earth. Professor L. S. Smith, of the University of Wisconsin, has shown that the effect on the lower sight line is much greater than that on the upper, and that readings taken at the time of maximum change of refraction give too small staff intercepts, but that the effect is negligible when the lower ray is 3 to 4 ft. above the ground. In consequence, the value of the constant,

deduced from observations taken during the midday hours is likely to be too great for the reduction of sights taken in the morning or evening, when the variation in atmospheric density is very small; and similarly a constant measured in the morning or evening is too small for observations in the middle of the day.

To determine that value of the constant f/i by which observations taken at any time of day may be reduced with sufficient accuracy, several observations of the base lengths should be made at different hours of the day. Furthermore, the conditions under which the constant is measured should be as nearly as possible those obtaining in the survey, particularly as regards weather and the nature of the ground over which the base line is laid out.

The same general methods and precautions are applicable to the determination of the constants for a movable hair instrument. In this case, the multiplier constant f/p is usually arranged by the maker to have a value of 1,000. It is not unusual to find that the micrometer readings are not exactly zero when the stadia hairs are brought to the axial hair. The amount of index error, e , to be applied as a correction to the sum of both micrometer readings may be deduced from the observation of the measured distances

by expressing each in the form, $D = \frac{fs}{(n+e)p} + (f+c)$. The index

error for each stadia hair may be similarly obtained from the same expressions for D , if s is the half intercept, and n and e are respectively the reading and index error of one of the micrometers.

For the evaluation of the single constant of an anallactic telescope, observations are taken over several measured distances with the same precautions as before. In the case of the internal focussing telescope, the aim should be to obtain values of the multiplier and the additive term which will suit for the determination of other than short distances. These are deduced by solving simultaneous equations corresponding to a series of sights ranging from, say, 200 to 800 ft. If the values so derived are used as constants, the errors introduced at measured distances of 20 to 100 ft. can then be determined, and, if appreciable, they may be eliminated by making the additive term a variable for short sights. In the case of telescopes designed to be virtually anallactic (page 484), it is practically impossible to detect any departure from the anallactic condition, since, with a constant multiplier, the additive term may never exceed an inch.

Value of Constant.—Although the constant C in the general formula, $D = Cs + C_1$, is designed to be a round number, it is a difficult matter for the surveyor to space the hairs exactly at the required interval after a breakage. Change of conditions further increases the likelihood that the value of the constant may be found other than was intended.

Since the reductions are considerably simplified if C is exactly 100,

instruments have been fitted with means for adjusting the stadia interval so that the hairs may be set to yield this value of the constant. Experience has shown, however, that there is greater possibility of error with adjustable hairs, by their accidental disturbance, and that better results are obtained with permanently fixed hairs. In the case of the anallactic telescope, the constant is adjusted by motion of the anallactic lens, and the adjusting screw must be adequately protected against accidental movement.

When the constant of a non-adjustable instrument is found to differ from 100, and the difference is too great to permit the assumption of a value of 100, two methods are employed to lessen the arithmetical work of reduction: (1) the use of a table of reduction; (2) the use of a stadia rod specially graduated to suit the instrument.

(1) The first method is the more satisfactory. The table required is one showing the values of Cs for various values of s , and can be prepared in a few minutes. It is sufficient to tabulate Cs for values of s increasing by 0.1 ft. Intermediate values are obtained with sufficient accuracy by taking each unit in the second decimal place of intercept as equivalent to 1 ft. of distance, since the constant will have a value in the neighbourhood of 100. Otherwise, a subsidiary table can be prepared for $s = .01, .02$, etc., ft. The tabulated distances fall to be increased by C_1 unless an anallactic telescope is used. This term is commonly included in drawing up the table, but, unless for normal staff holding, the tabulation of Cs only is preferable on the whole.

(2) The second method consists in graduating the rod so that when the observed intercepts are multiplied by 100 the correct values of Cs are obtained. This method has the serious objection that, since the rod is not graduated in feet, the determination of elevations involves additional calculation. It is also undesirable in that the same rod must always be used with the same instrument and that a change in the value of the stadia interval necessitates repainting of the rod.

In the case of the movable hair instrument, it is equally desirable to have the value of the constant $C = f/p$ a round number. Since in this case s is a constant, the quantity Cs should be made a round number, and this is easily effected by adopting a suitable value for s . Thus, if C is found to have the value, 1.019, the targets should be fixed on the staff at a distance of 9.813 ft. apart. Then, for level

sights, $D = \frac{10,000}{n} + C_1$, and the reductions may be performed with

the aid of a table of reciprocals. There is not the same objection to making the rod suit the instrument in movable hair tacheometry as in the fixed hair system, since the middle target from which elevations are obtained is placed at a known distance from the foot of the staff.

Observing with the Tacheometer.—In sighting the stadia rod, the pointing may be made to any part of the graduation. Readings are estimated to $\cdot 01$ ft. When the instrument constant has a value of 100, it is usual to dispense with the booking of the individual stadia hair readings and to read the staff in terms of Cs , or 100 times the actual intercept. The mental subtraction of the lower reading from the upper is facilitated by setting one of the hairs to a whole foot graduation. The reading of the axial hair must, however, be noted. The reduction is somewhat simplified if this hair is directed to the point on the rod corresponding to the height of the instrument axis above the ground. This will not be possible when the lower part of the rod is hidden, and should not be done if it brings the lower stadia reading near the ground. When the method is employed in conjunction with that of reading the intercept in terms of Cs , the latter is first observed with one of the stadia hairs at that whole foot graduation which brings the axial hair approximately to the instrument height. Thereafter, the axial hair is brought exactly to the instrument height graduation, and the vertical angle is noted. A small error is introduced by the circumstance that this angle differs slightly from that at which the intercept was read, but its effect is negligible except when locating station points by highly inclined sights. This simplified method is called the "Height of Instrument" method.

When intervening obstacles to sighting prevent the three hairs being brought simultaneously upon the staff, the intercept between the axial hair and one of the stadia hairs is observed, and the appropriate constant is used. If the constants for the half-intervals are unknown, the telescope is then tilted to enable the intercept between the axial hair and the other stadia hair to be read, the mean of the two vertical angles being used in the reduction.

In observing with the movable hair instrument, the centre target is first bisected with the axial hair. The micrometer milled heads are then simultaneously turned to move the stadia hairs from the positions they happen to occupy until they bisect the other targets. To read the distance of each hair from the centre, the number of complete pitches is counted on the comb scale by reckoning from bottom to bottom of the notches. The fractional parts are given on the drums, and the sum of the two complete readings is the quantity n .

Alternative Methods of Observing.—Several alternative methods of observing have been proposed at various times, and by different writers, and each has its own particular advantages to recommend it.

Mr. James Williamson, M.Inst.C.E., in his book *Surveying and Field Work* (Messrs. Constable & Co., London), describes a method of tacheometrical surveying in which only two cross hairs—the two outer—are used, the lower hair being always sighted on one particular point or zero index on the staff. The advantage of the method, therefore, is that, when the instrument has been set so

that the lower cross hair intersects the chosen point on the staff, only one staff reading has to be taken and booked, and this reading gives the actual staff intercept. The chief disadvantage appears to be that, by the ordinary method, the reading on the central hair is useful as a check against the occurrence of a gross error, due to an error in reading one of the other hairs, and in this modified method the check is not available.

No special equipment, other than that necessary for ordinary tacheometrical work, is required, but Mr. Williamson recommends the use of a special staff with a zero mark at about 4' 6" above the bottom of the staff, this height corresponding to the average height of the trunnion axis of the instrument above the ground. If such a staff is used, the reading of the upper hair gives the staff intercept directly, and this results in a saving of time and reduces the possibility of the occurrence of errors in reading. It will be realised, however, that, for a given length of staff, the loss of a length of 4' 6" at the bottom makes the possible length of sight less than what it would be if the ordinary methods of observing were used. Against this, is the argument that sights or pointings on very low portions of the staff are undesirable in any case.

Mr. Williamson shows that, for any given staff intercept, the locus of the zero point of the staff is a circle, which passes through the instrument, and has as its horizontal diameter a line, equal in length to the distance corresponding to the given staff intercept when the angle of elevation is zero, and which lies below the trunnion axis of the instrument at a distance equal to half the length of the staff intercept. This theorem makes it possible to construct a special chart, or diagram, by means of which the reduction of the observations can be made very quickly by graphical means. A copy of such a chart, on a large scale, is included in the book.

Prof. F. A. Redmond has given the name "The Even Angle Method" to a modification, suggested by him, of a method originally devised by Mr. M. E. York Eliot.* The characteristic feature of the method is that the vertical angle is never measured but is selected and "fixed" before the staff is read. The advantage of this procedure is that fine subdivisions of the vertical angles are not necessary, so that tables with a reasonably "open interval" can be computed, and the angle set to suit the interval of the tabulated angles, this interval corresponding with the divisions on the vertical circle.

The method of observing is as follows :

- (1) The telescope is pointed at the staff and clamped in such a position that the cross-hairs appear to be suitably placed for reading.
- (2) By means of the tangent screw, the zero of the vernier is set exactly opposite the nearest circle division and the value of the angle recorded.

* "The Use of 'Even Angles' in Stadia Surveying," by Professor F. A. Redmond, B.Sc. *Empire Survey Review*, Vol. III, No. 17, July, 1935

(3) The middle hair is read to the nearest tenth or hundredth, according to requirements, and the reading booked.

(4) One of the stadia wires is set, by means of the tangent screw, to cut the nearest foot mark on the staff, and either the readings on each stadia wire or the stadia intercept are booked.

It can be proved, either mathematically or by experiment, that the staff intercept is not appreciably changed by the small motion of the telescope needed to bring one hair from the position in which it was when the telescope was set to the fixed angle to the new position where one hair intersects the nearest foot mark.

The chief advantage of the method is, of course, that it is possible to compute tables for use with it which are of reasonable size and at the same time do not necessitate heavy interpolations. The main disadvantage is that the reading on the central hair cannot be used as a check against error of reading. Also, the method is not suitable for use with every type of instrument as the tables have been specially designed for use with readings on a 5-in. vernier instrument, with a circle divided to 20', and, when circles are divided to half degrees, as is the case with many plane table alidade circles, it cannot be used to the best advantage.

Professor Redmond has published special tables for use with this method, the reference being *Tacheometrical Tables*, by F. A. Redmond (Simpkin, Marshall and Co., Ltd., London).

A writer in the *Empire Survey Review* recommends the Height of Instrument method when it is used in conjunction with a "Differential Slide Rule" for the reduction of the observations, and, when so used, claims the following advantages for it as compared with the "Even Angle" method :*

(1) The Even Angle method involves more work in the calculation of heights.

(2) The use of the "Differential Slide Rule" simplifies the reduction of the readings in the uneven angle method, and does away with the necessity for the use of traverse tables when computing the co-ordinates of traverses in which distances are measured by tacheometer.

(3) It is more convenient in use when modern instruments, in which the reading eyepiece for the vertical circle is alongside the main telescope eyepiece, are used.

Relative Merits of Vertical and Normal Rod Holding.—The vertical system proves the more convenient on the whole, and is generally preferred. Normal staffing is, however, sometimes adopted, and a comparison may be drawn between the two systems.

Ease of Reduction of Observations.—The distance and elevation formulæ are somewhat simpler for vertical than for normal holding, and published reduction tables and diagrams are based on the

* "Tacheometrical Surveying and the Use of the Differential Slide Rule," by N. B. Favell, B.A. *Empire Survey Review*, Vol. IV, No. 29, July, 1938.

former. Practically, there is little difference in this respect since, except in the case of large values of θ , it is usually allowable in the normal system to assume $h \sin \theta = 0$ and $h \cos \theta = h$.

Facility of Holding.—Provided the rod is fitted with a spirit level, the vertical direction is more easily determined and maintained than the normal. Normal holding by the use of a staff director necessitates the rodman sighting the instrument, and this is at times impracticable in rough country, but the method has the advantage, particularly evident in high wind, that the rod can be swung as in ordinary levelling.

Effect of Careless Holding.—Errors of distance and elevation caused by deviation of the staff from its intended vertical or normal position are very much more serious with vertical than with normal holding. This would prove an important objection to vertical staffing, were it not that the errors are easily kept within allowable limits by the use of a spirit level or plummet.

Accuracy of Measurements by Stadia.—The principal sources of error in stadia observations fall into three classes: (1) Errors in reading the staff intercept; (2) Errors of instrument and value of the constant; (3) Errors due to natural causes.

(1) Since the actual intercept is multiplied by a constant of about 100, an uncertainty of .01 ft. in the reading produces an uncertainty of 1 ft. of distance. In estimating the reading of each hair to the nearest .01 ft., the greatest error which would occur under perfect conditions of sighting is $\pm .005$ ft., and the average error $\pm .0025$ ft. The average error in the estimated value of the intercept is therefore $\pm \sqrt{.0025^2 + .0025^2} = \pm .0035$ ft., corresponding to $\pm .35$ ft. of distance. Practically, such a good estimate of the intercept cannot be expected. Accuracy of reading depends greatly upon the length of sight, definition and magnifying power of the telescope, fineness of the hairs, elimination of parallax, clearness of the atmosphere, graduation of the rod, steadiness of holding and accuracy of the vertical or normal position of the rod, and the observer's personal factor. If the resulting uncertainty is expressed as a ratio of the distance, its value will be comparatively great for very short sights and also for distances beyond the range of clear sighting for the particular instrument used. The length of sight giving minimum value of the proportional error will generally lie between 400 ft. and 700 ft., depending upon the quality of the instrument.

In observations with the movable hair instrument on a staff with prominent targets, errors of sighting are reduced at long distances; but up to about 800 ft. the results are likely to be somewhat inferior to those obtained from fixed hair observations.

(2) The accuracy with which distances and elevations are measured is dependent upon the adjustment of the altitude level and the elimination or determination of index error, as well as

upon the accuracy of reading of the vertical circle. The effect of such errors as are likely to occur in the vertical angle measurement is relatively unimportant in distance determinations. Errors of levelling, however, may have serious effects on the elevations, and attention must be paid not only to the adjustment of the altitude level but also to maintaining the bubble central during observations.

Distance and elevation are affected by using an incorrect value for the constant, and the resulting errors can be kept within reasonable limits only by determining the constant from time to time under the same conditions as obtain on the survey. Appreciable error may be introduced by alteration in the value of the constant through change of conditions.

(3) The chief natural sources of error are wind and the effect of the difference of atmospheric refraction on the two stadia lines of sight. In high wind the rod cannot be held steady and plumb, and good reading is impossible. To avoid the second error, the lower sight line should not be made to pass within 3 ft. of the ground for any considerable portion of its length. This precaution becomes specially important during the midday hours. In consequence of the disturbed state of the lowest strata of air, it is not unusual for distances to be determined more accurately in moderately rough country than over very flat ground.

Experience shows that the error in distance from a single observation should not, on an average, exceed about 1 in 500 for moderate lengths of sights, and that the error of elevation need not exceed about .3 ft. for moderate inclinations. Since many of the errors to which the observations are subject are of a compensating nature, the closing error of a stadia traverse is proportional rather to the square root of the distance than to the distance. The linear error in traversing by stadia is usually less than $10\sqrt{D}$ ft., where D is the length of traverse in feet. The closing error of elevation in circuits between bench marks depends greatly upon the roughness of the country, but on average ground can be kept well within $.01\sqrt{D}$ ft.

THE TANGENTIAL SYSTEM

Distances and elevations may be deduced from staff readings taken by a theodolite without any additional fittings whatever.

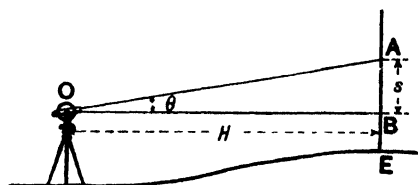


FIG. 323.

Fig. 323 shows the simplest case, that in which the ground is sufficiently level that the staff may be read with a horizontal line of sight. The reading at B is observed with the telescope levelled, and that at A by means of a sight

inclined at θ to the horizontal. Denoting the difference of the readings by s , then

$$H = s \cot \theta.$$

The elevation of E is derived from the reading B as in ordinary levelling.

When the ground will not permit of a horizontal sight, two vertical angles, θ and ϕ (Fig. 324), must be measured. For long sights the observations may be taken to prominent targets or marks on the staff at a known distance s apart. Otherwise a stadia rod is used, the two graduations observed being preferably an exact number of feet apart.

If θ is observed to the upper, and ϕ to the lower point on the staff, the reduction formulæ are as follows :

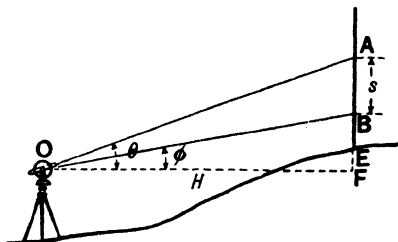


FIG. 324.

$$\begin{aligned} \text{Since } AF &= H \tan \theta, \text{ and } BF = H \tan \phi, \\ (AF - BF) &= s = H (\tan \theta - \tan \phi), \end{aligned}$$

$$\text{whence } H = \frac{s}{(\tan \theta - \tan \phi)}, \text{ for angles of elevation,}$$

$$\text{and } H = \frac{s}{(\tan \phi - \tan \theta)}, \text{ for angles of depression.}$$

Knowing H.I., the elevation of the axis of the instrument above datum, the elevation of E is given by

$$\begin{aligned} \text{Reduced level of E} &= \text{H.I.} + \text{FB} - \text{EB}, \\ &= \text{H.I.} + H \tan \phi - \text{EB}, \text{ for angles of elevation,} \\ &\text{and} = \text{H.I.} - H \tan \phi - \text{EB}, \text{ for angles of depression.} \end{aligned}$$

The evaluation of $\frac{s}{(\tan \theta - \tan \phi)}$ is somewhat laborious as, since two angles are involved, a table of the values of $(\tan \theta - \tan \phi)$ would be cumbersome. The reduction is simplified by adopting a length of 10 ft. for s and using a table of reciprocals. Various other suggestions have been made for lessening the labour of reduction, and a number of modifications of the theodolite have been devised so that part, at least, of the work may be performed mechanically.

Use of Particular Values of θ and ϕ .—If the inclined sights are taken to two targets on the staff, there is no choice as to the values of θ and ϕ , but if the staff is fully graduated, particular angles may be set on the vertical circle, and the variable s is obtained from the staff readings. Reduction is facilitated by using only those angles the natural tangents of which are .01, .02, .03, etc. If two

consecutive angles from a list of these are employed for an observation, $(\tan \theta - \tan \phi)$ is always $\cdot 01$, so that

$$H = 100 s,$$

and the quantity, $H \tan \phi$, required in the calculation of the levels, amounts to s , $2s$, $3s$, etc., according to the angle ϕ used. The method therefore enables reductions to be performed mentally.

By reference to a table of log. tangents, a list of the required angles may be prepared as follows:

Tangent.	Angle to nearest second.	Tangent.	Angle to nearest second.
$\cdot 01$	0 34 23	$\cdot 06$	3 26 1
$\cdot 02$	1 8 45	$\cdot 07$	4 0 15
$\cdot 03$	1 43 6	$\cdot 08$	4 34 26
$\cdot 04$	2 17 26	$\cdot 09$	5 8 34
$\cdot 05$	2 51 45	$\cdot 10$	5 42 38
		etc.	etc.

The difficulty of setting off these angles with sufficient accuracy by vernier explains why this system has not been applied in practice to any great extent. The method is a feasible one with micrometer reading of the vertical circle. It is, however, possible to have the vertical circle fitted with a special scale the graduations of which are placed at the above angles on either side of zero, while the figures represent 100 times their tangents.

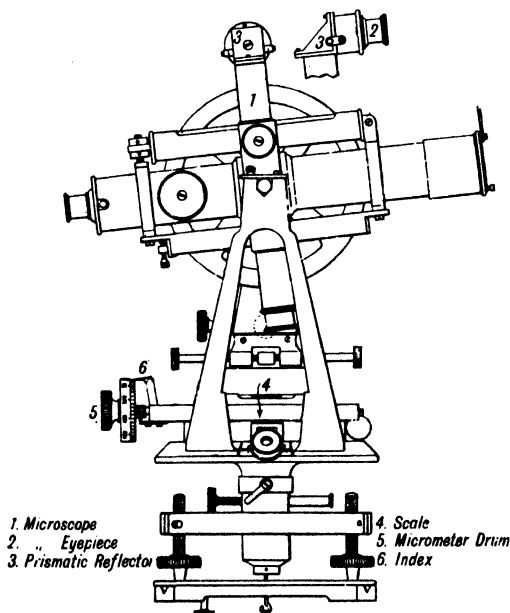


FIG. 325.—OMNIMETER.

The Omnimeter.—Eckhold's Omnimeter (Fig. 325) was the earliest of the special instruments for tangential tacheometry, but it is now little used, and is mainly of historical interest.

It consists of a transit theodolite fitted with (1) a powerful compound microscope permanently fixed with its axis accurately at right angles to the optical axis of the telescope and passing through the horizontal axis, (2) a graduated scale placed upon the vernier plate, the reading of this scale being performed by means of the microscope. As usually arranged, the scale is 4 in. long, and is divided into 200 parts. For the purpose of measuring fractional parts of these divisions, the scale can be given a small longitudinal motion by a micrometer screw of $1/50$ in. pitch, so that the graduation corresponding to the approximate reading may be brought into the line of sight of the microscope. Fractional parts are given on the micrometer head to $1/500$ th of a scale division. The zero of the scale is in the middle of its length, and should be the point sighted by the microscope when the telescope is horizontal. As before, an observation consists in making pointings to two graduations or targets on the staff, preferably 10 ft. apart, but, in place of reading θ and ϕ , the corresponding readings of the scale are noted. For convenience in reading, the microscope should be fitted with a prismatic reflector: in the later forms of the omnimeter the eyepiece of the microscope is alongside that of the telescope.

Reduction.—In Fig. 326, O represents the horizontal axis, and S the scale, of which the length and distance from O are greatly exaggerated. Oa and Ob are the lines of sight of the microscope normal to OA and OB, those of the telescope. The zero of the scale is at f vertically below O.

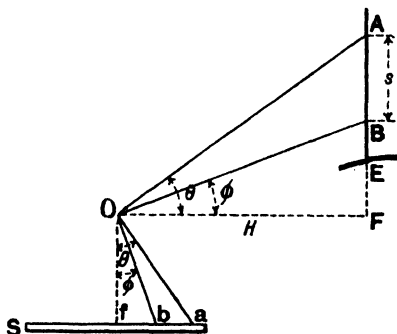


FIG. 326.

Let n_1, n_2 = scale readings at b and a respectively,

d = length of one scale division,

c = constant of the instrument = $\frac{Of}{d}$.

Since the triangles of OABF are similar to those of Oabf,

$$\frac{AB}{OF} = \frac{ab}{Of} \text{ so that}$$

$$H = \frac{Of \times s}{d(n_2 - n_1)} = \frac{cs}{(n_2 - n_1)}.$$

For the vertical BF, required in computing the elevation of E, we have

$$\frac{BF}{OF} = \frac{bf}{Of},$$

$$\text{so that BF} = \frac{n_1 dH}{Of} = \frac{n_1 H}{c} = \frac{sn_1}{(n_2 - n_1)}.$$

The instrument is designed to have c a round number, but the value of the constant should be determined by repeated observations of measured distances.

The Bell-Elliott Tangent-Reading Tacheometer is an improved form of omnimeter in which the scale is graduated to read directly the natural tangents of the two vertical angles.

The Gradienter.—If the tangent screw actuating the vertical circle of a theodolite is provided with a micrometer head for the measurement of parts of a revolution and a scale for counting the whole turns, the fitting is called a Gradienter. It is principally used in setting out gradients, but is also employed in tachemetry. The pitch of the screw must be accurate, and is commonly such that a line of sight originally horizontal is moved through $\tan^{-1} \cdot 01$ by one revolution, so that the horizontal hair is shifted 1 ft. on a vertical staff at a distance of 100 ft. In some instruments this angular motion is produced by two revolutions of the screw, but in either case the smallest division on the drum corresponds to $\cdot 01$ ft. on a staff 100 ft. distant.

In the case where a horizontal sight can be taken, the staff is first read with the telescope level. The screw is then given one turn (or two turns in the case of the finer pitch), and the staff is again read. Then

$$H = 100 \text{ s.}$$

where s is the difference of the readings.

For inclined sights, the staff may be held vertically or normal to one of the lines of sight. The former is the usual method. The formulæ derived below for that case may easily be modified for normal holding. In making an observation with the staff station

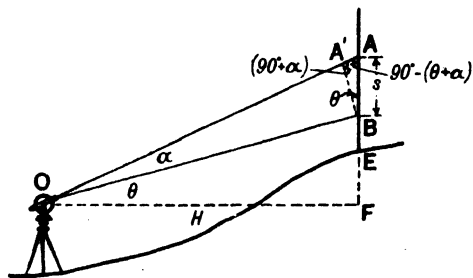


FIG. 327.

higher than the instrument (Fig. 327), the telescope is first directed towards a low point B on the staff, and the vertical angle θ is read. The line of sight is then moved through $\tan^{-1} .01$ by the grader screw, and the intercept $AB = s$ is noted. Alternatively, if A and B are fixed

targets, the number of turns, n , necessary to move the line of sight from one target to the other, is read on the drum.

To obtain the general formulæ, let the pitch of the screw be such that one revolution moves the line of sight from the horizontal through c ft. on a vertical staff 100 ft. distant, and, in the case of Fig. 327, let n turns of the gradienter move the line of sight from

B to A through $\alpha = \tan^{-1} \frac{nc}{100}$.

Draw A'B perpendicular to OB.

$$\text{Then } OB = \frac{100A'B}{nc},$$

but, from triangle A'AB,

$$A'B = \frac{s \cos(\theta + \alpha)}{\cos \alpha} = s(\cos \theta - \sin \theta \tan \alpha) = s(\cos \theta - \frac{nc}{100} \sin \theta);$$

$$\therefore OB = s \left(\frac{100 \cos \theta}{nc} - \sin \theta \right),$$

$$\text{and } H = OB \cos \theta = s \left(\frac{100 \cos^2 \theta}{nc} - \frac{1}{2} \sin 2\theta \right).$$

The vertical component, BF, of the sloping distance =

$$OB \sin \theta = s \left(\frac{100 \cos \theta \sin \theta}{nc} - \sin^2 \theta \right) = s \left(\frac{100 \sin 2\theta}{2nc} - \sin^2 \theta \right).$$

When θ is a depression, the above formulæ apply provided B is the higher of the two points on the staff. If, however, θ is measured to the lower point, the observed angle must be decreased by

$\tan^{-1} \frac{nc}{100}$ for insertion in the formulæ.

The use of a self-reading rather than a target staff facilitates reduction, for nc is then unity, and, since the second term can often be neglected, we have

$$H = 100 s \cos^2 \theta,$$

$$\text{and } BF = \frac{100 s \sin 2\theta}{2}.$$

Fergusson's Percentage Unit System.—Mr. J. C. Fergusson, M.Inst.C.E., has devised a novel system for the division of the circle with the object of reducing to a minimum all calculations involving trigonometrical functions. An unequal division of the circle is adopted, angles being reckoned in terms of their natural tangents, expressed as percentages. The application of this system to tangential tacheometry facilitates the reduction of observations.

Fig. 328 illustrates the method of division. A circle is inscribed in a square, and is divided into octants by the quadrant lines and by the diagonals of the square. Each of the eight tangents, of length equal to the radius of the circle, is divided into 100 equal parts.

Lines from the centre to these points divide each octant into 100 unequal parts, which each subtend $\frac{1}{100}$ of the tangent and thus form telemetric units. The points of division on the circle are

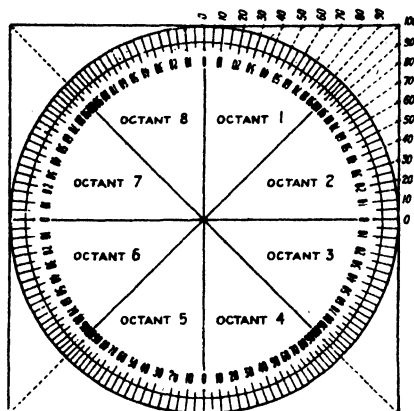


FIG. 328.

numbered from 0 on the quadrantal lines to 100 at the diagonals. The magnitude of any angle between a quadrantal line and a line in the octant on either side is given on the circle in percentage units, and its value in these units measures the length of the perpendicular or tangent in terms of the radius.

The percentage system is embodied in Fergusson's percentage theodolite, the circles of which are graduated both in degrees and percentage units. A vernier cannot be used for subdividing the unequal percentage divisions,

and its place is taken by a spiral drum micrometer, by means of which readings can be taken to .01 of a unit.

In the application of the system to tangential tacheometry, the observations may be made either on a graduated rod or on one with fixed targets. The former method is more convenient of reduction. A reading is taken of a low point on the rod with the vertical circle set to a whole percentage unit. The telescope is then elevated through an exact number, n , of units, usually 1 or 2, and the upper rod reading is noted. Then, corresponding to the

formula, $H = \frac{s}{(\tan \theta - \tan \phi)}$, we have

$$H = \frac{100s}{n}.$$

The elevation of the staff station may be determined as before by evaluation of $H \tan \phi$, which is readily obtained in this case. Alternatively, the line of sight is directed to the graduation representing the height of the instrument axis above the ground. If the vertical circle now reads $g\%$, this represents the gradient between the instrument station and the staff station, and their difference of

$$\text{elevation} = \frac{gH}{100}.$$

The Szepešsy Direct-Reading Tacheometer.—The objection to the tangential system, that two telescope pointings are required, has been overcome in a neat manner by Szepešsy, of Budapest, who has patented a direct-reading instrument. A scale of tangents projected

upon a circle, as in Fig. 328, is required. This may be engraved on a glass plate of cylindrical form attached to the vertical circle cover, so that it does not rotate when the telescope is tilted. Light transmitted through the scale is received by the objective of a microscope attached to the telescope and having its optical axis parallel to that of the telescope and passing through the horizontal axis of rotation. The rays undergo reflection by prisms, and are brought to a focus in the plane of the telescope reticule, the eyepiece of the telescope serving also as the eyepiece of the microscope. The image of the scale occupies nearly half of the field of view, and, when the telescope is directed towards a staff, the image of the latter is placed alongside that of the scale as in Fig. 329.

The scale of tangents is divided to $\cdot005$, and is figured at every $\cdot01$ in terms of 100 times the tangent, so that graduation 15, say, corresponds to $\tan^{-1} \cdot15$. To take a reading, the vertical circle tangent screw should be adjusted to bring a scale graduation, preferably a numbered one, opposite the axial hair. Since the scale graduations on either side represent angles whose tangents differ by $\cdot01$, the length of staff intercepted between them multiplied by 100 is the required horizontal distance. The scale reading opposite the axial hair being 100 times the tangent of the inclination of the line of sight, the vertical component is given by the previous staff intercept times the scale reading at the axis. In the case of Fig. 329 the horizontal distance is 199 ft., and the vertical component is $1\cdot99 \times 15 = 29\cdot9$ ft. The staff reading at the horizontal hair is noted as usual for application to the vertical component.

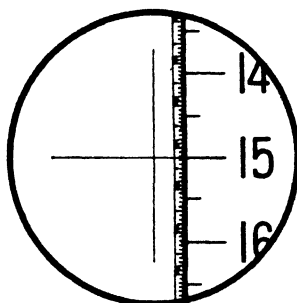


FIG. 329.

The instrument is made in England by Messrs. E. R. Watts and Son, of London.

The Subtense Bar Method.—In this method, distances are obtained by observation of the horizontal angle subtended by targets fixed on a horizontal bar at a known distance apart of from 2 ft. to 20 ft.

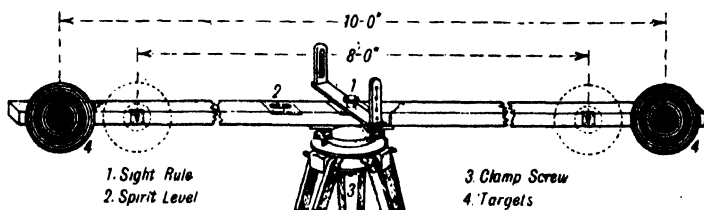


FIG. 330.—INDIAN SUBTENSE BAR.

PLANE AND GEODETIC SURVEYING

method is largely used in the Survey of India for measuring lengths of traverse courses in rough country.

The Indian subtense bar illustrated in Fig. 330 is 11 ft. long. It is mounted on a tripod, and is levelled by means of the small spirit-level 2 and quick levelling head. The sight rule 1 affords a line of sight perpendicular to the bar, which is thereby set normal to the line of measurement. When aligned and levelled, the bar is clamped by the screw 3. The targets are of 8½-in. diameter, and are painted red on one side and white on the other, in each case with a 3-in. black centre. The fittings are such that the discs can be placed either 10 ft. apart with the red faces showing or 8 ft. apart with the white faces showing, so that the colour seen indicates the interval being observed.

Observations may be made by means of a theodolite fitted with a horizontal eyepiece micrometer. The method then resembles stadia tacheometry with movable hairs, and the sloping distance is similarly obtained. An alternative system, due to Col. Tanner, is used in the Indian Survey for traversing. It consists in measuring the subtended angle on the horizontal circle of the theodolite, the method of repetition (Vol. II, Chap. III) being used to obtain the necessary refinement of angle reading with a vernier instrument. The smallness of the angle makes it possible in most cases to make the repetition by alternately manipulating the lower and upper tangent screws without unclamping.

In computing the distance in terms of the known subtense length s and the subtended angle a measured in this way, no reduction to the horizontal is required, since a is a horizontal angle, and we

have $H = \frac{1}{2} s \cot \frac{a}{2}$. The approximate formulæ, $H = s \cot a$, and

$H = \frac{s}{a'' \sin 1''} = \frac{206,265 s}{a''}$, are more convenient and of ample

accuracy. The elevation of the subtense station may be derived by measuring the vertical angle to the bar. In subtense traversing in the Indian Survey, however, it is found economical of time to measure the bearings and vertical angles between stations independently of the observations on the bar, which need not in consequence be plumbed exactly over the station mark.

To apply the subtense bar system to contouring, it is sufficient to use a very simple form of bar. The alignment and levelling are simply estimated by the rodman, who holds the bar at a constant height above the ground.

Double Image Tacheometers.—There has been developed on the Continent of recent years a type of tacheometer, called the double image telemeter, of which there are several patterns. A horizontal subtense staff is required, and this is supported by a vertical rod, which is also graduated. The optical arrangement of the instrument is such that two horizontal images of the staff are formed, one above

the other, and these are seen to be displaced relatively to each other in the direction of their length by an amount proportional to the distance to the staff station. The required displacement is based on an invention patented by R. Bosshardt in 1923 in which suitably placed glass wedges, or very acute angled prisms, were used for this purpose. In the Wild RDH Self-Reducing Tacheometer described in some detail in pages 616 to 621, one image of the graduated subtense staff is formed by rays falling, after refraction by a rhomboid prism, on the upper half of the object glass. The other image is formed by rays which pass through two rotatable glass wedges mounted in planes perpendicular to the optical axis of the telescope and then fall on the lower half of the object glass. As the telescope is rotated about its horizontal axis the tacheometrical angle is varied automatically by suitable rotation of the wedges in such a way that 100 times the displacement reading of the images gives the horizontal distance directly, and, when the setting of the wedges is altered by turning them as a unit through 90° , 100 times the displacement reading of the images gives the difference in elevation between instrument and staff. Metres are given directly by readings on the image of the staff and decimetres by the reading on the staff vernier which is seen as the lower image, while centimetres and (by estimation) millimetres are read off on a graduated micrometer head, rotation of which causes a slight rotation of the rhomboid prism about a vertical axis and hence a small horizontal displacement of one image.

The Bosshardt-Zeiss Redta and the Kern DK-RT self-reducing tacheometers are very similar to, and work on much the same principle as, the Wild RDH instrument but differences in elevation are not obtained direct as in the Wild instrument. In the Zeiss instrument the vertical circle carries a scale of tangents as well as ordinary angular graduations, and differences in elevation are obtained by multiplying the horizontal distance by the tangent of the angle as read on the scale. The Kern instrument has a vertical circle read through the same micrometer eyepiece as the horizontal circle and is provided with either angular divisions or a scale of tangents.

Results obtained with double-image tacheometers are much more accurate than those obtainable with ordinary stadia wires. Experience both in this country and elsewhere shows that, with reasonably good observing conditions, the accuracy to be expected is of the order 1 to 2 cms. in 100m., or 1/10,000 to 1/5,000.

The double-image tacheometer, using sights of medium length, is most suitable for work of moderate accuracy such as traverse or large scale work, but, when numerous sights from one station are required, as in contouring or in detail survey, the horizontal staff is an inconvenience.

Relative Merits of Tangential and Stadia Tacheometry.—In comparison with the stadia system, tangential tacheometry labours under the disadvantage that, except in the case of the Szepessy and double image instruments, two manipulations of the instrument

and two sights are required for each complete observation. There is always a risk of disturbance of the instrument between the sights, as well as the possibility of a change in atmospheric refraction occurring in the interval. Refraction error is least in the case of the horizontal bar method, which is well adapted for long sights, but in the vertical staff methods full advantage cannot be taken of the refinements of reading in the various instruments. Some of these instruments are also liable to instrumental imperfections, since they have not the simplicity of the stadia instrument with non-adjustable hairs. There is little difference between the two systems in the amount of reduction involved. In general, however, the tangential system must be regarded as inferior to stadia work, although comparative tests show that the difference in accuracy is not serious.

FIELD WORK

General Arrangement of Field Work.—Tacheometric surveys are chiefly confined to detailed contouring, such as is required in location surveys for public works. The field work consists in locating representative points over the area.

If to the observations for the distance and elevation of a staff station there is added the measurement of its bearing from the instrument, the three co-ordinates thus determined serve to locate the point. By fixing in this manner as many points as are necessary, a survey may be made of the area within the limit of convenient observation from the instrument. Except in the case of very small surveys, it will be necessary to occupy several instrument stations so situated that all the required detail can be located from them. These form stations of a controlling framework, the precision of which governs that of the whole survey.

When the tract to be surveyed is sufficiently narrow that the half-breadth is within sighting range of the instrument, the survey is controlled by a traverse approximately along the centre line of the strip. In difficult ground, subsidiary traverses, closed or unclosed, may have to be run out at intervals to reach parts, such as wooded gorges, which cannot be surveyed from the main stations. Intermediate stations on long traverse courses have frequently to be occupied for the taking of topography. Control from a central traverse is that generally adopted in location surveys for lines of communication, the main traverse roughly following the proposed route.

When the survey is too broad to be based on a single traverse, the control may be furnished either by a triangulation or by a series of traverses. The former is more suitable when the survey embraces a considerable area. A principal and a subsidiary system may be required, in which case the latter is of low precision and may be executed by plane table. For moderate areas, traversing is

usually adopted. The arrangement may consist of a single main traverse from which numerous circuits are projected, or several approximately parallel traverses may be run out and tied together at intervals.

The measurement of traverse courses may be performed either by tape or tacheometrically, and the elevations of the instrument stations are obtained by ordinary or by tacheometrical levelling according to the degree of accuracy required. For large scale plans it is desirable to use tacheometer methods for locating detail only, but in small scale work they may be used exclusively provided due precautions are taken to limit the propagation of error. It is, however, always advisable to check the levelling at intervals by closures on to bench marks established by ordinary levelling.

Field Party.—A small survey on easy ground can be performed quite well by a surveyor and a staffman, but in rough country their progress is slow. Where long sights have to be taken, there is apt to be uncertainty in the location of the contours, as it is difficult for the surveyor to judge whether the staff is being held on representative salient points.

Large surveys are best executed by a party consisting of : (a) the chief of party ; (b) the instrument man ; (c) the recorder ; (d) two to four staffmen ; (e) one or two labourers for clearing and portage ; (f) a draughtsman, if the survey is to be plotted in the field.

The *Chief of Party* has to perform the following duties in addition to superintendence :

- (1) The selection of the instrument stations.
- (2) The placing of the staffmen, the most suitable positions being ascertained by reconnoitring the area controlled by the instrument.
- (3) The preparation of a sketch showing the position of each staff station, the run of the contours relative to those, detailed measurements of buildings, etc. A carefully made pictorial sketch is of great value in facilitating the plotting, particularly of rough country.

The *Instrument Man* is responsible for the actual observations. The number of staffmen he is able to keep occupied depends upon his expertness,* but to a greater degree upon the time required by the staffmen to change their positions. In close contouring over country with considerable detail, the observer may be fully occupied in sighting a single staff. On the other hand, he may be able to keep six or eight staffmen employed if the required contour interval is large and the ground is precipitous.

The *Recorder* acts as assistant to the instrument man. He keeps the field book, entering the readings as they are called out. His

* Correspondence in *Engineering Record* for 1914 indicates that a thoroughly experienced observer, assisted by a recorder, can take over 1,000 sights in average country in a day of 8 hours.

system of numbering or lettering of points observed must agree with that adopted in the chief's sketches. When the instrument man is head of the party, the recorder must prepare the sketches.

Staffmen.—When several staffmen are employed, the most experienced should be sent farthest from the instrument. A definite code of signals, preferably by whistle, must be arranged, so that each may know when his rod is being sighted and when he is free to take up a new position.

Routine.—Taking the case of a location survey controlled by traversing, the first step is to obtain the elevation of the initial traverse station, A. This may be done either by independent spirit levelling or by tacheometric observations carried from a bench mark. In the latter case, the procedure is analogous to that of ordinary levelling except that the alternate backsights and foresights are in general inclined to the horizontal, the elevations being derived from vertical angles and distances. This preliminary levelling is checked either by working back to the bench mark or by adopting the method, to be used in the traverse itself, of observing the difference of elevation between successive points by a backsight and foresight from each. In this case the instrument is first set up either over the bench mark or near it.

When the instrument is set up at station A, the height of the axis above the ground is measured and noted. Orientation is effected in terms of the meridian adopted in the same manner as in ordinary traversing (page 218). The second traverse station, B, having meantime been selected, observations are made for its bearing, distance, and elevation. The instrument man then proceeds to locate such points round A as will enable the detail to be plotted and contours interpolated. The observations to those points are called side shots. In general, the accuracy required in their observation may be much lower than that necessary in the traverse, since errors cannot be propagated through them. It is usually sufficient to read the bearings to the nearest $5'$, but, in contouring, the vertical angles should be read to the nearest $1'$. The staffmen should be kept as far as possible in straight lines. Time is economised in sighting if they are placed roughly in radial lines from the instrument: mistakes in distance reading are, however, more easily detected if the lines are roughly perpendicular to the traverse courses. When the side shots at station A are completed, the bearing of B is observed anew both to check the previous reading and to test the orientation of the circle.

The instrument is now carried forward and set up at B, while the chief reconnoitres and selects the position of the next station, C. The first observation from B is upon A, not only for orientation, but also for distance and difference of elevation. The length of each traverse course and the elevation of each station are therefore determined twice. These observations should be reduced, and their

consistency verified, as soon as possible. Any of the methods of page 220 is available for carrying forward the bearing, and the observations proceed at B as at A.

Notes.—(1) When the traverse distances and levels are obtained independently, it is still desirable to check them tacheometrically as above.

(2) Traverse courses are better kept sufficiently short that the distance can be accurately read. If long courses are introduced, the observations along them must be obtained by dividing the length into sections.

(3) Bearings of traverse courses should be obtained by sighting on poles marking the stations. When poles are not available, the edge of the staff is sighted for bearing.

(4) As in ordinary traversing, no opportunity should be neglected of taking cross bearings, and checks on elevations should also be secured where possible.

(5) The detail taken from an instrument station should not include points which can be better commanded from the succeeding station, but important points may be fixed by sights from both stations.

Note-keeping.—The tabular arrangement of the field book depends upon the system of tacheometry used and upon the character of the survey, but is largely a matter of individual preference. Sufficient columns should be provided for the entry of the results of the field observations and for use in the subsequent reductions. In the case of stadia book-keeping, the number of columns used ranges from 6 to 16.

The accompanying example will serve as a guide. In it the individual stadia hair readings are not entered, the distance reading representing 100 times the actual intercept. The reduced entries are for constants of 100 and 1·2, but details of the reductions are not shown, since the use of a table or diagram is supposed.

Inst. Stn. and Ht. of Axis.	Staff Station.	Distance Reading.	Axis Hair Reading.	Bearing.	Vertical Angle.	Horizontal Distance.	Vertical Component.	Elevation.	Remarks.
A 4·2								261·3	Bearings from Mag. Mer. at A
	B	532	4·2	93° 21'	+5° 16'	529	+48·7	310·0	
	1	316	4·2	326° 30'	−3° 43'	316	−20·5	240·8	Top of ridge
	2	205	8·5	320° 15'	−6° 12'	204	−22·1	239·2	Bed of stream
	3	218	4·2	357° 10'	−0° 36'	219	−2·3	259·0	Top of ridge

OFFICE WORK

Reduction of Observations.—Reduction formulæ for the various telemetric systems have been given, and, in describing the routine of performing reductions, it will suffice to consider the case of fixed stadia hair observations only.

If the constant of the instrument is sensibly different from 100, the first step is to alter the distance reading proportionately before obtaining the horizontal and vertical components. The labour of computing the latter quantities is reduced by the use of tables, diagrams, or mechanical aids. Some approximations are also

justifiable. If, as is usual, distances are required only to the nearest foot, the exact evaluation of $(f+c) \cos \theta$ is unnecessary, and it is commonly taken either as $(f+c)$ or simply as 1 ft. In the case of nearly horizontal side shots, the reduction to the horizontal may frequently be omitted, the limits of vertical angle and distance up to which the correction may be ignored depending upon the character of the work. It is usual to regard the distance reading as representing the horizontal distance for vertical angles up to 3° and distances not exceeding about 400 ft. If, in conjunction with this approximation, the $(f+c)$ term is ignored altogether, the two errors tend to compensate, and the limits may be extended. Elevations may be taken only to the nearest foot, or may be required as close as 0.1 ft. In the former case, $(f+c) \sin \theta$ can be neglected for vertical angles up to about 15° , but in the latter only up to 2° . When the $(f+c)$ term is included, and the reductions are performed by calculation, the use of the approximate formula,

$$V = \left(\frac{f\theta}{1} + (f+c) \right) \frac{1}{2} \sin 2\theta,$$

saves time, and is allowable up to $\theta = 15^\circ$.

Reduction Tables.—Reduction by direct calculation is laborious, and the work is greatly facilitated by the use of a reduction table, of which various forms are published. A common form gives for every minute of vertical angle the values of $\cos^2 \theta$ and $\sin \theta \cos \theta$, or 100 times these values, the quantities being called "horizontal distance" and "difference of elevation" respectively. Such tables may include, for a few values of $(f+c)$, the values of $(f+c) \cos \theta$ and $(f+c) \sin \theta$ for every degree of vertical angle. Another type of table gives the correction factor, $\sin^2 \theta$, required to reduce the distance reading to the horizontal.

Reduction Diagrams.—Observations may be reduced still more rapidly by the use of diagrams. Various forms of these are published, but it is a simple matter to construct one on squared paper. The accuracy of the reduction depends upon the scale of the diagram, which should be as large as possible.

Fig. 331 illustrates in outline the simplest form of diagram. It shows both horizontal corrections and vertical components, which are represented clear of each other to avoid confusion. The range of 12° of vertical angle, as illustrated, embraces the great majority of observations, but it would be desirable to extend it to 20° or 25° . If this necessitates too small a scale for clearness, the diagram can be continued on a second sheet.

To prepare the horizontal correction diagram, the scale of distance readings up to 1,000 ft. is set out along the left-hand vertical, and on the horizontal line at 1,000 the values of $1,000 \sin^2 \theta$ are marked off for vertical angles increasing by $10'$, or by $5'$ if the scale is sufficient. Straight lines joining the points so obtained to the origin give the corrections for other distance readings, since the

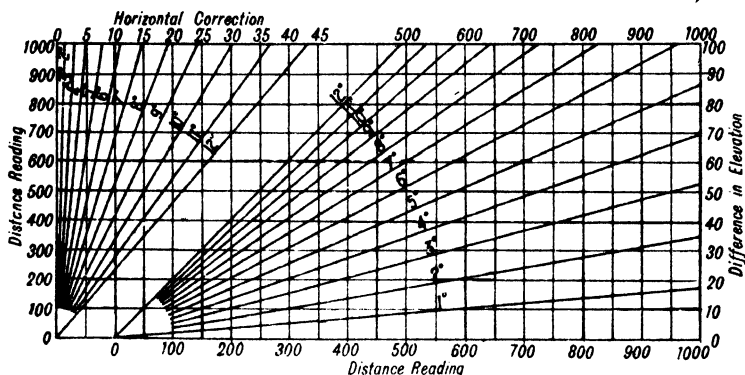


FIG. 331.—STADIA REDUCTION DIAGRAM.

correction varies as the distance. In constructing the diagram of vertical components, the range of the latter is limited to 100 ft. to keep the scale large. Distance readings are set out along the bottom, and on the right-hand vertical are marked off the values of $\frac{1}{2} \sin 2\theta \times 1,000$ for every $5'$ of vertical angle up to $5^\circ 45'$, the points being joined to the origin. Thereafter, the positions of the

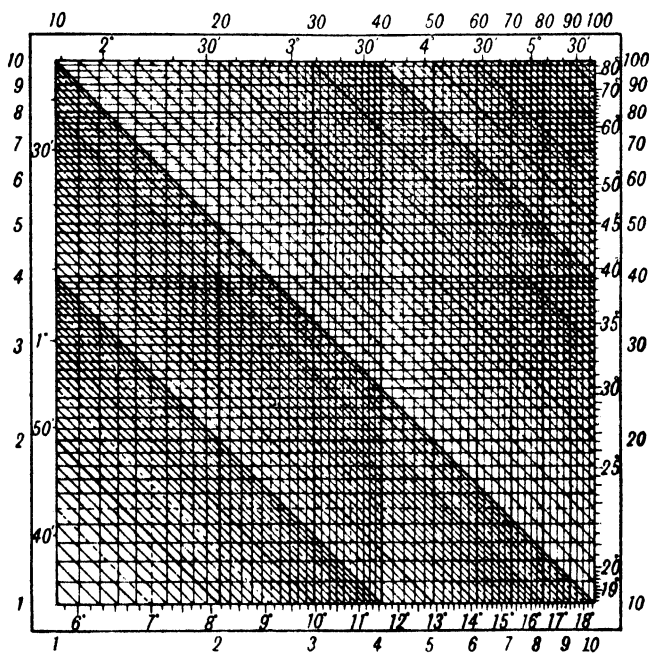


FIG. 332.—STADIA REDUCTION DIAGRAM.

radial lines are obtained by setting off on the top horizontal line the distance readings which give for each angle a vertical component of 100 ft.

Fig. 332 illustrates the McCullough diagram, which gives horizontal corrections for all vertical angles. On the left-hand and lower margins are scales of staff intercepts. The scales of vertical angles and of the required corrections extend round all four sides. The correction corresponding to any vertical angle and staff intercept is obtained by first following along the horizontal or vertical line through the value of the angle until an intersection is reached with the vertical or horizontal line corresponding to the observed intercept. The figure given at either end of the diagonal through this intersection is read, and, according as the vertical angle appears on the left, upper, bottom, or right-hand margin, it is multiplied by $\cdot 01$, $0\cdot 1$, $1\cdot 0$, or $10\cdot 0$ to give the actual amount to be subtracted from the distance reading.

Reduction by Mechanical Means.—Ordinary stadia observations may be reduced rapidly and with sufficient accuracy by using the stadia slide rule. Various forms of these are in use. Some are arranged to enable the horizontal and vertical distances to be read directly: others give the horizontal correction.

For the mechanical reduction of distances for plane table surveying with a tacheometric alidade, Dr. Louis* has devised a method whereby the horizontal component may be plotted by the use of proportional dividers. The scale on the dividers is one of vertical angles, and is such that when the centre mark is set to an observed vertical angle and the longer legs are opened out to scale the distance reading, the horizontal equivalent is given by the distance between the points of the shorter legs.

Plotting.—The framework of triangulation or traverse should, if possible, be completely plotted and checked before any side shots are laid down. If, however, the framework is built up as the survey proceeds, it is preferable to have the plotting of detail keep pace with the field work so that in difficult country the run of the contours may be verified on the ground. The main framework is best plotted by co-ordinates, but, when the measurements have been obtained by tacheometer, co-ordinate plotting is sometimes regarded as a refinement, and the protractor is used.

In plotting the side shots, the bearings are first marked off by means of a circular protractor. The centre is placed at the station point, and the circle is oriented with reference to the framework lines meeting at the station. The draughtsman then proceeds to mark off points round the circumference corresponding to the bearings, which should be read out to him. Opposite each point is written the reference number of the side shot. The protractor is then removed, and the distances of the several points are scaled

* "A New Device for Tacheometric Plane-Table Surveying." *Min. Proc. Inst. C.E.*, Vol. CCIII.

from the station in the directions of the appropriate marks. The points so obtained are surrounded with a small circle, and against each is written its elevation. Time is saved in plotting side shots by the use of one or other of the several forms of protractor fitted with a radial scale. This can be set to any bearing, so that each point may be plotted in one operation. When the side shots are plotted, contours are interpolated to lie appropriately amongst them.

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EXAMPLES

1. A staff is held at distances of 100 ft. and 300 ft. from the axis of a theodolite fitted with fixed stadia hairs, and the staff intercepts by level sights are 0.99 and 3.00 ft. respectively. Determine the constants of the instrument, and calculate the horizontal distance of the staff from the instrument when the intercept is 5.00 ft. if, in this case, the telescope is inclined at 10° to the horizontal, the staff being held vertically.

2. In the course of a tacheometrical survey the following observations were made for the normal cross-section of a stream, the instrument being set up on one bank with the telescope level:

Staff Point.	Readings.			Remarks.
1	4.62,	4.76,	4.90	edge of water
2	6.06,	6.27,	6.48	
3	7.25,	7.63,	8.01	
4	8.30,	8.82,	9.34	
5	8.36,	8.96,	9.57	
6	7.31,	8.05,	8.80	
7	5.67,	6.54,	7.41	
8	3.69,	4.75,	5.81	
				edge of water

The telescope was fitted with an anallactic lens, the value of the constant being 100. Compute the cross-sectional area of the flow at the time of the observations.

3. A tacheometer has a diaphragm with three crosshairs spaced at distances apart of $1/40$ inch.

The focal length of the object glass is 9 inches and the distance from the object glass to the trunnion axis is $4\frac{1}{2}$ inches. A staff is held vertically at a point the level of which is 80 feet A.D. The telescope is inclined at 9 degrees to the horizontal, and the readings taken on the staff are 6.65 feet, 5.14 feet and 3.63 feet.

Find the distance of the point from the telescope and the level at the telescope. The height of the trunnion axis of the telescope is 4' 6". (Univ. of Lond., 1918.)

4. Find up to what vertical angle sloping distances may be taken as horizontal distance in stadia work, so that the error may not exceed 1 in 300.

Assume that the instrument is fitted with an anallactic lens and that the staff is held vertically.

5. The cross and stadia hairs of a theodolite were replaced by a surveyor in the field after a breakage. The necessary instrumental adjustments having been made, the following observations for the determination of the multiplying tacheometric constant were taken upon a vertical staff held at measured distances from the instrument :

Observation.	Horizontal dist. to staff in feet.	Vertical angle.	Stadia Hair Readings.	
1	200	0	2.86	4.76
2	400	+1° 3'	4.00	7.83
3	400	+1° 12'	5.05	8.87
4	600	+1° 36'	4.00	9.76

Obtain the mean value of the constant given by these observations, that of the additive constant being known to be 1.2 ft. (T.C.D., 1930.)

6. A theodolite fitted with stadia wires and having an additive constant of 15 inches is used for contouring. At the first station the height of the instrument was 4.72 ft., and the following readings were taken :

Staff Station.	Top Wire.	Middle Wire.	Bottom Wire.	Vertical Angle.	Remarks.
1	1.50	3.45	5.40	5° 14'	Theodolite set up over B.M. 223.50 ft. above datum.
2	2.87	4.23	5.60	7° 23'	
3	3.95	5.96	8.00	8° 12'	

Calculate the heights of the staff at the stations above datum. Take the multiplying constant as 100. The staff is held vertically in each case. (Univ. of Lond., 1919.)

7. A tacheometer is set up at an intermediate point on a traverse course AB, and the following observations are made on a vertically held staff.

Staff Station.	Intercept.	Axial Hair.	Bearing.	Vertical Angle.
A	7.42	6.71	218° 37'	- 5° 30'
B	6.80	6.40	38° 37'	- 6° 20'

The instrument is fitted with an anallactic lens, and the constant is 100.

Compute the length of AB and the reduced level of B, that of A being 226.8. (R.T.C., 1921.)

8. Observations taken by means of a theodolite fitted with stadia hairs gave staff intercepts of 1.95 and 4.89 with the telescope horizontal and the staff held vertically at horizontal distances of 200 feet and 500 feet respectively from the centre of the instrument. The instrument was then set over a station having a reduced level of 264.3, the height of the instrument axis above the station point being 4.0 feet. The readings of the three hairs on a vertical staff were 3.00, 6.21, and 9.42, the telescope being inclined at 8° below the horizontal. Calculate the reduced level of the staff station and its distance from the centre of the instrument.

9. The elevation of a point X is to be determined by observations from two adjacent stations of a tacheometrical survey. The staff was held vertically upon the point, and the instrument constant was 100. Compute the required elevation from the following data, taking the two observations as equally trustworthy :

Inst. Stn.	Hit of Inst. above Stn.	Elevn. of Stn.	Staff Point.	Vertical Angle.	Axial Hair Reading.	Stadia Intercept.
G	3.8	490.9	X	+ 2° 27'	7.56	8.12
H	4.1	582.1	X	- 4° 51'	6.93	6.87

(T.C.D., 1929.)

10. The elevation of a peg marking a survey station *A* is 334.6 ft. A tachometer fitted with an anallactic lens, and having a constant of 100, is set over the station so that the line of sight is 4.1 ft. above the peg. An observation is taken on a vertically held staff with the line of sight directed downwards at $7^{\circ} 45'$ below the horizontal, and the observed readings of the three hairs are respectively 4.50, 7.37, and 10.24. The staff remains upon the same point while the instrument is transferred to station *B*. A backsight taken on the staff with an angle of elevation of $9^{\circ} 30'$ gives readings of 3.00, 5.56, and 8.13. Calculate the elevation of station *B* if the height of the line of sight above the station peg is 3.6 ft.

11. Determine the gradient from a point *A* to a point *B* from the following observations made with a fixed hair tachometer fitted with an anallactic lens, the constant of the instrument being 100 :

	Bearing.	Readings of Stadia Hairs.		Reading of Axial Hair.	Vertical Angle.
To <i>A</i> ,	345°	3.00	8.48	5.74	$+15^{\circ}$
To <i>B</i> ,	75°	2.50	12.20	7.35	$+10^{\circ}$

(R.T.C., 1914.)

12. The following notes refer to three of the observations in a tachometric survey. The elevation of the instrument station was 639.4, the trunnion axis of the telescope having been at 4.7 ft. above the station.

Staff Station.	Bearing.	Vertical Angle.	Hair Readings.		
1	$97^{\circ} 45'$	$- 6^{\circ} 12'$	3.19	4.70	6.21
2	$103^{\circ} 15'$	$0^{\circ} 0'$	7.00	8.92	10.84
3	$105^{\circ} 50'$	$+10^{\circ} 42'$	4.00	6.46	8.92

The instrument was fitted with an anallactic lens, the value of the constant being 100, and the staff was held *normal* to the line of sight. Find the horizontal distances to the staff points and their elevations.

13. Levels were carried from a bench mark to the first station *A* of a tachometric survey by tachometric observations. The instrument was fitted with an anallactic lens, and the value of the constant was 100. The following observations were made, the staff having been held vertically :

Inst. Stn.	Ht. of Axis.	Staff Point.	Vert. Angle.	Staff Readings.	Elevn.
1	4.4	O. B. M.	$-2^{\circ} 20'$	4.00, 6.11, 8.22	206.3
		Change Pt.	$+4^{\circ} 36'$	3.50, 5.07, 6.64	
<i>A</i>	3.8	do.	$-5^{\circ} 12'$	4.00, 5.90, 7.81	

Compute the elevation of station *A*.

(T.C.D., 1929.)

14. A traverse survey is run along the foot of a piece of precipitous ground, and the distance between two points *A* and *B* on the rough ground is to be obtained by observations from the traverse. *A* being visible from one of the traverse stations, *X*, and *B* from another station, *Y*, tachometrical observations from these stations are taken upon a vertically held staff at *A* and *B*, as follows :

Traverse Station.	Total Co-ordinates of Station.		Staff Point.	Bearing.	Vertical Angle. (Elevn.)	Hair Readings.
	Latitude.	Departure.				
<i>X</i>	1,075.6 N.	2,843.2 E.	<i>A</i>	$336^{\circ} 42'$	$18^{\circ} 23'$	5.00, 7.12, 9.24
<i>Y</i>	839.3 N.	3,609.5 E.	<i>B</i>	$12^{\circ} 27'$	$15^{\circ} 16'$	6.00, 7.95, 9.91

The tachometer is provided with an anallactic lens, and the instrument constant is 100. Calculate the distance *AB*. (T.C.D., 1927.)

15. Calculate the bearing and distance from A to D from the following notes of a traverse run by tacheometer. The telescope was fitted with an internal focussing lens, the constant being 100, and the staff was held vertically.

Line.	Bearing.	Vertical Angle.	Staff Readings.
AB	360° 0'	+5° 11'	3.00, 6.24, 9.48
BC	289° 54'	+2° 30'	3.13, 5.59, 8.05
CD	201° 12'	-2° 43'	2.97, 6.49, 10.02

16. The following notes refer to a tacheometrical traverse run from a station A to a station E, the horizontal distance between which is known to be 2,465.7 ft. Taking this value as correct, determine the constant of the instrument, an anallactic lens being fitted in the telescope, and the staff having been held vertically.

Line.	Bearing.	Vertical Angle.	Stadia Intercept.
AB	360° 0'	0	6.39
BA	180° 0'	0	6.39
BC	29° 36'	+2° 27'	5.12
CB	209° 36'	-2° 25'	5.10
CD	350° 8'	-1° 48'	7.01
DC	170° 8'	+1° 49'	7.02
DE	7° 54'	-3° 12'	7.14
ED	187° 54'	+3° 12'	7.13

17. The stadia intercept read by means of a fixed hair instrument on a vertically held staff is 3.73 ft., the angle of elevation being 6° 27'. The instrument constants are 100 and 1.1. What would be the total number of turns registered on a movable hair instrument at the same station for a 5 ft. intercept on a staff held on the same point, the vertical angle in this case being 6° 21' and the constants 1000 and 1.4 ?

18. A staff held vertical is observed with an ordinary theodolite, and the angles of elevation from the horizontal of two well-defined lines on the face of the staff, which are 10 ft. apart, are observed. What is the horizontal distance of the staff from the axis of rotation of the telescope, when the difference of the tangents of the angles of elevation is 0.033 ? (Inst. C.E., 1905.)

19. Two observations are taken upon a vertical staff by means of a theodolite, of which the reduced level of the trunnion axis is 154.3. In the case of the first the line of sight is directed to give a staff reading of 3.00, and the angle of elevation is 4° 58'. In the second observation the staff reading is 11.00, and the angle of elevation is 5° 44'. Compute the reduced level of the staff station and its horizontal distance from the instrument.

20. A surveyor, about to take a series of cross sections on rough ground by means of a theodolite, wishes to obtain his height of instrument by observing a staff held upon a bench mark which is at a lower level than the instrument. He takes two observations on the staff, the readings being 9.74 and 3.26, and the corresponding angles of depression 9° 20' and 10° 45'. Calculate the elevation of the instrument if that of the bench mark is 193.5. (T.C.D., 1925.)

21. An observation with a percentage theodolite gave staff readings of 3.12 and 7.86 ft. for angles of elevation of 4% and 5% respectively. On sighting the graduation corresponding to the height of the instrument axis above the ground, the vertical angle was 4.26%. Compute the horizontal distance and elevation of the staff station if the instrument station has an elevation of 213.1.

22. The horizontal angle subtended at a theodolite by a subtense bar with vanes 10 ft. apart is 18'. Compute the horizontal distance between the instrument and the bar.

Deduce the error of horizontal distance if the bar were 1° from being normal to the line joining the instrument and bar stations.

CHAPTER XII

HYDROGRAPHICAL SURVEYING

HYDROGRAPHICAL surveying is that branch of surveying which deals with bodies of water. It embraces all surveys made for the determination of water areas, volumes, and levels, rate of flow, and the form and characteristics of underwater surfaces. The usual methods of applying the fundamental principles of surveying and levelling have to be adapted to the conditions, and some of the operations and apparatus are of a specialised character.

Scope.—Extensive hydrographical surveys, directed to the preparation of charts for the use of navigators, are undertaken by various countries. Such surveys are conducted from specially equipped surveying vessels, and comprise the determination of depths available for shipping, the survey of currents, the location of shoals and other dangers, buoys, anchorages, and lights, as well as the mapping of conspicuous land features which will guide the navigator. An examination of the operations forming the routine on board a surveying vessel is outside the scope of this volume, but the civil engineer uses similar methods on a smaller scale in connection with the design and maintenance of certain classes of works. The relationship of hydrographical surveying to those branches of engineering which deal with harbours, docks, navigable waterways, coast protection, etc., is evident ; but the application of the subject to civil engineering is wider than might at first sight appear, since works connected with water supply, water power, irrigation, sewage disposal, flood control, land reclamation, viaducts, and river works generally, also involve the practice of hydrographical surveying.

That part of the subject which relates to the measurement of the discharge of rivers and streams, as required in water supply and similar projects, is dealt with in the latter part of the chapter. The more general branches of the subject may be classed as marine surveying, but they are just as applicable to inland waters as to the sea.

THE TIDES

The marine surveyor has to make such frequent reference to tidal movements that some knowledge of their general characteristics is indispensable. Detailed treatment of the subject is outside the scope of this work, and the reader is referred to the bibliography on

page 592, but a brief outline of the simpler phenomena is offered to explain the significance of some of the terms used in connection with the tides.

Tidal Theory.—Tides are periodical variations in the level of a water surface due to the attraction of celestial bodies. The sun and moon are the principal tide-producing agents.

No theory adequately explains all the phenomena of the tides. The most useful working hypothesis is the equilibrium theory, due to Newton, which forms the basis on which all subsequent work has been founded. This is a static theory which interprets the fundamental causes and character of the more general phenomena, but does not afford quantitative results regarding the range nor time of the movements at a place.

Equilibrium Theory.—Newton's law of universal gravitation states that every body in the universe attracts every other body with a force acting in the straight line between the bodies and of magnitude proportional to the product of their masses and inversely proportional to the square of the distance between them. In applying this law in the equilibrium theory, the chief assumptions made are : (1) that the earth is completely enveloped by an ocean of uniform depth ; (2) that the inertia and viscosity of the water and its attraction for parts of itself are negligible, so that the ocean may be supposed capable of assuming instantaneously the figure of equilibrium required by the tide-producing forces.

Tide-producing Force.—The moon is the principal tide generator, and the nature of tide-producing force is best followed by assuming that the earth and moon are the only two bodies in existence. These attract each other, the resultant force acting along EM (Fig. 333), the straight line joining their centres. Their separate positions are maintained by the earth and moon revolving monthly

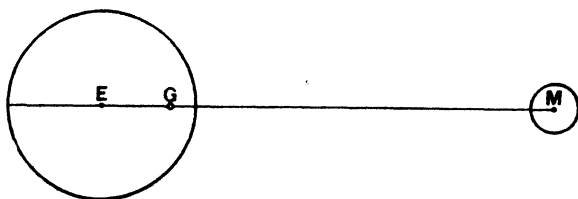


FIG. 333.

about G, their common centre of gravity. Let this be the only motion taking place, the daily rotation of the earth on its axis being supposed annulled since it exercises no influence on tide-producing force.

In consequence of the earth's revolution about G without rotation, all particles on the earth are subjected to centrifugal force of

uniform intensity, and the directions of these forces are parallel to the line joining the centres of the earth and moon at any instant (Fig. 334). While the total centrifugal force thus generated is

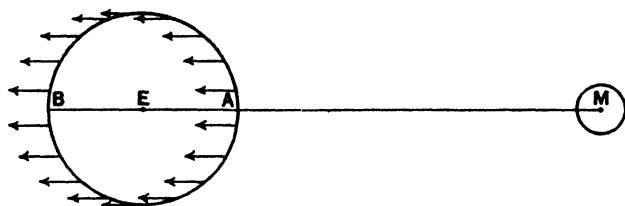


FIG. 334.

balanced by the moon's attraction, the attractive forces are not uniformly distributed over the earth. Since attraction is inversely proportional to the square of the distance, particles at A are subject to a greater force, and those at B to a smaller force, than the mean,

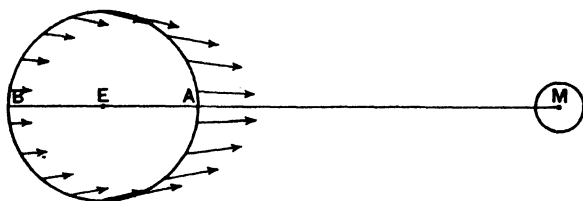


FIG. 335.

the distribution being represented diagrammatically in Fig. 335. On adding the two systems vectorially, the residual forces of Fig. 336 are obtained. These locally unbalanced forces are the tide-producing forces due to the moon.

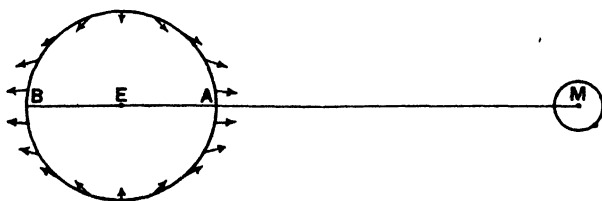


FIG. 336.

Equilibrium Figure.—In conformity with the assumption that the ocean is devoid of inertia and viscosity, it will assume under the action of these forces the equilibrium figure of a prolate ellipsoid of revolution with the major axis directed to the moon (Fig. 337). Two lunar tides are thus simultaneously produced. That at A, under the moon, is termed the superior lunar tide, or tide of the moon's upper transit; that at B, the inferior or anti-lunar tide, or

tide of the moon's lower transit. Low water simultaneously occurs at C and D.

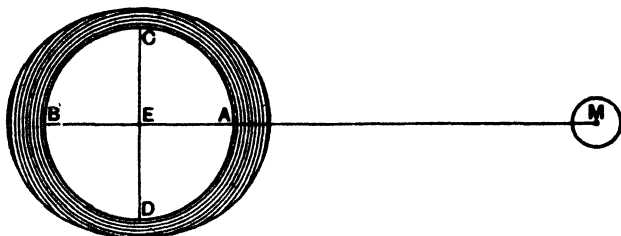


FIG. 337.

Tidal Day.—To explain the recurrence of high and low water at a place, consideration must be given to the effect of the earth's rotation on its axis. Let Fig. 337 represent a section in the plane of the earth's equator, the moon being assumed in that plane. As the earth rotates, a point such as A occupies the successive positions C, B, and D at intervals of 6 h., and returns to A in 24 h. If the moon remained stationary, the major axis of the lunar tidal figure would maintain a constant position, and A would experience during a rotation a regular variation in the level of the sea, and would encounter high water at perfectly regular intervals of 12 h., with low water midway between.

The effect of the moon's motion, assumed in the plane of the equator, may now be added. In a lunation of 29·53 days the moon makes one revolution relative to the sun, i.e. from new moon to new moon. This revolution is in the same direction as the diurnal rotation of the earth, from west to east, so that there occur 28·53 transits of the moon across a meridian during a period of 29·53 mean solar days. The interval between successive transits, or the length of the lunar day, is therefore on an average 24 h. 50·5 m. The axis of the lunar tide figure maintains its direction towards the moon, and therefore passes through a point such as A every 12 h. 25 m., which is the average interval between successive high waters.

Solar Tides.—In the same manner, the sun, acting alone, generates a superior solar tide and an inferior or anti-solar tide. The sun, however, is a less powerful tide-producer than the moon. The tide-producing force exerted by a celestial body may be measured by the excess in attraction at A (Fig. 337) over the mean, the deficit at B from the mean being of practically the same amount.

Let E , M , and S = the masses of earth; moon, and sun respectively,

m and s = the mean distances from the centre of the earth to that of the moon and sun respectively,

r = the radius of the earth,

k = the constant of gravitation.

The tide-producing force of the moon on unit mass at A

$$= kM \left(\frac{1}{(m-r)^2} - \frac{1}{m^2} \right)$$

$$= kM \left(\frac{2r}{m^3} \right) \text{ to a first approximation.}$$

i.e. Differential attraction or tide-producing force is proportional to the mass of the attracting body, and is inversely proportional to the cube of the distance. We therefore have

$$\frac{\text{Sun's tide-producing force}}{\text{Moon's tide-producing force}} = \frac{Sm^3}{Ms^3}.$$

Substituting $S = 331,000 E$,

$$M = \frac{1}{81} E,$$

$$s = 92,800,000 \text{ miles,}$$

$$m = 239,000 \text{ miles,}$$

$$\text{Solar tide} = .458 \text{ lunar tide.}$$

Since the moon and sun are acting simultaneously, the lunar and solar tides are superimposed, and the ocean assumes an equilibrium figure in obedience to the combined tide-producing forces.

Spring and Neap Tides.—At new moon the sun and moon have the same celestial longitude, and cross a meridian of the earth at

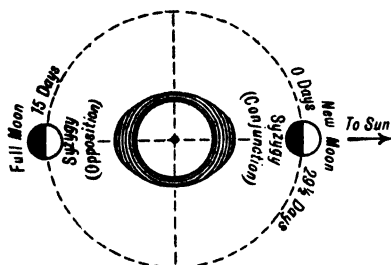


FIG. 338.—SPRING TIDES.

the same instant. The three bodies are in one plane, and the sun and moon are on the same side of the earth, the moon being said to be in conjunction (Fig. 338). The crests of the lunar and solar tide waves coincide, and the high water level of the resulting tide is above the average, and its low water level below the average. The tide is called *Spring Tide of New Moon*.

Thereafter, the moon falls behind the sun, and crosses the meridian about 50 minutes later each day. In about $7\frac{1}{2}$ days the moon is in quadrature, its elongation, or the difference between its longitude and that of the sun, being 90° (Fig. 339). The crest of the lunar tide then coincides with the trough of the solar tide. High water level is below the average, and low water level above the average, the tide being called *Neap Tide of the First Quarter*.

Full moon occurs at about 15 days from the start of the lunation. The moon's elongation is 180° , and the earth, moon, and sun are again in the same plane, but the moon is in opposition (Fig. 343). The moon and sun again act together, and the variation in sea-

level is great, the tide being *Spring Tide of Full Moon*.

In about 22 days the moon reaches quadrature with an elongation of 270° , and the conditions for small tides are repeated (Fig. 339). The tide is *Neap Tide of the Third Quarter*.

Finally, in about $29\frac{1}{2}$ days from the previous new moon, the moon returns to the meridian of the sun, and the cycle is recommenced with the *Spring Tide of New Moon*.

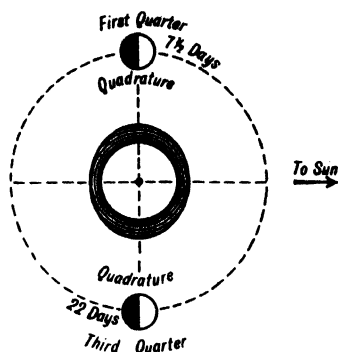


FIG. 339.—NEAP TIDES.

Priming and Lagging.—Besides giving rise to variation in the tidal range, varying relative positions of the sun and moon affect the regularity with which high water recurs at a place, so that the tidal day is not of constant length. The tidal figure at any time consists of the superimposed lunar and solar tides, and, since the moon is the more powerful agent, the crest of the composite tide lies nearer the meridian of the moon than that of the sun. It is therefore convenient to refer the time of high water to that of the moon's upper or lower transit. When high water occurs at a place before the moon's transit, the interval between successive high waters is less than the average, and the tide is said to prime: otherwise, it is said to lag.

At new moon the crest of the composite tide is under the moon, and there is a normal tide without priming or lagging. Between new moon and first quarter the axis of the resultant tidal figure lies

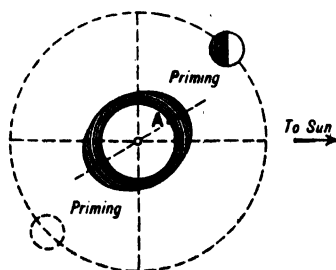


FIG. 340.

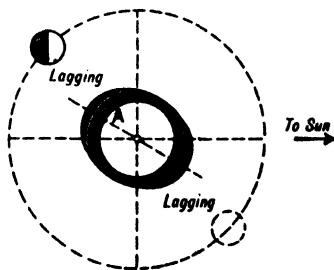


FIG. 341.

as in Fig. 340. The place A experiences high water before coming under the moon, i.e. before the moon crosses the meridian of A, and the tide primes. At quadrature the crest of the composite tide is again under the moon, and the tide is normal. Between first quarter and full moon the conditions are as in Fig. 341. A does

not encounter high water until some time after it has passed under the moon, and the tide lags. Normal tide prevails at full moon, and again at the third quarter. The tide primes in the third quadrant of the lunation, and lags during the fourth:

The effects of varying relative positions of the sun and moon may be summarised as follows :

Moon in	Moon's Elongation.	Moon's Phase.	Moon's Approx. Age. Days.	Tide.	Range.	Tide.
Syzygy	0°	New	0	Spring	Great	Normal
Quadrature	0°-90°	1st Quarter	7½	Neap	Decreasing	Priming
	90°				Small	Normal
Syzygy	90°-180°	Full	15	Spring	Increasing	Lagging
	180°				Great	Normal
Quadrature	180°-270°				Decreasing	Priming
	270°	3rd Quarter	22	Neap	Small	Normal
	270°-360°				Increasing	Lagging
Syzygy	360°	New	29½	Spring	Great	Normal

Further Sources of Inequality.—Several other factors affect the range as well as the interval which elapses between successive high waters at a place. The more important of these are the ellipticity of the orbits of the moon and the earth and the varying declination of the moon and sun.

Effect of Ellipticity of Orbits of Moon and Earth.—The lunar orbit is not circular, as has been tacitly assumed, but is an ellipse with an eccentricity of .055. The moon is said to be in perigee when nearest the earth and in apogee when most remote, the ratio between the least and greatest distances being $\frac{1-.055}{1+.055}$. Since tide-producing force is inversely proportional to the cube of the distance of the attracting body,

$$\frac{\text{range of perigean tide}}{\text{mean range}} = \left(\frac{1}{1-.055} \right)^3 = 1.17,$$

and

$$\frac{\text{range of apogean tide}}{\text{mean range}} = \left(\frac{1}{1+.055} \right)^3 = 0.84.$$

This disturbance has a period, between successive perigees, of the anomalistic month of 27.55 days, so that it does not bear a constant relation to the succession of spring and neap tides. The tides, whether spring or neap, are increased at perigee and decreased at apogee by the superimposition of this "lunar elliptic" tide.

The distance between the sun and the earth likewise varies on account of the earth's orbit being an ellipse. The sun is in perigee

on Dec. 31st and in apogee on July 1st, the period of the tidal disturbance being the anomalistic year of 365.26 days. The resulting inequality is small, since the eccentricity of the orbit is only .0166, so that

$$\frac{\text{perigean range}}{\text{mean range}} = \left(\frac{1}{1 - .0166} \right)^3 = 1.05,$$

and

$$\frac{\text{apogean range}}{\text{mean range}} = \left(\frac{1}{1 + .0166} \right)^3 = .95.$$

Effect of Declination.—The sun and moon have been assumed to remain in the plane of the earth's equator, but both bodies periodically vary their position on either side of the equator, or have

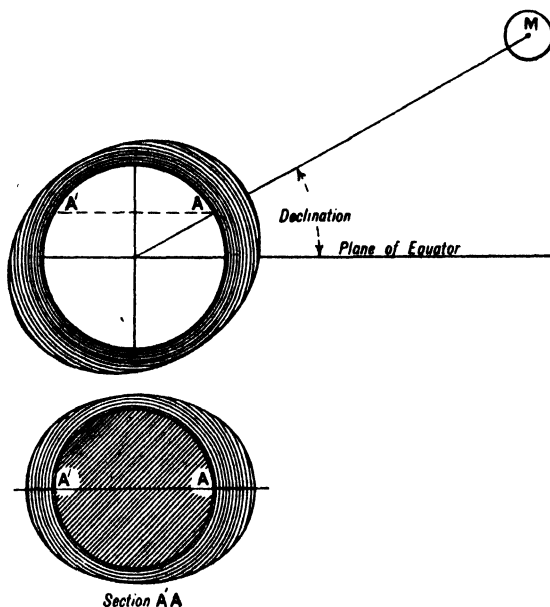


FIG. 342.

declination. The sun's declination (Vol. II, Chap. I) attains a maximum value of $23^{\circ} 27'$ N. or S. at the solstices, and is zero at the equinoxes, the period of the motion being the tropical year of 365.24 days. The plane of the moon's orbit is inclined to that of the ecliptic at a mean angle of about $5^{\circ} 8'$, and the moon undergoes a periodic variation in declination on either side of the equator during the tropical month of 27.32 days, or time of the moon's revolution relatively to the First Point of *Aries* (Vol. II, Chap. I).

The declination is N. for half of the month, and S. for the other half. The maximum N. or S. declination attained ranges from $23^{\circ} 27' + 5^{\circ} 8' = 28^{\circ} 35'$ to $23^{\circ} 27' - 5^{\circ} 8' = 18^{\circ} 19'$ in cycles of about 19 years.

The effects produced by solar and lunar declination are similar, the latter being the more marked. Fig. 342 illustrates a lunar declinational tide, the axis being towards the moon and therefore inclined to the equator. At places on the same side of the equator as the attracting body the superior tide is greater than the inferior, the reverse being the case for places on the other side of the equator. The conditions during a tidal day at the place A are exhibited on the section through A by a plane parallel to the equator. The inferior crest is encountered at A', and the level of high water is then sensibly less than that of the previous high water. This phenomenon, termed *Diurnal Inequality*, vanishes when the attracting body is on the equator.

Discrepancies between Theory and Observation.—In view of the assumptions involved in the equilibrium theory, close agreement between the results of theory and those of observation is not to be expected. The ocean is assumed to be able to arrange itself in obedience to the tide-producing forces, but it is not explained how the adjustment is effected. In reality, the time available for the formation of an equilibrium figure is insufficient on account of the inertia of the water, and, in consequence, the lunar and solar tides at a place do not occur at the instants of transit of the moon and sun. The average length of the tidal day does, however, agree with that of the lunar day, and spring and neap tides occur at about the times of syzygy and quadrature respectively. Inequalities due to declination and varying distance of the attracting bodies can also be traced.

The theory does not accord with observation as to the amount of tidal movement. On the assumption that the ocean completely envelops the earth, it can be shown that the greatest range of tide would be about 3 ft.; but very much greater ranges than this actually occur. The average range experienced at a place depends upon the form of the coast lines and the depth of the adjoining seas. On account of the irregularity of the obstructing land masses, the tide is heaped up in places, while the range is further increased by the attraction of the ocean for parts of itself and by the momentum of the water carrying it beyond the equilibrium position. In consequence, the prediction of the tides at a place must be based largely on observation.

Similar difficulties due to the influence of local conditions are encountered in the theories classed as dynamic or kinetic, which investigate the characteristics of the tidal oscillation. These need not be detailed, and we may pass to a consideration of the main characteristics of actual tidal phenomena.

The Primary Tide Wave.—The Southern Ocean, extending southwards from about 40° S. latitude, is the only great body of water which encircles the earth. It is free from large obstructions to the development of the equilibrium figure, except where it is restricted in width by the southern part of South America. In consequence, it is the only ocean in which the tides approximately obey the tide-producing forces of the sun and moon. From the primary tide wave generated there, derivative waves are propagated into the Pacific, Atlantic, and Indian Oceans.

Derivative Waves.—These waves proceed in a general north and south direction in the open oceans, and consequently are only indirectly traceable to the tide-producing forces of the sun and moon. The direction of propagation is greatly influenced by the form of coast lines and the intervention of land masses. The velocity of wave travel is great over areas of deep water, and may exceed 600 miles per hour, while the amplitude, or vertical range from crest to trough, is then not more than 2 to 3 ft. The velocity is greatly reduced in shallow water and by contact with coasts. When the wave has to pass through shallow water, or is contracted in width, the water is heaped up on account of its momentum, and the range between high and low water levels is much increased. The true wave motion may then assume the character of a wave of translation.

Age of the Tide.—On account of the direction of propagation of the tide wave, high or low water occurs at different times at various places on the same meridian. If the assumptions of the simple equilibrium theory were fulfilled, the spring tide of new or full moon would occur at each place at the instant of the noon or midnight transit of the moon. Actually, the greatest spring tide arrives several tides after the transit at new or full moon. The interval is called the age of the tide, and represents the time which elapses between the generation of the spring tide and its arrival at the place. It is obtained as the mean of several observations, and is usually reckoned to the nearest $\frac{1}{4}$ day. Its value varies for different places up to a maximum of about 3 days.

The progress of the tide wave and the varying age of the tide may be shown by means of a chart of co-tidal lines. These are lines drawn through all places having high water at the same time. Their form and distance apart exhibit the path of the wave and its varying speed.

In the case of the British Isles, the age of the tide on arrival at Land's End and the west of Ireland is nearly $1\frac{1}{4}$ days. The wave divides, one branch running up the English Channel, and the other along Ireland and the west coast of Scotland, sweeping round the north of Scotland into the North Sea. The former wave passes through the Strait of Dover with an age of $1\frac{1}{2}$ days. The latter attains that age off the north-east coast of Scotland in its passage

down the North Sea, and finally, when the age is 2 days, it encounters off Harwich the succeeding tide from the English Channel.

Lunitidal Interval.—The interval of time which elapses between the moon's transit and the occurrence of the next high water at a place is called a lunitidal interval. If the times of several consecutive tides are observed, and are referred to the times of the moon's upper and lower transits, the value of the lunitidal interval will be found to vary because of the existence of priming and lagging. The required local times of transit are derived from the times of transit at Greenwich given in the *Nautical Almanac* by applying 2 m. for every hour of longitude, adding if the place is west, and subtracting if east, of Greenwich.

On plotting the lunitidal intervals for a fortnight on a base of times of the moon's transits, a curve similar to Fig. 343 is obtained. This curve of semi-mensual inequality of time has approximately the same form for each fortnight, and may be used for the purpose of rough predictions of the occurrence of high water at the place.

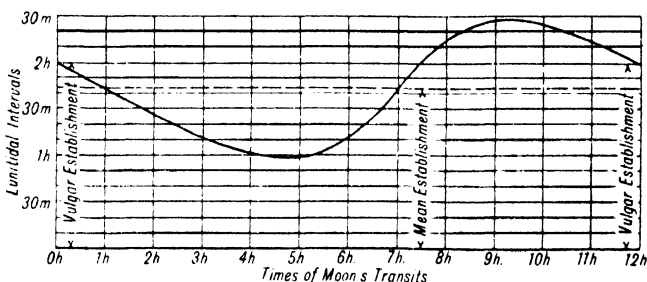


FIG. 343.

To obtain the local time of high water on any day for which the local time of the moon's transit is known, the appropriate lunitidal interval is obtained from the curve and added to the time of the preceding transit.

Mean Establishment.—The mean establishment of a place is the average value of all its lunitidal intervals.

When the mean establishment is known, a rough prediction of the time of high water may be made by deducing the lunitidal interval for that day from the mean establishment. This involves the application to the establishment of a correction representing the amount of priming or lagging at the generation of the tide, the evaluation of which requires an approximate knowledge of the age of the tide at the place.

The priming and lagging correction to mean establishment is as follows :

Hour of Moon's Transit	0	1	2	3	4	5	6	7	8	9	10	11	12
Correction in Minutes	0	-16	-31	-41	-44	-31	0	+31	+44	+41	+31	+16	0

To illustrate the manner of applying the correction, let it be required to find the time of afternoon high water at a place at which the mean establishment is 4 h. 12 m. and the age of the tide $1\frac{1}{2}$ days, if the moon transits at 5 h. 46 m. p.m. on the day in question.

Since the moon falls behind the sun by about 50 m. in 24 h., at the birth of the tide, $1\frac{1}{2}$ days before, the moon crossed the meridian at which the wave was generated $1\frac{1}{2} \times 50$ m. = 1 h. 15 m. earlier, *i.e.* at 4 h. 31 m.

By interpolation from the table, the priming correction corresponding to 4 h. 31 m. is 37 m., so that

Required lunitidal interval = 4 h. 12 m. - 37 m.	=	3	35	h. m.
Time of moon's transit	=	5	46	p.m.
Time of high water	=	9	21	p.m.

The results given by this method of prediction are only approximate, since it takes account only of the moon's phase and makes no allowance for variations in declination or distance of the moon and sun. At places with marked diurnal inequality the estimates may be seriously wrong, but otherwise phase prediction is sufficiently accurate for many purposes.

Vulgar Establishment.—The value of the lunitidal interval on the day of full or change of the moon is called the vulgar establishment, or simply the establishment, of the place. It is approximately the clock time at which high water occurs on those days. It exceeds the mean establishment by an amount depending upon the age of the tide at the place, since the parent tide, generated a day or two previously, was in the second or fourth quadrant and therefore lagging. The value of the establishment for nearly all ports and anchorages is published in the Admiralty Tide Tables under the name "High Water, Full and Change."

Mean establishment can be derived from vulgar establishment if the age of the tide is known, and the lunitidal interval for any day can then be obtained as before. Thus, if the vulgar establishment of a place is 4 h. 32 m., and the age of the tide there is $1\frac{1}{2}$ days, then at the generation of the tide the moon crossed the meridian at 12 h. - $1\frac{1}{2} \times 50$ m. = 10 h. 45 m. The lagging corresponding to this hour of transit is, from the table, 20 m. The mean establishment is therefore 4 h. 32 m. - 20 m. = 4 h. 12 m.

Height of Tides.—The rise of a tide is the vertical distance of the high water level above a fixed reference surface. The datum

surface in general use is that of low water of ordinary spring tides for the locality.

The range of a tide is the vertical distance from low water level to the succeeding high water level.

Observation of the high and low water levels of successive tides at a place shows the gradual diminution in rise and range from spring to neap tides (Fig. 344). Such a diagram could be used in conjunction with phase prediction of times to make rough predictions of heights.

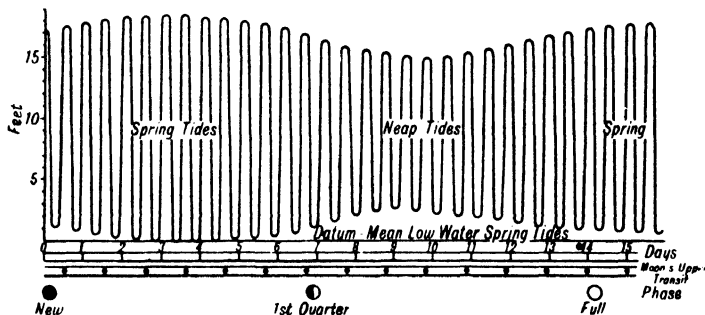


FIG. 344.

Rate of Variation of Level.—If the level of the water is observed every 15 or 30 m., it will be found that the rate of variation of height is small at high or low water and greatest at half-tide. On plotting the heights on a time base, the resulting tide curve is found on an average to approximate to the form of a harmonic curve.

For many purposes it is useful to be able to ascertain, in default of regular gauge readings, the approximate height of a tide of known rise or range at any time between high and low water.

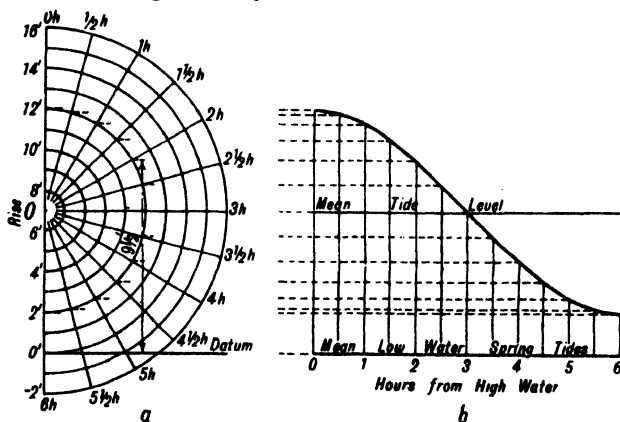


FIG. 345.

This is conveniently obtained from a diagram (Fig. 345a) constructed as follows.

Draw a line to represent mean low water level of spring tides or other datum adopted. At the appropriate height above datum scale off a point *O* on mean tide level. With centre *O*, construct a series of semicircles of radii representing in whole feet the semi-ranges of all tides likely to occur at the place. Divide the circumference of one of the semicircles into as many equal parts as there are half-hours between high and low water. If the place experiences normal tides, the average interval between high and low water is 6 h. 12 m., and for the present purpose the interval may be assumed to have a constant value of 6 h. Draw radii through the points so obtained, and designate them as shown. The elevation at any time of a tide of known range is approximately given by the ordinate from the datum to the point of intersection of the radius representing the time with the semicircle corresponding to the range.

In the example illustrated, the spring tide rise is taken as 14 ft. To ascertain the approximate height of 2 h. from high water of a tide having a rise of 12 ft., the height of the intersection of the radius marked 2 h. with the semicircle through 12 ft. is read, the result being 9½ ft. above datum. The construction of the harmonic curve for this tide is shown at *b* (Fig. 345).

The same results are obtained analytically from

$$H = h + \frac{1}{2} r \cos \theta,$$

where *H* = required height of tide above datum,

h = height of mean tide level above datum,

r = range of the tide,

$$\theta = \frac{\text{interval from high water}}{\text{interval between high and low water}} \times 180^\circ.$$

Deviations from Normal.—At many places the above methods of estimating the times and heights of the tides are of little service on account of the influence of local conditions. The amount of diurnal inequality is very variable over the earth. It is small in European tides, but large in the Indian Ocean and parts of the Pacific and on the coasts of Australia and China. At some places it is so pronounced that there is only one tide in 24 h. for several consecutive days. On the other hand, the tides may approach a place from two different directions. When the two tides arrive at different times, the place experiences twice the normal number of tides, but if the difference in time is small, the effect is to make the tide practically stand at high water, it may be for several hours. These phenomena occur at several places on either side of the English Channel.

Tidal Streams.—On account of the inequalities in sea level caused by the tide wave, the action of gravity produces along the sloping surfaces an actual flow of water, called a tidal stream or

current. In the open ocean the height of the tide wave is so small that the stream is inappreciable, but in shallow seas the gradients are much increased, and the resulting currents become an important factor in navigation.

The speed and direction of a stream vary during the passage of the tide wave causing it. The greatest velocity usually occurs at about half-tide, and there is a period of rest, or "slack water," at the turn of the tide. This does not necessarily take place at the times of high and low water, since in general the flood stream continues to run after the sea level has started to fall, and the ebb stream for some time after low water. The direction may remain fairly constant during flood and become reversed at ebb, but in many cases the direction undergoes a regular series of changes during flood and ebb. The actual regime depends upon local circumstances, and may call for careful study by the engineer in connection with the design and construction of works (page 572).

Meteorological Effects.*—Wind.—Considerable variation may occur in both the height and time of tides through the action of wind in heaping up the water. A strong wind blowing in the direction of the flood stream accelerates its progress, and increases the height of the tide. The same wind retards the ebb. On coasts swept by the wave high water occurs sooner, is higher, and keeps up longer than usual, while low water level is also above the average. The reverse effects occur when the wind blows against the flood stream. The effect of wind is least marked in the open ocean, and is most evident when the storm occurs in the region of a contracted area through which the tide pours.

Barometric Pressure.—The atmosphere resting on the sea exerts upon it a varying pressure as indicated by differences in the height of the barometer at different places. The sea level is depressed in regions of high pressure and is raised where the pressure is low, a difference of 1 in. in the reading of the barometer corresponding to a difference of sea level of over 13 in. Observation shows that the height of tides is affected thereby, but investigation is difficult since the effect is masked by the greater influence of the wind caused by a steep barometric gradient.

Harmonic Analysis.—The constants "Mean Establishment" or "Vulgar Establishment" for any port are called "Non-Harmonic Constants," and tidal predictions computed from them may be erroneous, and, at certain places and under certain conditions, the error may be so great as to render the predictions valueless. The modern tendency therefore is to avoid using these constants and to

* For an account of investigations on meteorological effects on the tides see "The Fluctuations of Mean Sea Level with Special Reference to those caused by Variations in Barometric Pressure," by Sir Charles Close in the *Geographical Journal*, Vol. 52, 1918, pages 51–58. Also, "The Second Geodetic Levelling of England and Wales, 1912–21."

replace them by others, known as "Harmonic Constants," whenever these are available. At present, non-harmonic constants for different ports are published in Section I of Part II of the Admiralty Tide Tables, and harmonic constants, for all ports for which they have been computed, in Section II of the same publication.* The Introduction to this volume contains a note advising mariners to use the harmonic method of prediction wherever possible, because it is probable that, in course of time, it will replace all others.

In the harmonic method of prediction, the tide is regarded as made up of a series of simple harmonic constituents of the type :

$$H \cdot \cos (nt - k)$$

where H is the amplitude or half-range of the constituent,

n its speed or rate of change in degrees per hour,

t is the time in hours from some fixed time,

k is an angle which is constant for any particular constituent and particular port.

The speed n depends only on the particular constituent and is the same for all ports, but the constants H and k for that constituent vary from port to port and are determined, by a process called "Harmonic Analysis," from a series of observations of the heights of the tide at intervals of time one hour apart. Observations covering a period of 15 or of 29 days are sufficient to enable a rough determination of the more important constituents to be made, but, for a really reliable determination, they should cover a period of 355 days (12 lunar months). When such a series has been taken, the analysis to determine the constants is a lengthy and laborious process and can best be done by an authority such as The Liverpool Tidal Institute, The Observatory, Birkenhead, which is prepared to undertake work of this kind for a nominal fee. Methods of making approximate analyses from observations covering periods of 15 or 29 solar days are described in the Admiralty publication *Instructions for Analysing Tidal Observations*, but, for an account of the methods used by the Liverpool Tidal Institute for a more complete analysis of observations covering a period of 355 days, reference may be made to a paper by Dr. A. T. Doodson on "The Analysis of Tidal Observations" in *Philosophical Transactions of the Royal Society*, No. A.227, pages 223-279, in which full instructions for the computations are given.

There are a large number of the tidal constituents, and as many as 36 may be used for the computation of the tidal predictions given in Part I of the Admiralty Tide Tables, although 10 only are included in the tables of harmonic constants given in Part II,

* The Admiralty Tide Tables are published in two parts. Part I is published annually and gives times and heights, for each day of the year, of high and low waters at "Standard" ports. Part II is published at intervals of about five years and contains non-harmonic constants, tidal differences and harmonic tidal constants. Supplements to this part are published as required.

this number being sufficient for all predictions required for navigational purposes. The constituents are distinguished by special letters :

$$M_2, S_2, N_2, K_2, K_1, O_1, P_1, M_4, MS_4, \text{etc.},$$

where the suffixes 1, 2, 4 indicate whether the constituent is diurnal, semi-diurnal or quarter-diurnal. Mean Sea Level, referred to the port datum, is represented by the letter and suffix A_0 .

The following table gives the description or name, the period and the speed, in degrees per hour, of the harmonic constants given in Section II of Part II of the Admiralty Tide Tables :

Symbol for Constituent.	Description or Name.	Period.	Speed in degrees per hour.
M_2	Lunar semi-diurnal	$\frac{1}{2}$ lunar day	28.984
S_2	Solar semi-diurnal	$\frac{1}{2}$ solar day	30.000
N_2	Larger elliptic semi-diurnal	—	28.440
K_2	Luni-solar semi-diurnal	$\frac{1}{2}$ sidereal day	30.082
K_1	Luni-solar diurnal	Sidereal day	15.040
O_1	Lunar diurnal (declinational)	—	13.943
P_1	Solar ditto	—	14.959
M_4	First overtide of semi-diurnal	$\frac{1}{4}$ lunar day	57.968
MS_4	Compound luni-solar $\frac{1}{4}$ diurnal	—	62.032

Prediction of Tides from the Harmonic Constants.—The times and heights of high and low waters cannot be determined directly from the harmonic constants and it is necessary to compute the heights at hourly intervals. The times and heights of high and low waters can then be obtained by inspection of the hourly heights, or, better still, by scaling them from a curve in which heights are plotted against time.

For purposes of prediction, the formula adopted is :

Value of constituent at zero hour on the day in question

$$= f \cdot H \cdot \cos (E - g),$$

a formula which replaces the simpler one already given. Here,

H denotes the mean amplitude (half-range) of the constituent at the port in question,

f is a factor, differing slightly from unity, which varies slowly from year to year,

E is an angle which is the same for all ports.

g is a constant angle special to the port and to the constituent.

The harmonic constants for any particular port are therefore, the mean sea level A_0 and the values of H and g for the several constituents.

The first step is to determine E for zero hour on the given date and, for this purpose, E is regarded as being made up of two parts so that :

$$E \text{ (at zero hour) } = m + d$$

where :

m = Value of E at zero hour on the first day of each month.

d = Increment in E from zero hour of the first day of the month to zero hour of the day in question.

The tables for assisting prediction are of two kinds, annual and permanent. The annual tables (Tables I and III) give values for those quantities— f and m —which vary from year to year, while the permanent tables give values of d (Table II), and of $\cos (E-g)$ (Table IV), for a sufficient number of values of $(E-g)$. Having found f , H and $\cos (E-g)$, a fifth table gives their product to one decimal place.

When $(E-g)$ has been determined for zero hour, its cosine is found in Table IV, and, for the following 23 hours of the day, the values of $\cos (E-g)$ are the 23 values which appear in the table under the entry for zero hour. Hence, the hourly heights can easily be obtained for each constituent, and the height of the tide above the port datum at any hour is then A_0 plus the sum of the heights of the different constituents for that hour.

A specimen computation is given in Section II of Part II of the Tide Tables, and forms for harmonic tidal predictions (H.D. 288) may be obtained from the Admiralty Chart Agent, Mr. J. D. Potter, 145 Minories, London, E.C.3, or from any of the other agents.

Tide-Predicting Machine.—When tidal predictions have to be made on a large scale, it is usual to use a “Tide-Predicting Machine.” This consists of an apparatus in which a number of separate harmonic motions are traced out by a suitable mechanism and combined in such a way that the total effect of all the individual motions is transmitted to a moving pencil, which traces out, on a revolving drum, a curve similar to the curve that would be obtained on an automatic self-registering tide gauge. Each separate harmonic motion can be adjusted to suit the amplitude and phase of the constituent which it represents. Hence, the operation consists in setting up on the scales of the machine the readings corresponding to the H and g of each constituent. Then, when the machine is set in motion, the tide curve is traced out, and heights and times of high and low waters are subsequently scaled from the resulting curve.

A tide-predicting machine is an exceedingly complicated and very expensive piece of mechanism. Consequently, machines of this kind are not in common use, and, when a long series of predictions is required, it is usual to get it made by some central authority, such as the Liverpool Tidal Institute, which specialises in this class of work and has a tide-predicting machine at its disposal.

TIDE GAUGES

The vertical movements of tides are measured by means of a tide gauge. When a prolonged record of the tides of a locality is required,

the gauges at the observation station are of a permanent character. In the execution of a marine survey it is necessary to ascertain the variation of sea level during the operations, and a tide gauge must be established temporarily if there is no existing one in the vicinity.

Selection of Site.—The gauge may be required to record the tides at a particular point, in which case there may be little choice as to its position. On the other hand, if it is desired to register the average conditions prevailing over a wide area, the site must be selected with a view to obtaining a representative record for the locality. Abnormal conditions prevail in shallow water, in narrow creeks and straits, and near the mouths of rivers. Deep water and shelter from storms are the chief desiderata, so that frequently the most suitable situation is on the side of a bay with a wide entrance. In the case of a survey covering a limited area, the choice is more restricted, and the gauge should be placed as near the work as possible, consistent with obtaining a trustworthy record.

Forms of Tide Gauge.—Gauges may be classed as non-self-registering and self-registering. The former require the attendance of an observer to note the elevations of the water surface and the times of reading. The latter are automatic, and produce a continuous record, or marigram.

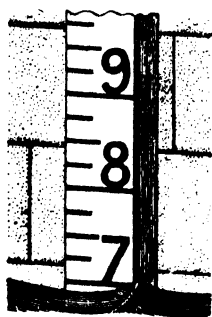


FIG. 346.
STAFF GAUGE.

Non-self-registering Gauges.—*The Staff Gauge* (Fig. 346) is simply a graduated board, 6 in. to 9 in. broad, firmly fixed in a vertical position, and is the most commonly used form. Its length should be more than sufficient to embrace the highest and lowest tides known in the locality. The graduations and figures should be very bold, as it may be necessary to read them from a distance. The division is sometimes carried to tenths of a foot, but, for ordinary engineering purposes, there is no advantage in closer graduation than quarter feet.

Piers, etc., form suitable supports for the staff, since the zero must always be under water. If such a support is not available, the gauge must be erected below low water mark, and securely strutted or guyed. In such a case, if the tidal range necessitates a long staff, it may be preferable to arrange the gauge as a series of posts from above high water mark outwards, these being set by levelling so as virtually to constitute one staff. In setting any gauge, the zero may be fixed at a predetermined level, or, more usually, having erected the gauge, the elevation of its zero is observed by levelling and noted.

The *Float or Box Gauge* (Fig. 347) is designed to overcome the

objection that accurate reading of a staff gauge is at times difficult on account of the wash of the sea. It is enclosed in a long wooden box about 12 in. square, in the bottom of which are bored a few holes. The surface of the water thus admitted is comparatively smooth, particularly when the holes are small and well below the surface. The float carries a vertical rod, which may itself be graduated or which may carry a pointer over a fixed scale. In the former case the graduations increase downwards, and the reading is obtained against a fixed index. With the latter arrangement the rod may be made quite short, and the attached pointer is brought through a narrow continuous slit in the face of the box so that readings are obtained on a staff gauge attached to the outside of the box.

The *Weight Gauge* is suitable for situations where the above gauges would be liable to disturbance or would not be conveniently accessible for reading. In the simplest form, a weight is attached to a graduated wire, chain, or tape, and observations are made by lowering the weight to touch the surface of the water and reading against a fixed mark. Alternatively, the gauge may take the form shown in Fig. 348, in which the chain carrying the weight passes over a pulley and along the surface of a graduated board, to which it is hooked when not in use. The chain is furnished with an index, and readings are obtained by unhooking the chain, lowering the weight to touch the water surface, and noting the graduation opposite the index. The length of chain must be such that readings can always be obtained at the lowest state of the tide, but the graduated board need not be unduly long as a second and third index can be attached to the chain at intervals of, say, 10 ft. In referring to these, the reading is increased by 10 or 20 ft., as the case may be.

The reduced level of the water surface corresponding to zero reading must be determined. This can be obtained by reading a levelling staff held with its foot opposite the bottom of the suspended weight and noting the corresponding gauge reading. Otherwise it may be deduced from the level of the pulley, the length along the chain from the bottom of the weight to the index, and the horizontal distance between the vertical part of the chain and the zero graduation. The value of the zero may change through stretching of

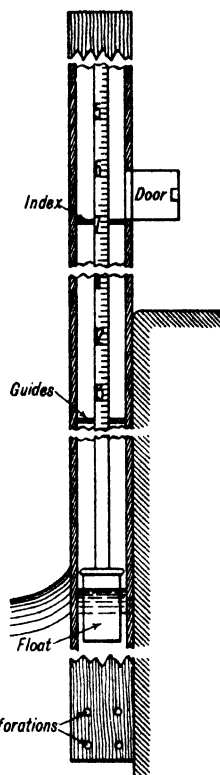


FIG. 347.
FLOAT GAUGE.

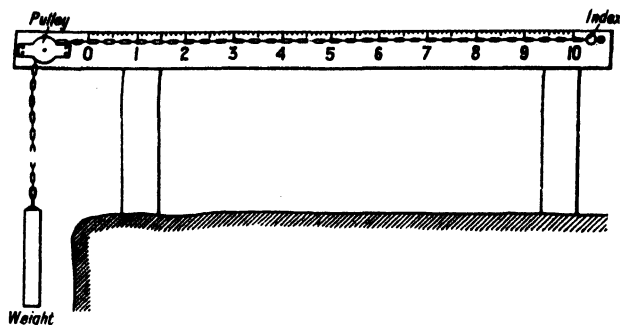


FIG. 348.—WEIGHT GAUGE.

the chain, which should be tested occasionally by steel tape, and adjusted if necessary.

Self-registering Gauges.—Many types of automatic gauges have been constructed, but, while they vary in detail, the essential parts are similar in all forms. A float, protected from the action of wind waves, has attached to it a wire or cord which is coiled round a wheel and is maintained at a constant tension by means of a counterweight or spring. The vertical movement of the float, transferred through the float wheel, is reduced in scale by means of gearing, and is finally communicated to a pencil or pen which traces a curve on a moving sheet of paper. In the commonest form the paper is mounted upon a drum which is rotated once in 24 hours by clockwork. A week's record can be received on the sheet without confusion between the different curves. In more elaborate gauges the recording mechanism is arranged to accommodate a band of paper sufficiently long to contain a month's record. The paper is paid out from one cylinder, passes over a second, against which the pencil or pen is pressed, and is wound upon a third drum.

In the housing of automatic gauges it is frequently necessary to lay piping from below low water level to a well, constructed under the building and in which the float operates. The effect of wave action outside is communicated but slightly to the water in the well because of friction in the pipe and the provision of a grid at its seaward end.

Sometimes, when a tide gauge has to be erected in an exposed position, and the well in which the float rides is deep, there is difficulty in obtaining the exact height of the water for a given reading on the gauge. Wave motion outside the well may prevent accurate readings of the water level at any instant from being taken, while, inside the well, it may be difficult to judge the exact depth to which the float is immersed or when a rod let down from the top touches the surface of the water. In this event, the "soundings"

—that is, observations of the height of the water level in the well below a fixed point—can be taken by means of an electrical device, which was designed by Capt. C. H. Ley, R.E., for determining the zero correction of the Ordnance Survey tide gauge at Newlyn, and which is described in *The Second Geodetic Levelling of England and Wales*, 1912–21. This device consists of an electrical buzzer connected to a pair of ear-phones and a dry battery. One terminal of the apparatus is earthed and the other is connected, by means of a long flexible lead, to a weighted pointer or index. When the pointer is lowered and makes contact with the water in the well a loud noise is heard in the buzzer. This apparatus, which is very simple, is extremely sensitive and enables the exact level of the water in the well to be determined with great accuracy. The metal pointer, used for making contact with the water, can be a piece of silver wire which may be sealed into a glass tube, about half an inch internal diameter and twelve to fifteen inches long, so that the bottom of the wire projects very slightly below the bottom of the Plaster of Paris used to seal the wire into the tube. The space above the Plaster of Paris, between the flexible wire lead and the walls of the tube, can be filled with lead shot to weight the tube and make it hang vertical. The reason for having the glass tube is to prevent the pointer coming into electrical contact with the walls of the well or the side of the float. A fairly thick rubber ring passed round the tube, about half-way along its length, will also assist in preventing electrical contact between pointer and wall through the outside of the tube getting damp.

For an interesting paper on "Tide Gauges," by Commander H. D. Warburg, R.N., see *Report of Proceedings. Conference of Empire Survey Officers*. 1935.

MEAN SEA LEVEL AS A DATUM PLANE FOR LEVELS

For small local surveys it is often quite sufficient to assume an arbitrary horizontal plane as the reference or datum plane from which all heights are measured, and to define this plane as being a certain distance below some fixed point or bench mark. For more extended surveys, or where other surveys have to be joined together, such a reference plane is not convenient, and it is more usual to assume Mean Sea Level at a certain place as the plane from which heights are to be measured. In this case, mean sea level is defined as the mean level of the sea, obtained by taking the mean of all the heights of the tide, as measured at hourly intervals over some stated period covering a whole number of complete tides.

One advantage of adopting mean sea level as a datum for levels is that it is a natural "level" surface which can always be re-established if bench marks on the land should be disturbed or moved in any way. Also, some connection between the levels on the land and the level of the sea is often required for different engineering

purposes, such as drainage, reclamation, hydro-electric, harbour works, etc. For scientific purposes, mean sea level, when used in conjunction with geodetic levelling, may be used as a fixed datum to study such matters as small and gradual subsidences or elevations of the land relative to the sea. For all such work, mean sea level is the best datum to use as it is not so variable as the high or low water marks or the mean level of high and low tides.

Variations in Mean Sea Level.—Mean sea level, as determined in the manner defined above, shows appreciable variations from day to day, from month to month and even from year to year. Consequently, the period of time to be covered by the observations from which it is to be derived as a datum for levelling will depend on the purpose for which the levels are required. For many purposes quite a short period will suffice, but, if a datum is needed for levels approaching geodetic standards of accuracy, a period covering twelve lunar months can only be accepted as a minimum, and it is more usual, and much more satisfactory, for the observations to cover a period of several years.

The differences between extreme values of mean sea level may amount, in ordinary cases, to a couple of feet or more in the daily values, to about a foot and a half in the monthly values and to almost a foot in the yearly values. The monthly variations are usually more or less periodic, so that, at any given place, mean sea level tends to be low on certain months of the year and high on others. Daily values often change irregularly owing to irregular variations in the force and direction of the wind and in the atmospheric pressure.

As an example of the variations of monthly and yearly values of mean sea level we may take the figures for Takoradi, on the West Coast of Africa, as this port is on an open and fairly regular coast line on the Atlantic, with no large or irregular land masses in front or at the side of it and no deep inlets of the sea near it. Moreover, there are no great variations of the atmospheric pressure at localities in its neighbourhood, nor are there ever any abnormal or sustained gales or winds. Consequently, it is well placed from the point of view of obtaining information regarding the oceanic tides, uninfluenced by local and unusual physical, topographical, or meteorological conditions.

The following are the means of the monthly values for the eight years 1931 to 1938 inclusive :

Month.	Mean Sea Level.	Month.	Mean Sea Level.
January	2.963	July	2.580
February	2.981	August	2.551
March	3.062	September	2.747
April	3.048	October	3.080
May	2.862	November	3.209
June	2.687	December	2.996

Mean = 2.897

Here, the heights given are heights above the datum used for setting the tide gauge.

It will be seen from these figures that mean sea level is low in August and high in March and November, a feature which is repeated in the monthly heights for each year. The total range of the means is from 2.551 to 3.209—a difference of 0.658. The lowest value for monthly mean sea level was recorded in August 1930, when it was 2.494, or 0.403 below the mean, and the highest in November 1937, when it was 3.361, or 0.464 above the mean—a range of 0.867, or almost a foot.

In other parts of the world similar variations in the values of monthly mean sea level occur, even when lunar, and not calendar, months are taken. Thus, at Galveston, in the U.S.A., the monthly mean sea level, as worked out from observations covering a period of 23 years, shows a mean range of about 0.86 ft. during the year, the maximum value occurring in September and the minimum in January.

Mean sea level for the year may also show fairly considerable variations, and the following are the results for nine years' observations at Takoradi :

Year.	Mean Sea Level.	Year.	Mean Sea Level.
1929-30	2.831	1934-35	2.902
1930-31	2.792	1935-36	2.936
1931-32	2.764	1936-37	2.927
1932-33	2.835	1937-38	2.993
1933-34	2.948		

Mean = 2.881

These figures show a total range of 0.23 ft., with a maximum value 0.112 ft. above the mean and a minimum value 0.117 ft. below the mean, but even greater variations are met with elsewhere. At Galveston, for instance, M.S.L. for the year 1917 was 0.24 ft. below the general mean, and in 1921 it was 0.38 ft. above—a total range of 0.62 ft.—while for two consecutive years, 1918 and 1919, there was a difference of almost 0.4 ft.

On account of these differences in the annual values, and the great accuracy of modern geodetic levelling,* it is usually considered desirable, when adopting mean sea level as a datum for geodetic levels, to base it on observations extending over a period of about nineteen years, by which time the moon's nodes will have completed one entire revolution. In practice, however, this is not always feasible and a datum, based on observations covering a few years only, is often accepted and used.

* In the English Geodetic Levelling, 1912-21, the maximum closing error in a closed circuit was 0.4771 ft. in a perimeter of 290 miles and the smallest 0.0073 ft. in a perimeter of 60 miles. The longest circuit was 301 miles and the closing error in it was 0.1612 ft. (See "The Second Geodetic Levelling of England and Wales, 1912-1921," page 57.)

Primary and Secondary Tidal Stations.—It sometimes happens that, having established mean sea level at one point, it is desirable to establish it at another, so that a line of levels starting at the first point can be tied in at the second. If the two points are on an open coast line and are not too far apart, and if the observations at the first point cover a long period of time, it may not be necessary to have such an extensive series of observations at the second point, provided simultaneous observations over a relatively short period indicate that the ordinary tides at the two places are very similar. In this case, the station for which a long series is available may be called a primary station, and the one for which a short series is available a secondary station.

Suppose that *A* is the primary station and that *B* is the secondary one, and that, at *A*, the result of 19 years' observations gave 15.785 ft. as the height of mean sea level above the datum of the gauge and that the mean for one particular year was 15.648 ft. For the same year, the height of mean sea level above the datum of the gauge at *B* was 20.469 ft. Then, at *A*, the mean sea level for this year was 0.137 ft. below the normal. Consequently, this amount may be treated as a correction to be applied to the readings at *B* to reduce the year's observations there to the mean for a number of years, so that the corrected mean sea level at *B* becomes $20.469 + 0.137 = 20.606$ ft. above the tide gauge datum.

Owing to the complicated nature of the tidal waves and streams round coast lines such as those of Great Britain, and the resulting large local variations in the tides at different places, it would not be possible to apply this method successfully to stations on such coasts; but it can be applied to stations, such as places on the eastern coast of America or the western coast of Africa, which are not more than 300 or 400 miles apart and which lie on an open seaboard where abnormal tides or streams do not exist, as observations have indicated that, in such circumstances, the long period oscillations of the tides are very similar, both in amplitude and phase, over considerable areas.

If a line of levels is to be connected to two or more tidal stations and a long series of observations is not available at any one of them, the times and periods covered by the observations from which the mean sea levels are to be computed should, if possible, be the same for all stations.

Some interesting articles on tidal observations, with particular reference to levelling operations and written in a "popular" and non-technical style, are contained in Chapters I-IV and in Chapter XIV of the *Bulletin of the National Research Council*, No. 78, February 1931, which is published by the National Academy of Sciences, Washington, D.C. There is also an interesting discussion of English tidal observations in *The Second Geodetic Levelling of England and Wales, 1912-1921*.

SHORE LINE SURVEYS

The survey of a shore line is usually performed by traversing along the shore, the edge of the water being located by taped offsets, tacheometer, or plane table, according to circumstances. In the survey of narrow rivers both banks may be located by tacheometry from a traverse on one side. In the case of wide rivers it will be necessary to traverse along both banks and connect the two systems at intervals. Otherwise a triangulation net is run along the river, as in Fig. 349. In the most rapid shore line surveys the banks are sketched with reference to a compass traverse executed from a launch, linear distances being deduced from the speed of travelling.



FIG. 349.

In the case of tidal waters it is necessary, for plotting on other than small scales, to locate both the high and low water lines. These are taken for average spring tides. The position of the former may be estimated roughly from deposits and marks, but to locate the lines carefully it is necessary to ascertain the elevations corresponding to high and low water of ordinary spring tides and trace the corresponding contours along the shore as in direct contouring. Owing to the limited time available for the survey of the low water line, it is best located by interpolation from soundings.

SOUNDING

The operation of sounding corresponds to that of levelling in land work, and has for its object the determination of the levels of the submarine surface. This is accomplished by measuring from a boat or a launch the depth of water at various points, the measurements being termed soundings. The water surface is thus used as a level surface of reference, which, however, in tidal waters undergoes continual change of elevation. But by observation of a series of tide gauge readings during the period of sounding this variation may be ascertained, and it is then possible to subject the observed values of the soundings to a process of reduction whereby the levels of the submarine surface may be referred to a fixed datum.

Scope.—Sounding is the most commonly required operation in hydrographical surveying, and is undertaken for engineering, navigational, and scientific purposes, these in many cases being closely interrelated. For the design of such works as breakwaters, groynes, wharves, river viaducts, etc., a knowledge of the levels of the bottom over the site is obviously necessary. In the maintenance of harbours and docks, periodical soundings are taken to detect the progress of silting and so to ascertain where, and in what volume,

material should be removed by dredging, and where it can safely be deposited. The measurement of river discharge and the examination of changes in the bed due to scour or silting also involve the running of lines of soundings between the banks.

A very important application of sounding is in the preparation of charts exhibiting the depths available for navigation. Harbour and river engineers are concerned with this aspect of the subject in the location and maintenance of navigable channels. In another direction sounding is applied in the investigation of scientific problems relating to the rate of accumulation of bars, growth of coral formations, etc.

Soundings required for civil engineering purposes seldom exceed about 25 fathoms, and a description of the methods applicable to deep sea sounding (beyond the 100 fathom line) is outside our scope. These observations may be accompanied by investigations of the nature of deep sea deposits and ocean temperatures, and are undertaken mainly for scientific purposes, but are of immediate practical importance in cable laying.

Sounding Party.—Setting aside the preliminary work of establishing shore signals, the constitution of the party depends upon the manner in which the soundings are to be located.

When the location is controlled by angular fixes taken on the boat, a full sounding boat complement comprises the surveyor, instrument man, recorder, leadsman, and oarsmen. The surveyor superintends the operations, sees that the boat is being kept on the proper course, and verifies the soundings as called by the leadsman. The instrument man is responsible for the sextant or compass observations to shore marks. Usually the surveyor himself performs the duties of instrument man, but, in locating by two simultaneous sextant observations on three shore objects, the chief and an instrument man share the work. The recorder acts as timekeeper, and notes the soundings as called out by the leadsman as well as the results of angular and other measurements.

When soundings are located by angular observations from shore, the instrument man remains there. Two shore observers are required in certain methods. They measure bearings to the boat on receipt of a signal from it, and, unless the surveyor makes the signals, an extra man is required in the boat to act as signaller.

For cross rope sounding the instrument men and signaller are dispensed with, but a crew is required to man the reel boat.

In tidal waters gauge readings must be made at regular intervals during the sounding operations. A trustworthy man is stationed at the gauge with instructions to note the reading at 10 or 15 minute intervals, his watch being set to correspond with that of the recorder. If a gauge is established close to the survey, it may be possible to have the readings observed through a field glass by a member of the boat party.

Equipment.—The essential equipment comprises a boat or launch, apparatus for taking the soundings, and instruments for their location. The transit theodolite is used when the location is based upon angular measurements from shore, but observations from the boat are made by sextant or, less commonly, and with less accuracy, by prismatic compass.

Sounding Boat.—The chief qualities desired in a row-boat for sounding are roominess and stability. A flat-bottomed cobbler is suitable for sheltered waters, but for sea work a boat of either the whale or ship's lifeboat type is more easily handled. Row boats for regular sounding may with advantage be provided with a well



FIG. 350.—SOUNDING BOAT.

(Fig. 350), through which the soundings are taken. For harbour and river sounding two oarsmen are sufficient, but four are frequently necessary for sea work. In localities where the currents are strong, much time is saved by the use of a motor or steam launch. The equipment of a sea boat should include a compass, grapnel, and spare oars.

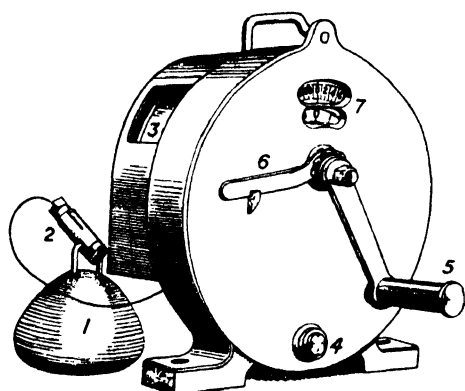
Sounding Rod.—The sounding rod is a pole of tough timber, usually circular in section, 2 to 3 in. in diameter, and 15 to 25 ft. long. Long rods are awkward to manipulate when sounding in shallow water, and should be formed of two or three lengths screwed together so that the top may be removed when necessary. The rod is sometimes graduated in feet and tenths, but division to quarter feet only is generally preferable. To facilitate plunging it, the pole is weighted at the base, which is sometimes enlarged to prevent excessive sinkage into a soft bottom. If samples of the surface material of the bottom are desired, the base is made concave at the end, and the cavity is filled with tallow, to which the material adheres.

Lead Line.—The lead line consists of a graduated line or chain to which is attached the lead or sinker. A line of either hemp or cotton is commonly employed, but is liable to elongate with prolonged usage. To minimise this effect, the line must be thoroughly stretched when wet before graduation. A satisfactory method is to coil it tightly round a tree trunk or post, secure the ends, and wet it thoroughly. The shrinkage caused by the wetting induces tension, and, when dry, the line is stretched. The process should be repeated until the stretch becomes inappreciable, and the line is then soaked and graduated by attaching cloth or leather tags at foot intervals. Every fifth foot mark is distinguished from the others, and every tenth is made to exhibit its distance from the bottom of the lead. Such a line should have its length tested periodically.

For regular sounding, a chain of brass, steel, or iron is preferred since, although subject to elongation through wear, it maintains its length much better than the line. The links may be 1 in. long, and should be welded or brazed. The attached tags are sometimes of brass, but may be of leather or cloth as in the line.

For ordinary sounding, the weight of the lead sinker ranges from 5 to 20 lb. according to the strength of current and depth of sounding. A weight of 8 to 10 lb. is sufficient for harbour work. When samples of the submarine surface material are required, a lead is used with a recess in the bottom for the reception of tallow as in the sounding rod.

Sounding Machines.—The work of sounding is simplified by the use of one of the various forms of sounding machines suitable for engineering surveys. Fig. 351 illustrates Weddell's Sounding Machine, which is manufactured by Messrs. George Russell and



- | | |
|-------------------------------|-------------------|
| 1 Lead Weight | 5 Winding Handle |
| 2 Swivel and Hook | 6 Pawl Arm |
| 3 Barrel | 7 Recording Dials |
| 4 Plunger for Zero Adjustment | |

FIG. 351.—WEDDELL'S SOUNDING MACHINE.

tenths of a foot. The lead is raised by means of the handle 5, and it may be suspended at any height by means of a pawl and ratchet. The standard machine is designed to record a maximum depth of 100 ft.

At the commencement of a series of soundings, the dials are set to read zero by means of the handle 5. By depressing plunger 4, the reading is held at zero, and the handle is again turned until the bottom of the lead is raised or lowered to touch the surface of the water. On releasing the plunger, the machine is ready for use, no correction to the readings being required to allow for the height

Co., Ltd., of Motherwell. The instrument is intended to be bolted over the well or to the gunwale of the sounding boat, and consists of a galvanised malleable cast iron casing carrying on a spindle a gun-metal barrel, 2 ft. in circumference. The lead, of about 14 lb. weight, is suspended by a flexible wire cord from the barrel, and can be lowered at any desired rate, the speed of the drum being controlled by means of a brake. The indicating dials are driven by gearing from the spindle through a friction device, the outer dial recording the depth in feet, and the inner showing

of the machine above water level. It is unnecessary to raise the weight to the surface after each sounding in sheltered water, but it should be wound up when moving from one section to another.

The Sounding Sextant.—The form of sextant used in coast surveying is known as the sounding sextant (Fig. 352), and differs somewhat from the astronomical sextant (Vol. II, Chap. II). It is of specially strong construction, and is fitted with a large index glass to minimise the difficulty of sighting from a small boat, due to its motion. The horizon glass has no transparent part, and is entirely supported by a metal tray. The object viewed directly is therefore sighted over the top of the horizon glass, and the reflected image is separated from the direct one by the small thickness of the frame. The arc is boldly graduated, and is read by the vernier to single minutes. The ring 5 carries a disc with pin-hole sight, but the instrument is also provided with a Galilean telescope of wide field. This is substituted for the disc when the sights are long, merely to afford resolution. The telescope is non-adjustable, but the testing and adjustment of the perpendicularity of the mirrors to the plane of the arc and for index error are conducted as for the astronomical sextant, except that test sights may be taken on distant terrestrial objects instead of celestial bodies.

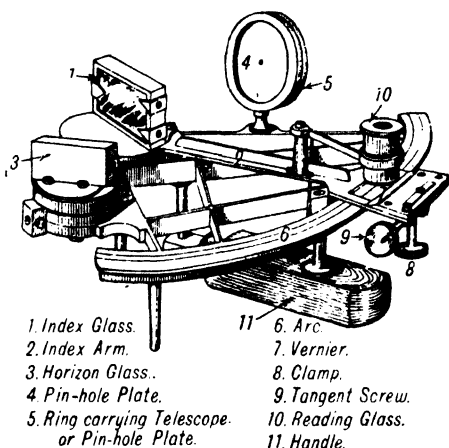


FIG. 352.—SOUNDING SEXTANT.

Water Glass.—During sounding, an inspection of the submarine surface or of underwater construction is sometimes required, and may be made by means of the water glass or water telescope (Fig. 353). The instrument consists of a watertight box of wood or sheet iron open at the top and having a sheet of plain glass fitted at the bottom. Two handles are provided for steadying the instrument. The observer, leaning over the side or stern of the boat and holding the instrument with both hands, lowers the bottom a few inches below the surface of the water, and looks in at the upper end. The clearness of the view presented depends upon the depth and transparency of the water and the brightness of the day.



Glass Plate
FIG. 353.

Shore Signals.—Shore signals are required to mark ranges defining section lines and as points to which to observe in making angular

measurements from a boat. For the former purpose, signals have in general to be specially set out and erected, but for angular observations existing landmarks may be available, especially in the vicinity of ports. It is, of course, necessary that objects, such as tall chimneys, spires, lighthouses, etc., used as signals, should be capable of identification on the map.

Temporary signals consist of poles, piles of stones, or white-washed marks on rocks. Unless the observations are made from short distances, ordinary ranging rods are too small, and longer poles of 2 to 4 in. diameter, or battens, are required. These are whitewashed or painted, and, if practicable, sufficient height should be given that they can be viewed against the sky. They should be flagged, or have discs or spars nailed on at the top. Different ranges may be distinguished from each other by using flags of various colours or by fixing timbers to form various geometrical figures, such as a triangle, cross, etc. The reference number of each range is sometimes exhibited by nailing on spars to form Roman numerals when read laterally.

It may happen that one at least of a pair of range signals has to be placed in the water. If the depth is such as to preclude the erection of an ordinary signal, a short flag-pole may be attached to a float formed of a block of light wood weighted at the bottom and anchored in position. The action of current in displacing the buoy may, however, lead to considerable error in the direction of the range. If there is no tide, this error may be avoided by guying the float from three anchor boulders, but in tidal waters guying is troublesome, as the guys must be effective at low water. If there is a considerable tidal range, a long flag-pole must be used, so that a sufficient length may show above the surface at high water.

Whitewashed marks are useful on rocky foreshores, and are suitable for observing to in making angular fixes from a boat. For the accurate definition of ranges, it is generally inadvisable to use two such marks, unless they are at such elevations that they appear as one continuous line when viewed from a boat in the range.

Sounding by Rod.—In the absence of strong current, soundings are taken by rod up to depths of 20 to 25 ft., beyond which the lead line is more convenient. Currents may reduce the limiting depth for rod sounding to 15 ft.

Except when sounding through a well, the leadsman stands in the bow. He plunges the rod at a forward angle, depending on the speed of the boat, so that when it reaches the bottom it can be read in a vertical position. He must read the rod quickly owing to the onward motion of the boat, and the surveyor should satisfy himself by a glance that the sounding as called is correct. Unless samples are being collected, the rod is then simply allowed to float up loosely in the hand, and is grasped at the right place for the next stroke.

Sounding by Lead.—The leadsman in the bow of the boat heaves the lead forward such a distance that it will reach the bottom at the point where the sounding is required. As the boat progresses, he takes in the slack, so that the line or chain is vertical when being read. The soundings are, for engineering purposes, read to $\frac{1}{4}$ ft. by estimation. As soon as the reading is called, the leadsman must coil in the line for the next cast in a manner which will prevent its becoming entangled.

Spacing of Soundings.—Since the desideratum that observations should be made at points of change of slope is impossible of fulfilment, soundings should be so spaced that no important irregularity in the submarine surface will go unrecorded. With this object, it is best to sound along a series of straight lines at right angles to the shore line, so that the successive contours may be intersected normally or nearly so.

The interval between lines of soundings and the spacing of the casts thereon depend primarily upon the object of the survey but also upon the nature of the bottom. On a rocky foreshore the possibility of the existence of pinnacle rocks necessitates closer sounding than is required in the case of a soft bottom. The spacing of soundings should not be made to depend altogether upon the intended scale of plotting, as the finished drawing may be made to exhibit only a selection of typical observations.

For general engineering purposes, 100 ft. is the most common interval between lines of soundings, and the casts may be made at 25 or 50 ft. apart under favourable circumstances. When close estimation of excavation is required, the lines may be as little as 20 ft. apart, with soundings at 10 ft. intervals.

When indication is obtained of a sudden inequality in the bottom, additional lines should be interpolated to locate its limits. These lines are sometimes projected radially from a buoy marking the suspected position of the irregularity, the operation being termed "starring." On the other hand, the interval between casts may generally be increased with safety as the water deepens.

Methods of Locating Soundings.—If the lines of soundings are defined by range signals to serve as steering marks, the position of the boat on the line can be determined by one measurement, linear or angular. When the lines are not marked out, the boat is kept on an approximately straight course, and two observations are then necessary to locate its position from time to time. Fix observations may be made: (a) entirely from the boat; (b) entirely from the shore; (c) from both.

The methods of location are:

- (1) By cross rope.
- (2) By range and time-intervals.
- (3) By range and angle from shore.

- (4) By range and angle from boat.
- (5) By intersecting ranges.
- (6) By two angles from shore.
- (7) By two angles from boat.
- (8) By one angle from shore and one from boat.
- (9) By tacheometry.

Cross Rope Sounding.—The most accurate method of locating soundings is that by cross rope. It involves stretching across the line of sounding a rope marked off by equidistant tags, the soundings being taken opposite the tags. The method is very suitable for sounding in harbours and across rivers if the sections do not exceed about 1,500 ft. in length, but it can also be adapted to offshore work.

The Cross Rope.—For short lines a hemp rope will serve the purpose, but it cannot be heavily stressed, and for lengths over 500 ft. a steel strand wire rope of $\frac{1}{4}$ to $\frac{1}{2}$ -in. diameter should be used. Phosphor-bronze rope is much more durable, but more expensive. The tags are of brass or leather, and are marked with their distances from one end. They are spaced at 10, 20, or 25 ft. intervals. The rope is wound upon a reel, which is mounted in a boat called the reel boat, usually similar to the sounding boat.

Procedure.—The zero end of the cross rope is first made fast at one end of the section to a spike or other attachment previously located. The reel boat is then rowed across the line of sounding, unwinding the rope as it goes. The course of the reel boat should be as straight as possible, and, if a range is not available in which the oarsmen can maintain themselves, their progress should be directed by the surveyor in the sounding boat waiting at the beginning of the section. If the reel boat is not rowed straight across the section, the uncoiled rope will lie in an irregular curve along the bottom, and will most probably foul when being tightened preparatory to sounding. On arrival of the reel boat at the other side, her anchor is taken ashore, and the men proceed to wind in the rope as tightly as possible. If anchoring is impracticable, the reel is taken ashore and spiked down.

On receiving a signal, the sounding boat now starts to cross the section. The oars are shipped, and the oarsmen propel the boat by hauling on the cross rope. As each tag approaches the man in the bow, he warns the leadsman, who takes the sounding when the mark reaches him.

When a section is completed, the sounding boat is rowed to the starting-point of the next, while the reel boat proceeds back along the line, winding in the rope. This proves more expeditious than dragging the rope along the bottom to the next section line.

Use of Floated Cross Rope.—The rope, instead of being allowed to lie along the bottom as it is uncoiled, may be suspended from a

series of floats. These are usually pieces of light wood which are clipped on in the reel boat as the rope is first being paid out. A float may be attached at each tag, and the 100 ft. distances distinguished by specially painted floats. When the rope is tightened, the points at which soundings are required are thus clearly visible. The sounding boat is propelled as before, but the rope is not taken on board.

The advantages of this method, as compared with the previous one, are :

(a) The rope can be reeled in more tightly, so that the error due to vertical sag is reduced.

(b) It is more rapid, since, in changing from one section to the next, the rope need not be reeled up, as there is no chance of fouling the bottom.

It has, however, the disadvantages that :

(a) It is quite unsuitable for situations where the operations are liable to sudden interruption by shipping.

(b) The action of wind and current on the floats increases the lateral sag.

Cross Rope Offshore Sounding.—In calm weather or in sheltered situations, offshore sounding may be performed by cross rope with or without floats, provided the sections are comparatively short. The operations differ from those described above only in the manner of stretching the rope. One end is fixed on shore, and the reel boat is anchored at sea with the other end, or, alternatively, the seaward end of the rope may be attached to a buoy or simply anchored.

Location by Range and Time-intervals.—In this method the sounding boat is kept in range with two shore poles defining the section line, and is rowed at a uniform rate, the soundings being taken at regular time-intervals. The method is not susceptible of great accuracy, but is useful in conjunction with other methods. It is seldom adopted alone except for comparatively short sections of known length, such as between the banks of a river or from a coast line to buoys of known position. If the total length of section is unknown, the precision of the method is greatly reduced on account of the influence of wind and current on the speed of the boat.

While proceeding along the section, the oarsmen must maintain a steady stroke so that equal time-intervals may correspond to equal distances. The surveyor, watch in hand, warns the leadsmen when to sound. Alternatively, the soundings may be spaced by counting a constant number of oar strokes between each, or, in the case of a launch, a constant number of revolutions of the screw. When the total length of section is known, it is only necessary, for purposes of plotting, to divide it out into a number of equal parts. When the distance is unknown, the speed of the boat or launch must be very carefully rated beforehand by travelling

over a measured length and noting the average number of oar-strokes to a given distance or the actual speeds of the launch corresponding to various engine speeds.

Location by Range and Angle from Shore.—The boat is again steered in range with guide poles, and its position in the line is observed from a theodolite station on shore by measurement of its bearing or the angle between it and a located shore object. The method is a useful one, capable of considerable precision, and proves simple in plotting. It has the disadvantage that the surveyor in the boat has not the entire operations under his immediate control.

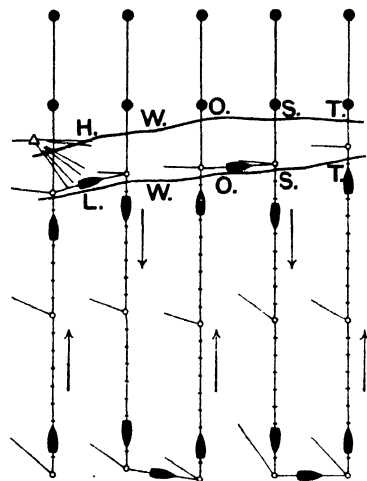


FIG. 354.

Some surveyors fix each sounding by an angle, but this is not recommended as the rapidity with which the angles have to be read and booked may necessitate a notekeeper at the instrument, and is very likely to lead to confusion. It is sufficient to locate, say, every tenth sounding by angle and the intermediates by time-intervals. Fig. 354 shows angular fixes applied only to the end soundings and one midway in each section, and illustrates the course of the boat, seaward and shoreward on alternate sections.

Suitable instrument stations should be set out, and their positions determined beforehand, due consideration being given to the quality of the intersections which will be made between the lines of sight and the range lines. The nearer the intersection is to a right angle the better: a new instrument station should be occupied when the angle diminishes to about 30° .

The routine of the instrument man on setting up at a station consists in first orienting on the back station or on a reference point which can be reproduced on the plan. The line of sight is then directed towards the leadsman or the bow of the boat, which has been brought into position at the start of the section. The signaller holds up a flag for a few seconds to warn the instrument man to prepare, and on the fall of the flag the sounding and angle are observed simultaneously. The telescope is kept pointing towards the boat as it moves along the section, and the remaining angles are taken in the same way.

The recorder in the boat distinguishes in his notes those soundings which are fixed by angle, but in transcribing the angle book notes

into the sounding book at the close of the day's work, it may happen that there is difficulty in identifying particular angles. The possibility of confusion is eliminated if the time of each fix is noted both by the recorder and the instrument man, their watches having previously been compared. When frequent fixes are made on each section, the possibility of discrepancy between the records is increased, particularly through the omission of an angle observation. The time check may then involve reading to seconds, and unless the instrument man has a notekeeper, his work becomes very hurried. An alternative and satisfactory method for the detection of mistakes is to make successive signals by flags of different colours. At each fix, the colour is noted in the angle book and also by the recorder.

Notes.—(1) If the lines of soundings have previously been drawn on the plan, suitable instrument stations may be selected, and note made of the sections to be dealt with from each so as to avoid acute intersections.

(2) There is no check on the accuracy of the fixes other than that afforded by the time-interval location. As a precaution, the instrument man, before leaving a station, should check his orientation by again sighting the reference point.

(3) It is unnecessary to read angles to parts of a minute. In many cases it is sufficient to estimate readings to the nearest 5 min. without consulting the vernier.

Location by Range and Angle from Boat.—In principle this method is similar to the last, but the angular fix is made between the range line and a shore object, and is observed from the boat by sextant (Fig. 355). A compass bearing may take the place of the sextant angle, but proves less accurate.

The boat is rowed outwards and inwards on alternate sections as before. Fixes are taken at the ends of the sections and at as many intermediate soundings as are considered necessary, but the majority of the soundings are located by time-intervals between the fixes. In observing with the sextant, the telescope is directed on the range signals, and the side object is brought into coincidence.

Compared with the previous system, this method has the advantages that

(1) The surveyor has better control of the work, since the party is not divided. Frequently the instrument man is dispensed with, and the angles are observed by the surveyor.

(2) The angles are booked by the recorder directly they are measured, so that there is less probability of mistakes in booking.

(3) At important fixes a second angle to another shore object may be observed as a check on the first when plotting (Fig. 355).

(4) On one section different shore objects can be used in the various fixes so as to maintain good intersections throughout.

There is little to choose between the two methods as regards accuracy and ease of plotting.

Location by Intersecting Ranges.—Angular observations are avoided if the position of each sounding is defined by the inter-

section of two ranges. The method is suitable for the location of a few isolated soundings, and proves highly accurate if the intersections are good.

The system is sometimes employed when it is required to determine by periodical sounding the rate at which silting or scouring

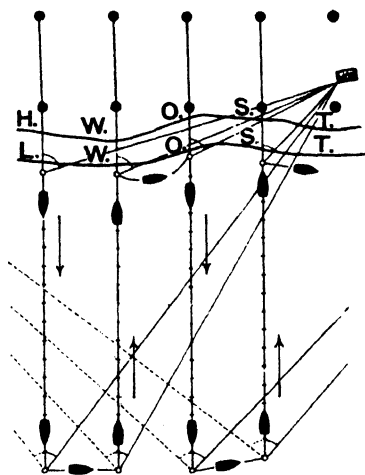


FIG. 355.

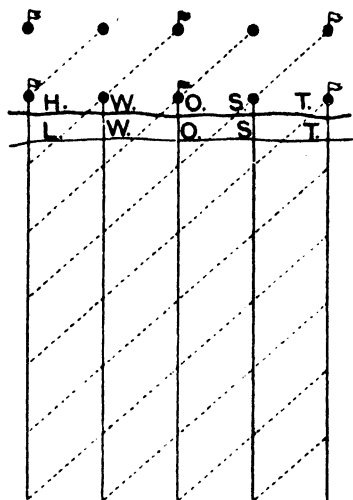


FIG. 356.

is occurring at a place. For such observations it is essential that the repeated soundings should be made at the same spots, and permanent range signals are established. When applied to section sounding, as in Fig. 356, precautions must be taken to obviate confusion between successive intersections, and a system of flagging the range poles is necessary.

Location by Two Angles from Shore.—In this system the fix is made independent of a range by having simultaneous observations to the boat taken by theodolite from two shore stations, the soundings being located by the intersections of the sight lines. The method may be applied to the location of a few isolated soundings: if it is used on an extensive survey, the boat should be run on a series of approximate ranges.

The routine followed by the instrument men is similar to that adopted in locating by a range and one shore angle. They must observe simultaneously on the fall of the signal flag in the boat, and should note their watch times. New instrument stations must be occupied by one or both observers when the intersection angle falls below about 30° .

The principal merit of the system is the elimination of the preliminary work of setting out and erecting range signals. It also

avoids the difficulty, in localities where the currents are strong, of maintaining the boat exactly on a range. It is, however, a serious disadvantage that two instrument men are required on shore.

Location by Two Angles from Boat.—By observation of the two angles subtended at the boat by three suitable shore objects of known position (Fig. 357), the boat can be located by solution of the three-point problem (page 563). This method is largely used, particularly when periodical soundings are not required. It not only possesses the merit of concentrating the party, as in the range and one boat angle method, but, if a sufficient number of landmarks are exhibited on an existing map, no preliminary shore work is required.

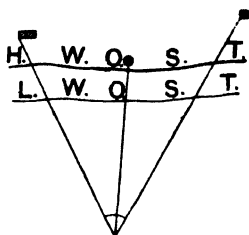


FIG. 357.

It is important that the angles should be observed as nearly simultaneously as possible, particularly for large scale plotting, and the surveyor and the instrument man should each have a sextant. If the surveyor is observing alone, he should allow as little time as possible to elapse between his observations by using two sextants, which are successively set to the angles and read afterwards. If the boat is stopped, a check can be secured by measuring the total angle subtended by the extreme objects. Refined reading is not required, but considerable care is necessary in selecting suitable reference points on shore (page 566).

If the compass is used instead of the sextant, the position of the boat is defined by bearings to two shore objects, but with considerably less accuracy.

Location by One Angle from Shore and One from Boat.—This combination of the last two methods is not much used. For convenience of plotting, one of the shore points sighted with the sextant should be the theodolite station.

Location by Tacheometry.—A tacheometric observation on a staff held in the boat, since it gives the bearing and distance from a shore station, affords a simple method of location, which, however, can be used only in smooth water. Soundings have been located with considerable precision in river work by simultaneous observations from tacheometers on both banks. This fix affords a twofold check since each observation gives the position by angle and distance, while together they yield an intersection. If the water is shallow over the area being sounded, the stadia rod may be dispensed with by reading the intercept on the sounding rod at the moment it is held on the bottom. Instrument stations should be selected near the edge of the water so that no correction for vertical angles may be necessary.

Note-keeping.—*Recorder's Notes.*—These are entered in the sounding book, the form of which varies according to the system of location adopted. In any case columns are provided for measured depths, location observations, watch times, and remarks. Columns are reserved for the subsequent entry of tide gauge readings from the gauge reader's notes, reduction corrections, and the reduced soundings.

The location measurements, if made from the boat, are simply entered as distances in the case of cross rope sounding (Fig. 358). With sextant observations, the shore stations must be sufficiently described, and the entries may be made in one column or separated as in Fig. 369.

The times of start and finish of each section line must be noted, as well as those at which intervening fixes are observed. In cross rope sounding, one or more intermediate times are entered opposite the corresponding distances. In cases of interruption or delay, the times of stoppage and resumption must be noted, and fixes made at these times. Any ranges which intersect the line of sounding are remarked in their proper places, as this forms a useful check in plotting.

When the location is controlled from shore, the remarks column will show which soundings have been located, and, if successive signals are distinguished, the signals should be described.

Instrument Man's Notes.—The record kept by an instrument man observing on shore includes the angles or bearings taken to the boat, their watch times, and a note of the colour of the distinguishing signal. Before leaving the ground, he should compare his entries with those in the sounding book for the discovery of possible discrepancies.

Gauge Reader's Notes.—These simply consist of a list of the gauge readings with the corresponding watch times. It is usually sufficient to read to the nearest $\frac{1}{4}$ ft. at intervals of 10 or 15 minutes, but in special cases readings may be taken to 0.1 ft., and observed every 5 minutes.

Reduction of Soundings.—The object of reducing soundings is to convert the observed depths to the values they would have if measured from a water surface of unvarying and known elevation forming the datum of reduction. The reduced soundings are the reduced levels of the submarine surface in terms of the adopted datum.

The datum most commonly used is the mean level of low water of spring tides, written L.W.O.S.T. (low water, ordinary spring tides), or M.L.W.S. (mean low water, springs). The local value is adopted, as the datum has not a constant elevation over a wide area, but in practice this is no objection, and this datum is the most

Section No. 4.

.....Surveyor.

.....Recorder.

Date.....

Time.	Gauge.	Distance.	Sounding.	Correc- tion.	Reduced Sounding.	Remarks.
A.M.						
10-3		25	5½	-8½	-2½	
		50	9		¾	
		75	13½		5½	
		100	15½		7½	
		25	18		9½	
		50	20½		12	
		75	21½		13½	
10-5	14½	200	22		13½	
		25	22½		14½	
		50	23½		15	
		75	25½	-8½	17½	
		300	26½		18	
		25	27		18½	
		50	27½		19½	
		75	27½		19	
		400	28½		20½	
		25	29		20½	
		50	27½		19½	
		75	26½		18	
		500	26		17½	
		25	26½		17½	
		50	24½		16	
		75	22		13½	
		600	20½		12½	
		25	17½		9	
		50	14½		5½	
		75	14		5½	
10-10	14½	700	11½		3½	
		25	10½		1½	
						Lt. Ho. and C's Chy. Stk. in range.

FIG. 358.

useful one for navigational charts. It is commonly worked to in engineering surveys, but not exclusively, and H.W.O.S.T., mean sea level, or any arbitrary, but defined, datum, such as dock sill level, may be used instead.

Reduction is performed by computing and applying corrections to the measured depths. In tideless waters a constant correction, equal to the difference of level between the actual water surface and the datum, could be applied to all the soundings of a series, but in tidal waters the amount of correction changes with the varying water level as read on the tide gauge.

Having interpolated the appropriate gauge readings, and entered them in the sounding book, the corrections are deduced as the differences between the gauge readings and the value of the datum as it would be indicated on the gauge. These are entered as positive

or negative quantities according as the latter is greater or smaller than the former. If, as is usual for engineering purposes, the reduced soundings are to be expressed to $\frac{1}{4}$ ft., corrections are applied for every $\frac{1}{4}$ ft. variation of tide level, and the observations are corrected in groups corresponding to this variation.

Fig. 358 shows the reduction of a line of soundings located by cross rope. The datum of reduction is L.W.O.S.T., which corresponds to 6 ft. on the gauge used. The gauge readings were 14 and $14\frac{1}{4}$ at 10-0 and 10-10 a.m. respectively, and the height $14\frac{1}{4}$ is interpolated for 10-5. The corrections are obtained by subtracting 6 ft. from the gauge readings, the $8\frac{1}{4}$ correction being applicable from the start of the section to 450 ft., which has been reached about midway between 10-5 and 10-10. The negative value of the first reduced sounding shows that point to be above the datum.

PLOTTING SOUNDINGS

Reduced soundings may be plotted in section form in the ordinary manner, provided the soundings have been taken in lines. This method of representing the variation in level of the bottom is required in river work, and is sometimes used in marine surveying for the design of engineering works. For navigational and general engineering purposes the reduced soundings are shown in plan. They are exhibited as spot depths, the values of the reduced soundings being written at the points representing their positions. The interpolation of contours increases the value of the chart to the navigator, and is a necessity for engineering purposes, as the plan then exhibits the topography of the bottom, and is available for the calculation of volumes of dredging.

Locating Soundings in Plan.—Having laid down the shore survey and from it the positions of landmarks and range signals, the section lines are drawn (in the case where the soundings have been taken in lines), and the positions of the soundings are spaced out. In plotting cross rope soundings to a large scale, vertical and lateral sag of the rope may occasion difficulty in spacing the soundings when the total length of section is known. In such a case, the distance should be divided up into a number of equal parts corresponding with the number of soundings. If the recorder has noted the rope distance at which the line of section is cut by the range of two landmarks represented on the plan, the spacing out can be performed in two parts with increased accuracy.

The plotting of angular fixes and intermediate soundings does not require explanation, except in the case of location by two sextant angles from the boat. This is an application of the three-point problem, and differs from its use in plane table surveying only in that the measured values of the two subtended angles are known.

The Three-point Problem.—The problem may be stated: Given three points A, B, and C, representing the shore signals, and the values a and b of the angles APB and BPC, subtended by them at the boat P. Required to plot P. The problem may be solved mechanically, graphically, and analytically.

Mechanical Solutions.—(1) *By Station Pointer.*—The station pointer or three-arm protractor (Fig. 359) consists of a graduated circle with one fixed and two movable arms, the fiducial edges of

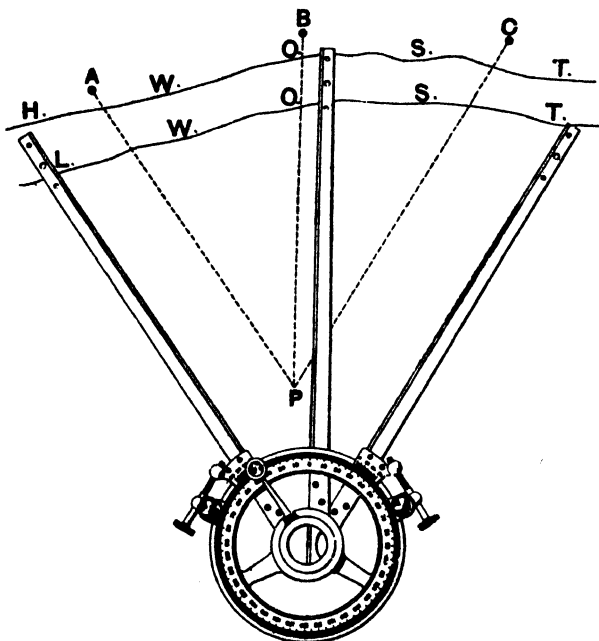


FIG. 359.—STATION POINTER.

which pivot about the centre of graduation. The edge of the fixed arm passes through the zero division, and the movable arms can be set so that their fiducial edges subtend the observed angles with that of the fixed arm. For large scale plotting the arms can be extended by clamping on lengthening pieces.

To use the station pointer, set off the angles a and b , and clamp the movable arms. Move the instrument over the plan until the bevelled edges of the three arms simultaneously touch the points A, B, and C. The centre of the protractor then marks the position of P, which is recorded by a prick mark.

(2) *By Tracing Paper Protractor.*—On a piece of tracing paper protract a and b between three radiating lines from any point.

Apply the tracing paper to the plan, and move it about until A, B, and C simultaneously lie under the lines; then prick through the point P at the apex of the angles.

Graphical Solutions.—(1) Join AB and BC (Fig. 360), and at A and B set off AO and BO respectively, making angles of $(90^\circ - a)$ with

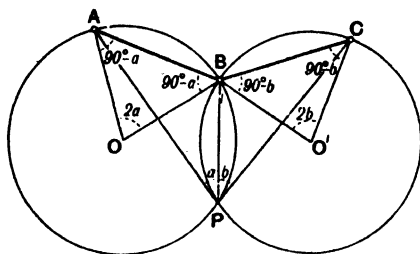


FIG. 360.

AB on the side next P. From B and C similarly set off BO' and CO' at $(90^\circ - b)$ with BC. With centre O, describe a circle through A and B, and, with centre O' , describe a circle through B and C. The intersection P of the two circles is the point sought, for angle APB at the circumference = $\frac{1}{2}$ AOB at the centre on the same chord. But $AOB = 2a$, so that $APB = a$; similarly, $BPC = b$.

Note.—Observed angles exceeding 90° have their excess over 90° set off on the side of AB or BC remote from P.

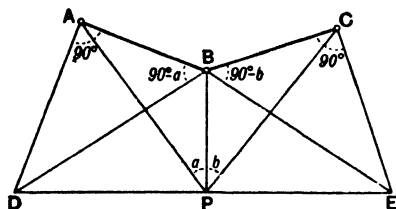


FIG. 361.

(2) Join AB and BC (Fig. 361), and from B set off BD and BE making angles of $(90^\circ - a)$ and $(90^\circ - b)$ with BA and BC respectively and on the side next P. From A erect a perpendicular to AB to cut BD at D, and from C a perpendicular to CB to cut BE at E. Join DE. From B drop a perpendicular BP on DE. This will intersect DE at the required point P, for the quadrilaterals BADP and B CEP can be circumscribed by the circles used in the previous construction, so that $APB = ADB = a$, and $BPC = BEC = b$.

Note.—Observed angles exceeding 90° have their excess over 90° set off on the side of AB or BC remote from P.

(3) Join AC (Fig. 362), and at A set off AD, making $CAD = b$ on the side remote from P. From C similarly set off CD, making $ACD = a$. Let these lines intersect at D. Join DB. The circumscribing circle through A, D, and C cuts DB, produced if necessary, at P, for $APD = ACD = a$, since they are circumferential angles in the same segment; similarly, $DPC = b$.

In place of drawing the circumscribing circle, it is more convenient to set off a and b on either side of any point on DB and to draw through A and C parallels to these lines to intersect at P.

Analytical Solution.—Given the lengths AB and BC and angles a , b , and c , the values u and v of angles BAP and BCP and the

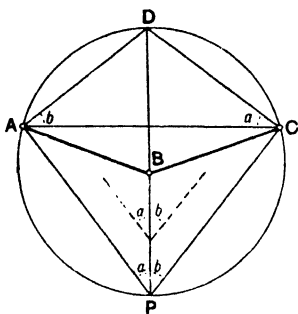


FIG. 362.

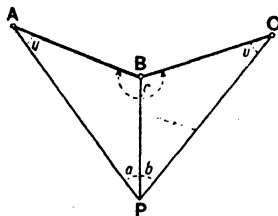


FIG. 363.

distances AP, BP, and CP may be computed trigonometrically (Fig. 363).

$$\text{By sine ratio, } BP = \frac{AB \sin u}{\sin a} = \frac{BC \sin v}{\sin b},$$

but $u + v = (360^\circ - (a + b + c)) = \text{say, } s$, so that, by substitution,

$$\frac{AB \sin u}{\sin a} = \frac{BC \sin (s - u)}{\sin b} = \frac{BC (\sin s \cos u - \cos s \sin u)}{\sin b},$$

whence

$$\cot u = \frac{AB \sin b}{BC \sin a \sin s} + \cot s.$$

For logarithmic calculation, find the auxiliary angle θ from :

$$\cot \theta = \frac{AB \sin b}{BC \sin a \sin s}.$$

Then

$$\cot u = \frac{\sin (\theta + s)}{\sin \theta \sin s}.$$

Angle v is now obtainable as $(s - u)$, and AP, BP, and CP by sine ratio. For a numerical example of this solution and also of one by using an assumed approximate value for u or v , see Appendix V.

The three point, or resection, problem is often of great use in ordinary survey work when it is desired to fix a point from three or more trigonometrical points and it is not convenient, or it is impossible, to take observations from these points. There are a number of different solutions, of which the above is only one. Others, including a semi-graphical solution which is particularly useful when observations are taken to more than three points, are given in Winterbotham's *Survey Computations*. The question of errors in resection is very fully discussed by J. E. Jackson in the *Empire Survey Review*, Vol. III, No. 18, October, 1935.

The Indeterminate Case.—If A, B, C, and P can be circumscribed by a circle, any point on the arc on which P lies subtends with AB and BC the observed angles a and b , and the position of P is indeterminate. In this case it will be found that a station pointer or tracing paper protractor can be moved about on the plan without swinging the legs off A, B, and C. In geometrical plotting the two circles of Fig. 360 merge into one, D and E coincide in Fig. 361, as do D and B in Fig. 362. Since $s = 180^\circ$, the analytical method yields the indeterminate result that $\cot u = (\infty - \infty)$.

Care must therefore be exercised to select for observation such shore stations that the circumference of the circle through them will not pass through the boat.

Strength of Fix.—If P is near the circumference of the circumscribing circle ABC, the fix, although theoretically determinate, is unsatisfactory, since errors of observation and plotting have then considerable effect in displacing P. Bad fixing is evidenced when in using the station pointer a small movement of the circle can be made without appreciably affecting the contact of the arms with A, B, and C (so that the experienced draughtsman finds the plotting of such a fix troublesome, while the beginner finds it simple). Bad fixes are indicated in the geometrical constructions by oblique intersection of the circles of Fig. 360, by D and E falling close together in Fig. 361, and by D falling near B in Fig. 362. If plotting is carried on at sea, bad fixes can be discovered at once and eliminated by check observations. Otherwise, the surveyor must exercise care in the selection of shore objects from which to fix, and the following precepts will serve as a guide.

Good fixes are in general obtained :

(1) When the three objects lie on a straight line or on a curve convex to the observer, the middle object being nearest, provided the angles are not both less than 30° .

(2) When the three objects lie on a curve concave to the observer, provided (a) the observer is within the triangle formed by them, (b) they are practically equidistant from the observer, and the observed angles are not less than 60° , (c) the observer is well outside the circumscribing circle.

(3) When one of the outside objects is much more distant than the others, and the angle between the near objects is much greater than that between the middle and remote objects.

Finishing the Sounding Plan.—The values of the reduced soundings are written up neatly in black on the points representing their respective positions. In engineering plans they are given to the nearest $\frac{1}{4}$ ft., and in navigational charts to the nearest foot or $\frac{1}{4}$ fathom. If the scale does not permit of showing all the soundings clearly, a selection is made, keeping in view the importance of the shallower soundings, and the sounding book is kept for reference.

Contour lines are interpolated amongst the figures in the usual manner. Since the submarine surface is in general characterised by flatter slopes than obtain on land, a smaller contour interval is necessary to exhibit its form. The Admiralty charts display contours at 1, 2, 3, 4, 5, 6, 10, 20, 50, and 100 fathoms, but for engineering purposes contours are usually shown at 2 ft. or 3 ft. intervals, while on large scale plans an interval of 1 ft. is sometimes adopted.

The contours are first sketched in pencil, and their positions relative to the soundings are verified by re-examination. A careful inspection should be made for indications of suspicious areas, and additional soundings may be required to remove uncertainty regarding the direction of contours where sudden inequalities occur between sections. Finally, the contours are inked in with Prussian blue. An effective method of colouring the water area consists in applying successive washes of pale Prussian blue to the areas contained within each contour, so that the tint is gradually deepened towards the deeper zones.

ECHO-SOUNDING

In the surveying ships of the Royal Navy soundings are now taken regularly by means of an echo-sounding apparatus, the latest types of which appear to have considerable potentialities from a civil engineering point of view.

The fundamental principle of echo-sounding is very simple and consists in the recording of the interval of time between the emission of a sound impulse directed to the bottom of the sea and the reception of the wave, or echo, reflected from the sea bottom. If v is the velocity of the sound wave and t the interval of time between the emission of the wave and the reception of the reflected wave, then the depth of the water is given by $d = \frac{1}{2}vt$. When v is known and t observed, d can thus be calculated.

In practice, two systems are in use. In the first—the “sonic” system—the impulse consists of waves of low frequency and long wave length which are audible and can thus be received in an ordinary head-phone. The second system is the “super-sonic” system, in which the waves are of high frequency and short wave length (about 4 inches), and are inaudible, so that a special apparatus is necessary for receiving and recording them. Among its other advantages, the super-sonic system has the property that the sound wave can be transmitted in the form of a narrow beam and is thus “directional.” This simplifies the problem of “screening” the receiving from the transmitting apparatus, so that the former is not affected by impulses received directly from the transmitter which would otherwise tend to conceal, or interfere with, the reflected waves. Another advantage is that the soundings are taken directly, or almost directly, beneath the ship, so that little or no sound is transmitted sideways, and the receiver is therefore not sensitive to

echoes reflected from submerged banks or other objects lying to one side of the point to which the sounding is to be taken. On the other hand, the sonic system has the advantage that it has greater penetrating power so that it is still used for deep-sea sounding.

As the time taken for a sound impulse to traverse the distance to the bottom and back is only $\frac{1}{40}$ of a second when the depth is 10 fathoms (60 feet), it will be realised that the time interval has to be measured very exactly, especially when soundings are required in shallow water where greater accuracy is required than in deep water. In modern machines not only is this interval measured electrically, but also, as the ship moves forward, a continuous record of the profile of the sea bottom is obtained on a travelling roll of paper.

The general arrangement of a complete super-sonic echo-sounding installation is shown in Fig. 364.

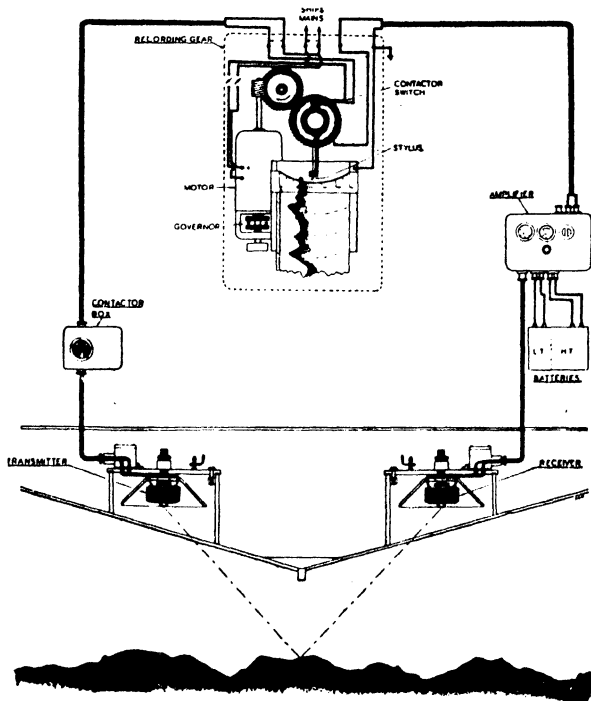


FIG. 364.

DIAGRAMMATIC VIEW OF A SUPER-SONIC ECHO-SOUNDING APPARATUS.

(By courtesy of Messrs. Henry Hughes & Son, Ltd.)

This illustration shows the installation connected to the ship's mains, but some forms of apparatus can be obtained, for use in a small launch, to work from storage batteries.

In engineering work, soundings of more than about 25 fathoms (150 feet) are seldom required and machines specially designed for work in shallow water can be obtained. Fig. 365 shows a diagrammatic view of the special survey type M/S. X. receiver and recorder manufactured by Messrs. Henry Hughes and Son, Ltd., 59 Fenchurch Street, London, contractors to the Admiralty for the supply of

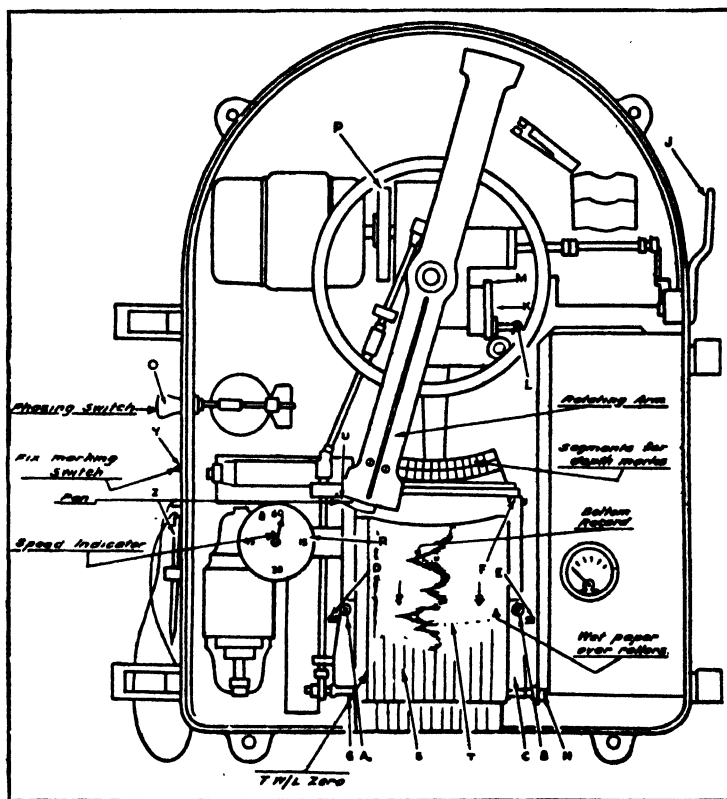


FIG. 365.

DIAGRAMMATIC VIEW OF M/S. X. ECHO-SOUNDING RECORDER.

(By courtesy of Messrs. Henry Hughes & Son, Ltd.)

echo-sounding apparatus. This recorder can be supplied with one or more scales, according to what is desired by the purchaser, and, by means of a system known as "phasing," the original scale can be extended at will by 75%, this operation being controlled by means of a variable switch. The largest scale supplied has a range of 0 to 40 ft. and this allows a space of $\frac{1}{4}$ in. on the graph to 1 ft.

of water, but the same machine can be geared down so that, when running at slow speed, the range is 0 to 40 fathoms, with a scale of $\frac{1}{2}$ in. = 1 fathom. The system of phasing advances the moment of transmission of the signal by a predetermined amount before the stylus commences to traverse the chart, thus allowing additional time for the signal to reach greater depths before it is recorded. The machine can be fitted at any convenient point in the launch or boat and there is no necessity to cut the plating. Alternatively, the transmitting and receiving oscillators can be supplied in a special

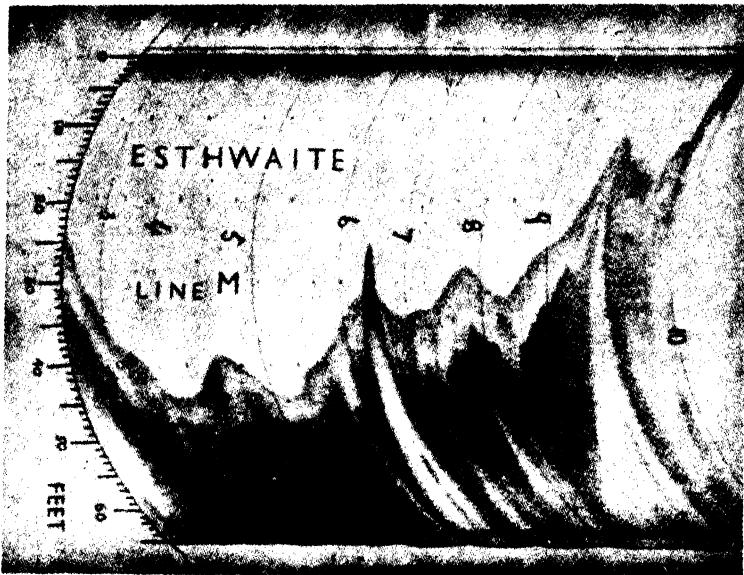


FIG. 366.

ECHO-SOUNDING RECORD FROM SURVEY OF ENGLISH LAKES, SHOWING TOOTH OF ROCK PROTRUDING THROUGH TWO LAYERS OF OTHER MATERIALS.

(By courtesy of Dr. E. B. Worthington, Freshwater Biological Association, and Messrs. Henry Hughes & Son, Ltd.)

mounting which can be fitted outside the ship. This is often a convenience in survey work or in boats navigating rivers, as the mounting can be fitted at the end of a boom in the bow so that oscillators can still be used in positions which the boat itself cannot reach.

In the machine illustrated, a button can be pressed at any instant when observations are being taken to fix the position of the boat, and this causes a line to be marked on the record chart.

These "fixes" can be numbered for identification purposes by means of an "electric contact pen" without interrupting the record.

Different types of echo-sounding instruments are also manufactured by Marconi and other makers.

With certain kinds of oscillators, two separate echoes are often received when the bottom consists of soft material, such as mud or sand, lying on top of rock or other hard material. Consequently, in this case, a record is obtained not only of the surface of the mud or sand, but also of the surface of the underlying bed of harder material. The usefulness of such a record when foundations for bridges, docks, wharfs, as well as dredging contracts, etc., are involved is obvious. Fig. 366 is a reproduction of a record from an echo-sounding survey of the English Lakes, made by the Freshwater Biological Association, which shows a tooth of rock protruding through two layers of other materials.

Another special use to which the apparatus can be put is to locate the position of wrecks. The record will show the position of the wreck and it may also give indications of any scouring action of the sea bottom, due to currents in the neighbourhood of the wreck.

Echo sounding has now established itself as an invaluable aid to the hydrographical surveyor and is in extensive use by many harbour, river conservation and improvement authorities all over the world. It has the following advantages over ordinary sounding methods :

- (1) It is more accurate than the lead line, as a truly vertical sounding is obtained, and swiftly moving water, or the motion of the vessel, does not cause it to be deviated appreciably from the vertical.

- (2) It can be used in strong currents or streams, where accurate soundings with the lead line are almost impossible.

- (3) It is, as a rule, more sensitive than the lead line method.

- (4) It can be used on days, or in weather, when the ordinary lead line method would be impossible.

- (5) It provides a continuous record of the bottom as the ship moves forward and this record can be examined at leisure later.

- (6) It often enables information concerning the sea or river bed, which could not be obtained by ordinary simple means, to be obtained simply and cheaply.

- (7) It is much more rapid in use than the ordinary methods.

For further particulars regarding echo sounding, including a description of the working of a super-sonic receiving machine, see a paper by Commander P. S. E. Maxwell, R.N., in the *Empire Survey Review*, Vol. III, Nos. 17, 18, and 19 (July 1935, October 1935, and January 1936). Also, one by Capt. J. F. Hutchins, D.S.O., O.B.E., R.N. (ret.) on "Echo-Sounding Equipment for Ships" in the *Transactions of the Institution of Engineers and Shipbuilders in Scotland*. 1936. (Paper No. 951).

THE SURVEY OF TIDAL CURRENTS

Observations of the direction and velocity of tidal currents have to be undertaken by the engineer in connection with certain harbour and coast protection projects. They are also required to aid in the selection of suitable points at which to discharge crude sewage into tidal waters, to ensure its being carried seawards. Such observations must be made at all states of the tide, and a complete set should therefore extend over a series of spring and neap tides.

Methods.—The most satisfactory method of ascertaining the direction and velocity of the tidal stream over an area is by immersing floats, which drift with the current, and are located from time to time. If, however, it is required to determine the characteristics of a current at a particular point, the measurements may be made by current meter (page 580). A form of meter suitable for marine work is fitted with a compass which can be locked before the meter is brought up, and which registers the direction of the current. Description of meter observations is reserved for page 582.

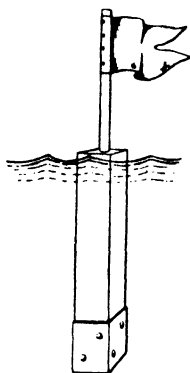


FIG. 367.
ROD FLOAT.

Floats.—The requirements of a float for tidal current observations are that (1) it should be carried along by the sub-surface current, and therefore should present as little surface as possible to the action of wind and waves, (2) it should be easily identified from a distance.

Fig. 367 shows a wooden rod float, 3 or 4 ft. in length, and weighted at the bottom with sheet lead. The double float of Fig. 368 consists of a surface

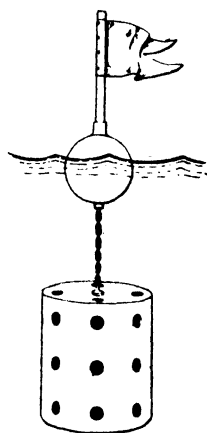


FIG. 368.
DOUBLE FLOAT.

float from which is suspended a perforated cylinder. The float and cylinder are of sheet iron, and the chain or wire connecting them is made adjustable in length so that the direction and velocity of a current at various depths may be indicated. A pail is a useful substitute for the cylinder shown. It may be loaded with stones until the float is nearly submerged. Flags attached to floats should not be larger than is strictly necessary: different colours should be used as a means of distinguishing the various floats of a series.

Surveying Course of Float.—The position of a float at various points of its course can be measured: (1) by two angles from a boat

TIDAL CURRENT OBSERVATIONS AT.....

Date.....

.....Surveyor.

.....Recorder.

Flood Tide.

Float.	When Immersed.	Tide Gauge.	Station.	Angle.	Station.	Angle.	Station.	Time of Observa- tion.	Time between Observa- tions.	Distance Travelled.	Speed of Current.	Remarks.
No. 3 Red 6 ft. long	a.m. 8.45							a.m. 8.45	h. m.	Feet.	Knots.	Sea smooth Gentle breeze S.E.
			B's Ch. Stk.	60° 56'	Lt. ho.	61° 47'	Cairn	9.25	0 40	1,460	0.36	
			do.	Check	do.	79° 52'	Farm ho.	10.42	1 17	3,400	0.44	
			Cairn	23° 14'	do.	73° 22'	Cairn	11.45	1 3	2,430	0.38	
			Farm ho.	75° 2'	Farm ho.	36° 7'	Pole	p.m. 12.10	0 25	550	0.22	
			do.	47° 8'	Pole	34° 0'	Hut					
			do.	45° 30'	do.	37° 53'	do.					
				Check	Farm ho.	83° 27'	do.					
				42° 0'	Pole	36° 2'	do.	12.29	0 19	240	0.13	
				39° 56'	do.	32° 3'	do.	1. 0	0 31	550	0.17	
				Check	Cairn	82° 2'	Pole					Float with- drawn

FIG. 369.

alongside the float; (2) by an angle from the boat and one from the shore; (3) by two angles from the shore. The first method is the most satisfactory, and is that usually adopted. The others are not recommended, as it is difficult to secure simultaneous observations unless the sea is very smooth.

If the object of the survey is to ascertain the characteristics of the current at a particular point, successive floats are placed in the water on the upstream side of the point at intervals of about half an hour. If the survey is to extend over a considerable area, the floats must be placed at different distances from the shore. In either case, the boat or launch takes up a position alongside each in turn, and sextant angles to three shore objects are observed at each position, the time of each observation being noted. When the floats are moving steadily, it is sufficient to observe their positions at about 1 hour intervals, but at the time of change of current they should be located every 15 minutes or so to define clearly the period of flood or ebb tide.

Note-keeping.—The notes will include not only the results of the sextant observations, but also a record of the wind, weather, and tidal conditions during the survey, as these are necessary for the interpretation of the results. Fig. 369 shows a form of field book provided with columns for the entering up of the computed velocities. A separate page is reserved for each float being followed. The appropriate tide gauge readings are subsequently entered from the gauge reader's notes.

Plotting and Reducing.—The sextant fixes are plotted mechanically or geometrically. From the scaled distances between the points so obtained and the watch times of the fixes, the speed of the current from point to point is computed, the results usually

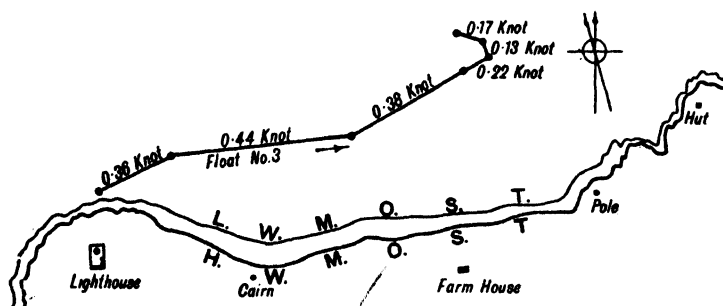


FIG. 370.

being expressed in knots (of 6,080 ft. per hour). The speeds are recorded in the current book, and are also written up in plan as shown in Fig. 370. It is useful to have the tide curves for the period of the observations plotted for reference.

STREAM MEASUREMENT

The measurement of stream discharge is an operation which falls to be performed by the engineer in connection with the design of water supply, irrigation, and power schemes. For certain special purposes a single measurement may be all that is required, but, since stream flow fluctuates from day to day, costly mistakes may occur by basing designs on insufficient observations. A complete investigation should extend over a considerable period, and should include measurements of the maximum and minimum flow and the duration of each at a time.

Methods.—The methods of measuring the discharge of a stream at a place may be classed as follows :

- (1) By measuring the area of cross section of the stream at the place and determining the mean velocity of flow.
- (2) By introducing a weir or dam across the stream and observing the head.
- (3) By chemical means.

The results are most commonly expressed in cubic feet per second or gallons per minute or per day.

Gauges.—For the comparison of measurements made at different times, the result of an observation should be accompanied by a statement of the stage of the stream in terms of the elevation of its surface at the place. This is obtained from the reading of a gauge erected at the discharge station and connected by careful levelling to a permanent bench mark.

In its simplest form the gauge may consist of a stake firmly driven into the bed and carrying a nail of known level from which measurements are made to the water surface. Otherwise, the gauge takes one of the forms described on page 539, but with the graduation sufficiently close to enable readings to be estimated to 0.01 ft. if required. The float gauge is particularly useful. Oscillations of the water are most effectively damped out by admitting the water to the float chamber through a cock, and refined readings may then be made against a vernier fixed on top of the chamber. The self-registering form is sometimes used to obtain a continuous record of stage. The most precise reading is possible by the use of the hook gauge (Fig. 371), which is employed in refined observations for the measurement of small heads over weirs. The sharp-pointed hook is carried by a scale which can be moved relatively to a fixed index and vernier by means of a screw. To take a reading, the hook is lowered into the water, and is then



FIG. 371.
HOOK
GAUGE.

slowly raised by the screw until the point just touches the surface. The reading can be estimated to the nearest 0.001 ft., if the water surface at the hook is thoroughly protected from wave action.

AREA-VELOCITY METHOD OF DISCHARGE MEASUREMENT

To avoid abnormal results, the site at which the measurement is to be made—called the discharge station—must be selected so that the stream lines are as regular as possible. The stream should be nearly straight for some distance on either side of the station, and the channel should be free from obstructions, and have its bed fairly uniform in shape and character.

The cross section is measured by ordinary levelling or by sounding. Soundings are located by cross rope or by means of a steel tape stretched across the stream, and are taken at intervals of from 2 ft. upwards. If the width is great, location by the methods used in offshore sounding (page 553) will be required. It is advisable to check the results by making the measurements twice.

The average velocity of the current is obtained :

- (1) By means of floats.
- (2) By current meter.
- (3) By calculation from the measured inclination of the surface.

Distribution of Velocity.—The velocity of the stream lines varies throughout the cross section in a manner depending upon the shape of the section, the roughness of the bed, and the depth of water. A typical distribution of velocity over the cross section is illustrated



FIG. 372.

in Fig. 372 by means of curves of equal velocity. The minimum velocity occurs at the bed, due to the retardation produced by friction: the maximum velocity occurs a little below the surface and away from the banks.

Fig. 373 shows the typical manner in which velocity is distributed along a vertical. Observation of numerous streams shows that the curve approximates to a parabola the axis of which is horizontal and coincides with the stream line of maximum velocity. This line is usually situated between the surface and 0.3 of the depth at the vertical, and approaches the surface as the depth decreases. In the case of moderately smooth channels, the mean velocity along the vertical has a value of from 0.7 to 0.95 of that of the surface,

the coefficient increasing with increase of velocity and depth. The ratio is less than 0.7 for channels having very rough beds or much obstructed by weeds. The stream line having a velocity equal to the mean velocity in the vertical occurs at about 0.6 of the depth. The mean velocity is also closely given by the mean of the velocities at 0.2 and 0.8 of the depth. These figures are applicable to normal cases only. The distribution of velocity is changed if the stream is covered with ice or if the water is being drawn through a sluice.

The distribution of surface velocity or of mean velocity in the verticals from one side of the stream to the other depends upon the shape of the cross section. If the bed were very regular, the variation would approach that of a parabolic law with the maximum velocity in mid-stream. Practically, the distribution of velocity across a stream cannot be satisfactorily predicted. In order to measure the mean velocity throughout the cross-section by floats or meter, it is therefore necessary to divide the area into a number of sections bounded by verticals and to measure the mean velocity in each.

Floats.—Floats may be classified as (a) surface floats, (b) double or sub-surface floats, (c) rod floats.

Surface Floats.—Surface floats are most commonly employed, but they should not be used in wind. They are particularly suitable for rough determinations and for gauging streams in high flood, when the use of other methods is difficult. Where the course of the float will be easily seen, it is sufficient to use a flat piece of wood or a corked bottle weighted so that the surface exposed to the wind is not too great. In circumstances where such objects are difficult to observe, a larger sealed vessel carrying a flag should be used.

Double Floats.—The form of double float of Fig. 368, as employed in sea work, is suitable for use in deep rivers, but the chain should be replaced by a thin wire to present a minimum of area to the action of stream lines of different velocity from those at the depth of the submerged float. Even so, when the sub-surface float is adjusted to the required depth, such as that of mean velocity, the indications are likely to be affected by the speed of the surface float and by the exposed area of the wire when the depth is great. Such floats are unsuitable for use on small streams.

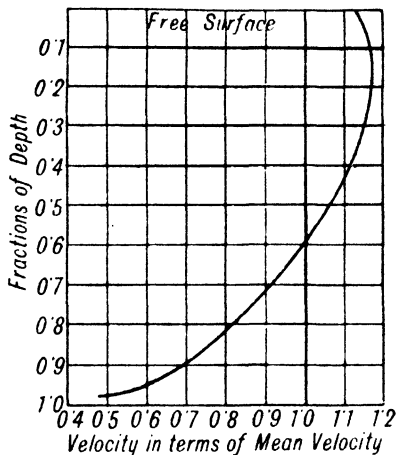


FIG. 373.

Rod Floats.—These consist of wooden rods or hollow tubes weighted at the bottom so that they float vertically with only a short length exposed above the surface of the water. The total length is such that the clearance between the lower end and the bed of the stream is as small as can be secured without danger of fouling the bottom. A number of rods of different lengths is required to suit the various depths across the stream. Rod floats are intended to indicate directly the mean velocity in the vertical, but are likely to give too large results because, on account of the necessary clearance at the bottom, they are not acted upon by the stream lines of least velocity. This error is greatest, and rod floats are unsuitable, when the bed is irregular, since the clearance is then excessive over part of the run. Francis investigated the effect of clearance in the case of a rectangular channel, and deduced the formula,

$$v_m = v_r \left(1.012 - .116 \sqrt{\frac{c}{d}} \right),$$

where v_m = the mean velocity in the vertical of the rod,

v_r = the velocity of the rod,

c = the clearance,

d = the depth of the stream.

Rods are likely to give better results than other types of float. They are very satisfactory if the bed is smooth, as in artificial channels.

Measurement by Floats. Field Work.—To measure velocity by means of a float, observation is made of the time it takes to travel a measured distance down the stream. A base line is set out on one bank as nearly as possible parallel to the axis of the channel and of length from 50 to 300 ft. At each end a line is set out across the stream at right angles to the base, and is marked by ranging poles or, in narrow streams, by a rope. The cross section of the stream is measured at each of these ranges, and, if the base length exceeds 100 ft., intermediate cross sections are taken at equal intervals of 50 or 100 ft. The area to be used in computing the discharge is the average area over the length of the run.

To determine the mean velocity, a number of float observations is required at different points across the width of the stream. When the ends of the run are marked by ropes, distances from one bank are conveniently marked by means of equidistant tags affixed to the ropes. When rod floats are used, it will be necessary to select lengths to suit the depth of water in which each run will be made. In the case of double floats, the wire must be adjusted so that the centre of the immersed float is situated at 0.6 of the depth. The float is placed in the water at some distance above the upper range, and, when it crosses that range, the time is taken, and note is made of the position of the float relatively to the tags. The same observations are made when it reaches the lower range. The time of

the run is sometimes taken by stop watch. The operations are repeated with the float at various positions from one bank to the other.

If the stream is too wide to be spanned by a rope, the passage of the float across a range is timed by sighting along the marking poles. For the greatest accuracy a theodolite is used. The distance of the float from the bank as it crosses each range is determined by angular observation. A theodolite or other angular instrument is used at the mid-point of the base line or at any other point whose position is obtained from the base line and from which good inter-sections are available on the ranges. As the float approaches the upper range, the line of sight is kept upon it, and, when it crosses the range, an assistant stationed there makes a signal, at which the bearing of the float is read. Both observers should take the watch time. The assistant then proceeds to the lower end of the base, and the float is again located and timed at its passage across the second range. Alternatively, a theodolite is set up at each end of the base. The observer at the upper one signals the instant of the passage of the float across that range, and the other locates it as before. At the lower range the duties of the observers are reversed. In the case of very wide rivers, the location of floats is best performed by sextant observations from a boat as in tide work.

Measurement by Floats. Calculation of Discharge.—The mean cross section throughout the run is first determined from the results of the section measurements at the upper and lower ranges and at the intermediate positions, if any. Since the width of the stream should be nearly constant over the base length, the same number of observations usually defines the form of the bed at each cross section. Each cross-sectional area is therefore divisible into the same number of trapezia, the parallel sides of which are the observed depths, and their width the constant distance between the soundings. Triangles may take the place of trapezia at the sides, and the width of these may vary from one section to another. The dimensions of the partial areas for an average cross section are readily obtained by averaging the dimensions of the corresponding areas of the individual sections.

The next step is to obtain the mean velocity throughout each of those partial areas. The velocity of a float is the distance between the ranges divided by the time of run, notwithstanding that the path of the float may not be parallel to the base line. In the case of a surface float, the mean velocity in the vertical throughout its path is obtained by multiplying the velocity of the float by a coefficient of from 0.7 to 0.95 according to the conditions (page 576), but surface floats should not be used unless the value of this factor can be carefully determined, preferably by means of a current meter. The velocity thus derived represents the mean velocity in the average cross section along a vertical whose position in the width

of the stream is the corresponding mean position of the float throughout its run. This is obtained by averaging the observed positions of the float at which it passed the two ranges.

On a base representing the width of the stream the average positions of the different runs are set off, and from them ordinates are erected to represent the mean velocities given by the floats. The resulting curve exhibits the variation of mean velocity across the stream, and is utilised to interpolate the value of the mean velocity for each partial area. These are represented by the ordinates corresponding to the positions of the centroids of the areas, which in the case of trapezia may usually be taken with sufficient accuracy at the middle of their width. The product of each partial area by its mean velocity gives the discharge through each section, the sum of which is the discharge of the stream.

The Current Meter.—As a means of measuring the velocity of flowing water, the current meter (Fig. 374) proves more convenient

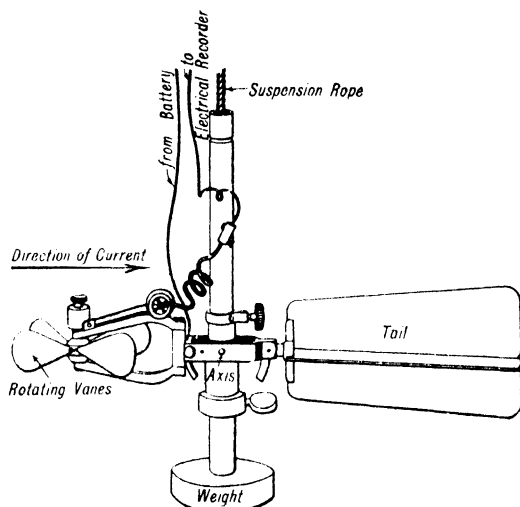


FIG. 374.—CURRENT METER.

than floats, and with careful usage gives better results. Various forms differ considerably in detail, but the instrument consists essentially of a spindle mounted on a fork and carrying a wheel with cup-shaped or helical vanes, which is rotated by the action of the flowing water. The spindle is vertical in some forms and horizontal in others, and is constructed to rotate with a minimum of friction. The number of revolutions made by the wheel during the time the meter is in operation is recorded by gearing or other means. The instrument is suspended by a rope, wire, or jointed

tube, and the weight of the mechanism is balanced by a two- or four-bladed tail, which keeps the instrument facing the current. A lead weight may be fixed at the bottom to assist in maintaining the meter in position. The weight is sometimes made torpedo shaped, and may be provided with a rear blade. Provision is made for the attachment of additional weights when the instrument is used in swift currents.

Several methods are employed for counting the revolutions of the wheel. In a simple form of meter the number of revolutions is shown on a dial on the instrument itself. When the meter is immersed at the required depth, the recording mechanism is thrown into gear by the operator pulling a cord. After a noted interval it is released by a similar pull, and the meter is then drawn to the surface and read. It is usually more convenient to be able to note the revolutions without having to pull up the meter at every observation, and this is effected in various ways. In the acoustic type the gearing is so arranged that a tap is made at every fifth or tenth revolution, and the sound is communicated to the observer through a tube. In the electrical type the revolutions are indicated by a sounder consisting either of a telephone receiver or a buzzer. For prolonged observations and high velocities electrical registration is most convenient, the revolutions being recorded upon a dial or dials above the surface.

Rating Current Meters.—Since the results given by meter represent the number of revolutions in a given time, it is necessary to know the relationship between the velocity of the current and the number of revolutions per second made by the meter. This ratio is obtained by rating the meter.

Rating is performed by running the meter at a uniform speed through still water over a measured distance and noting the number of revolutions and the time taken. Runs are made at speeds varying from the least which will cause the wheel to rotate to the greatest likely to be encountered in gauging. At specially equipped laboratories or rating stations the meter is suspended from a trolley, which is run at a sensibly uniform speed. In ordinary practice, rating is usually performed by hanging the meter from the prow of a dinghy, which is towed repeatedly along the course at different speeds.

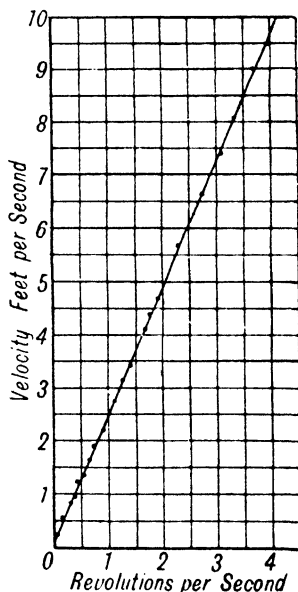


FIG. 375.—METER
RATING CURVE.

On plotting the mean velocities of the runs against the revolutions per second, a curve similar to that of Fig. 375 is obtained. For all meters the curve is practically a straight line, and is taken as such. The slope depends upon the type of meter. The ordinate corresponding to zero revolutions represents the velocity of current required to overcome initial friction and start the wheel. From the rating curve the velocity corresponding to any observed number of revolutions per second can readily be obtained.

In place of using the curve directly in the reduction of observations, it is more convenient to prepare a rating table giving velocities corresponding to revolutions per second. This is compiled from the equation,

$$V = c_1 R + c_2,$$

where V = the velocity of the current in feet per sec.,

R = the number of revolutions per sec.,

c_1, c_2 = constants, the former representing the slope of the curve, and the latter the ordinate for zero revolutions.

The constants are obtained from the curve, or their most probable values may be computed directly from the rating observations.

Use of Current Meter.—In measuring discharge from velocities taken by current meter, one cross section only is required. It is divided into partial areas as before, and the meter is used at different points across the width to give the mean velocity over each area. The velocity determinations are made either by using the meter in the verticals of the centroids of the partial areas or by taking observations at any points across the width, fixing their positions by angles or on a cross rope, and interpolating the values for the partial areas.

If the water is sufficiently shallow, observations can be secured by wading, the meter being fixed to a graduated rod. If there is a bridge conveniently near, and situated on a straight reach of the stream suitable for a discharge station, the meter should be suspended from successive points along it. Otherwise, the observations are made from a boat, or, if repeated measurements are required, a cableway may be thrown across the stream, and observations taken from a suspended car or platform. When a boat is used, the meter is attached to a rod or tube which is held vertically and clear of the prow. When the depth is considerable, the meter is suspended by a wire or rope, preferably from an outrigger provided with a pulley. The boat is headed upstream, and attention must be paid to keeping in the range at each observation. Unless the width is too great, the boat is brought into position and held against the current by ropes from each bank.

In taking an observation with the meter, the sounding is first taken, and the meter is then lowered to the required depth. If the meter is one in which the recording mechanism is thrown into

and out of gear by a cord, the interval between the two pulls is measured by means of an ordinary or stop watch and entered against the revolutions shown by the dial. With acoustic or electric meters, the count may be taken over two periods of one or two minutes each.

The object of the meter observations is to ascertain the mean velocity in the verticals at various distances across the width of the stream. Different systems of measurement are available, according as one or several observations are made in each vertical or the integration method is adopted.

Single Observations.—Single observations are taken either at the surface or at the depth of mean velocity. Surface measurements are not usually made in normal cases, but the method is useful in swift currents and in times of flood, when it is difficult to maintain the meter in a desired position much below the surface. In taking the observation, the meter is held six inches or more below the surface in order that it may be wholly submerged and below the wave line. The mean velocity in the vertical is deduced by multiplying the observed result by a coefficient having an average value of 0.9.

Mean velocity is obtained directly by immersing the meter to a depth of 0.6 of the depth of the vertical. This method is commonly used in ordinary circumstances, and, although the ratio, 0.6, is only an average value, the accuracy of the results is sufficient for most practical purposes.

Multiple Observations.—The best results are obtained by making measurements at a sufficient number of points in the vertical to enable the velocity curve to be plotted similarly to Fig. 373. The area contained between the complete curve and the depth axis divided by the total depth equals the mean velocity. This method is too laborious for ordinary measurements, but is that required in the evaluation of coefficients for the reduction of single point observations.

Good results are obtained by two observations in each vertical. These should be made at 0.2 and 0.8 of the depth of the vertical, the mean of the results giving a close approximation to the mean velocity.

Integration Method.—In this method the meter is moved slowly and at a uniform rate along the vertical from the surface and back, the number of revolutions and time being observed. The meter is thus exposed to all the velocities in the vertical, and these are integrated and averaged mechanically in the result. Certain types of meter are not quite suited to the method since the vertical motion, unless very slow, causes the wheel to revolve. The correction to be applied to the indications of such meters may, however, be ascertained by moving the meter vertically at the same uniform speed through still water.

The integration method is sometimes extended to give the mean velocity over an entire cross section. The meter is moved up and down at an angle of about 45° to the horizontal from one bank to the other and back (Fig. 376). This system is suitable for the rapid gauging of streams of moderate depth.



FIG. 376.

Measurement by Current Meter. Calculation of Discharge.—Except when the method of Fig. 376 is employed, the total discharge is obtained as the sum of the discharges through the partial areas into which the soundings divide the cross section. The mean velocities over those areas, if not directly observed, are derived by interpolation in the same manner as described for float measurements (page 579). In the case where the integration method is applied to give the mean velocity for the entire cross section, the discharge is simply the product of the observed velocity by the cross-sectional area.

Thrupp's Ripple Method.—If a high degree of accuracy is not required in discharge measurements, rapid determinations may be made from surface velocities observed by the method described by Mr. E. C. Thrupp.* It is based upon the circumstance that, if a small obstruction is placed in the surface of a stream, ripples are formed if the velocity exceeds about 9 in. per sec., and, as the velocity increases, the angle between the diverging lines of ripples becomes more acute. To afford a simple means of measuring the rate of divergence, Mr. Thrupp used two 3-in. wire nails—about $\frac{1}{8}$ in. in diameter—at a fixed distance d apart (Fig. 377), and found that the velocity could be derived from the distance l from the base line so formed to the point of intersection of the last ripples. He obtained for the velocity in ft. per sec.

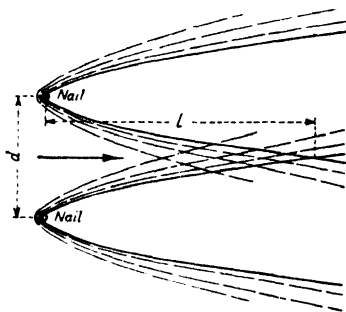


FIG. 377.

$$V = 0.40 + 0.206 l, \text{ for } d = 6 \text{ in.},$$

$$V = 0.40 + 0.28 l, \text{ for } d = 4 \text{ in.},$$

where l is measured in inches.

The method would appear to be quite as accurate as that of surface floats and much more convenient.

Velocity by Formula.—In this method of estimating discharge—sometimes known as the slope method—measurement is made of the average cross-sectional area of the stream on a straight

* *Min. Proc. Inst. C.E.*, Vol. CLXVII, p. 217.

reach as well as of the longitudinal inclination of the water surface. To obtain the mean velocity of the stream, the formula in general use is that deduced by Chezy, *viz.*,

$$V = c\sqrt{ri},$$

where V = the velocity in ft. per sec.,

r = the hydraulic mean depth,

i = the inclination of the water surface,

c = a variable coefficient.

The hydraulic mean depth, or hydraulic radius, is the cross-sectional area of the stream divided by the wetted perimeter or length of bed under water. The inclination is the ratio of the fall in a measured distance to that distance. The value of the coefficient c depends principally upon the roughness of the bed, but also upon the inclination and hydraulic mean depth. In practice, the value of c is commonly derived from tables or diagrams based upon the formula of Kutter and Ganguillet, *viz.*,

$$c = \frac{41.65 + \frac{1.811}{n} + \frac{0.00281}{i}}{1 + \left(41.65 + \frac{0.00281}{i}\right) \frac{n}{\sqrt{r}}},$$

in which the coefficient n depends upon the character of the bed. The value of n varies from 0.020 for irrigation canals with well-trimmed bed in perfect condition to over 0.035 for canals in very bad order with much weed and stones. For rivers very uniform in alignment, slope, and cross section, and with a smooth sandy or gravel bed, n is taken as 0.025. As the irregularities of the bed, etc., increase, n increases to 0.035, and may reach 0.050 if there is an excessive amount of weed or in the case of torrents bringing down much detritus.

In estimating discharge by formula, a straight reach of river should be selected with as nearly uniform a cross section as possible. The fall and the distance between the points at which it is measured should be sufficiently great that the inclination can be determined without serious error. Cross sections are taken at intervals along this distance, and an average section is deduced. The slope of the water surface is measured by simultaneous readings of gauges placed one at each end of the reach and similarly situated with respect to the current. Gauge readings are taken to 0.01 ft., and the zeros of the gauges are connected by careful levelling.

The results obtained by the slope method are inferior in precision to those in which the velocity is actually observed, principally on account of uncertainty in assigning a suitable coefficient in the formula and in measuring the slope. The method, however, is useful in making an isolated rough estimate of flood discharge from the flood marks left on the banks.

WEIR METHOD OF DISCHARGE MEASUREMENT

A weir is an artificial barrier built across a stream, which flows over it in a cascade of definite form, from the dimensions of which the discharge is computed. The weir method is specially applicable to the gauging of small streams when accurate results are required. The cost of construction usually prohibits its use on large streams, but in such cases existing dams are sometimes utilised in a similar manner.

Varieties of Weirs.—The various forms of weirs differ in certain particulars affecting the rate of discharge over them.

Crest.—Weirs constructed specially for the measurement of stream flow are of the sharp-crested type. In these, the crest, or edge over which the flow takes place, is virtually a line. In the case of dams or broad-crested weirs, the water passes over a surface, and a different condition of flow obtains.

Shape of Opening.—The opening or notch through which the water flows may be of rectangular, triangular, trapezoidal, or stepped form (Fig. 378). The rectangular weir is that most commonly used in stream gauging, the length of the notch being at least three times the head of water over it. Other forms provide for an increase in the breadth of the flow as the discharge increases. The triangular or V-shaped notch has the merit of giving a constant shape to the issuing stream at all heads. It is well adapted for the accurate measurement of small discharges, but is not so suitable as the rectangular form for use in very shallow streams. The trapezoidal form with side slopes of 1 in 4 was proposed by Cippoletti with a view to balancing by the increase of breadth upwards the loss due to increased contraction with increased head. The stepped form is designed to yield good measurements in dry weather and in flood. The centre notch is sufficiently small that the minimum flow can be measured with greater accuracy than is possible with a very small head over a long crest.

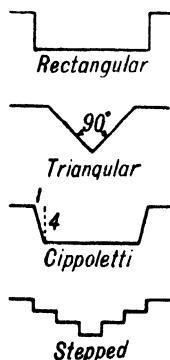


FIG. 378.
FORMS OF
WEIRS.

End Contractions.—The form of the stream issuing from a rectangular notch depends greatly upon the position and size of the opening relatively to the cross section of the channel on the upstream side. In Fig. 379, which represents the most common arrangement, the breadth of opening or length of crest is less than that of the stream. The stream lines at the sides are sharply curved, with the result that the stream is contracted in width just after passing over the notch. Such a weir is said to have end contractions. When, as in Fig. 380, the breadth of the weir is the same as that of the

approaching stream, the weir is without end contractions. Intermediate conditions arise when insufficient room between the sides

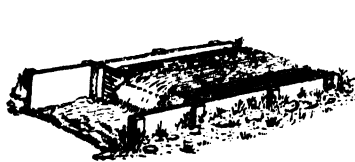


FIG. 379.

WEIR WITH END CONTRACTIONS.

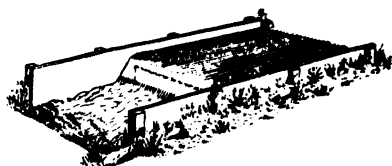


FIG. 380.

WEIR WITHOUT END CONTRACTIONS.

of the notch and those of the stream prevent the proper development of the curved stream lines, and the end contractions are then incomplete, a condition to be avoided in gauging.

Fig. 381 illustrates diagrammatically the normal form of the issuing stream or nappe in longitudinal section. The stream lines near the crest are curved upwards causing a vertical contraction, which is completely developed when the distance d is greater than $2H$. When the height of the crest is so small relatively to the head of water over it that the tail water is higher than the crest, the weir is known as a drowned or submerged weir.

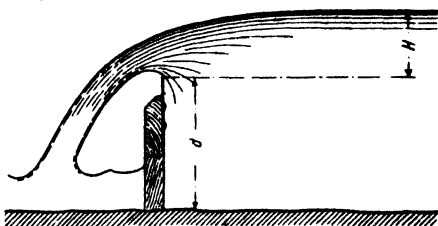


FIG. 381.

Construction of Weirs.—The site for a weir should be selected so that on the upstream side the channel is straight and free from great irregularities in order that the water may approach the weir with as uniform velocity as possible. The weir will be constructed of stout uprights and planking or tongued and grooved boarding, and the wall must be vertical and normal to the flow. Sheeting may have to be used to prevent leakage below or round the sides. The timbers forming the crest and sides of the notch are chamfered on the downstream face, the edge not exceeding $\frac{1}{4}$ in. in breadth. Otherwise, an $\frac{1}{4}$ in. thick metal strip is fixed, as in Fig. 386. The crest, in other than V notches, should be set accurately horizontal. If the weir is one with end contractions, these should be complete, so that the formulæ based on that condition may be strictly applicable. This is ensured by making the distance between the sides of the upstream channel and the sides of the opening not less than twice the head. The use of the standard formulæ also assumes that the nappe is free from contact with the weir below the crest, to secure which there must be free admission of air under the nappe.

Measurement of Head.—The dimensions of the weir being known, it is only necessary to measure the head of water over it to enable the rate of discharge to be computed. By the head is meant the difference of level, H (Fig. 381), between the crest, or the bottom of the V in the triangular form, and the surface of the sensibly still water several feet above the weir and beyond the influence of the depression of surface caused by it. This upstream distance should always exceed $3H$.

The measurement must be made with great care, more especially when the head is small. It is generally made up by means of a stake driven into the bed at the still water and with the top at the level of the crest. The head is then measured with a thin-edged scale or steel rule. The accuracy of the observation is improved if the head is measured not in the flowing stream but in a small gauge pit in communication with the stream. The connecting pipe, of 1 or 2 in. diameter, must have its end at the stream placed at right angles to the direction of flow and at an adequate distance from the weir. For the best results the hook gauge is used, the instrument being accurately set so that at zero reading the hook is at the same level as the crest. Various forms of float gauge are also used, and recording mechanism may be added so that the varying level of the float can be plotted automatically.

Discharge Formulæ.—There are several formulæ available for computing the rate of discharge over weirs, and the reader is referred to text-books on Hydraulics for a discussion of the subject. The simplest and most commonly used formulæ are those of Francis.

Rectangular Weirs.—For a sharp-crested rectangular weir without end contractions, and neglecting the effect of the velocity of the approaching water, Francis obtained

$$Q = 3.33 LH^{3/2},$$

where Q = the discharge in cubic feet per sec.,

L = the length of the crest in feet,

H = the observed head in feet.

The effect of the velocity of approach of the stream is equivalent to an increase of head. This velocity, v , need not be measured directly, but is obtained approximately by dividing the approximate discharge obtained as above by the cross-sectional area of the approach channel where the head is observed. The velocity head,

$h = \frac{v^2}{2g}$, was allowed for by Francis by putting

$$Q = 3.33L[(H+h)^{3/2} - h^{3/2}].$$

The effect of velocity of approach is more commonly dealt with by increasing the observed head by ah , where a is an experimental coefficient having a value between 1.2 and 1.68, and which may be taken as 1.5, so that

$$Q = 3.33L(H + 1.5h)^{3/2}, \text{ approximately.}$$

From the new value of Q a closer value for v can be obtained, and a second approximation may be made for Q , but this is seldom necessary since the velocity effect is small in most cases.

For a weir with end contractions, experiment shows that each complete end contraction shortens the effective length of the weir by about $0.1H$, so that for two end contractions,

$$Q = 3.33(L - 0.2H)H^{3/2},$$

or, allowing for velocity of approach,

$$Q = 3.33(L - 0.2H_1)H_1^{3/2},$$

where $H_1 = [(H + h)^{3/2} - h^{3/2}]^{2/3}$ or $= H + 1.5h$, according to the method of treating velocity head. The velocity correction is usually insignificant in the case of weirs with end contractions, since the approach channel is of much greater sectional area than the nappe.

Triangular Weirs.—In the case of a sharp-edged notch, with apex angle θ , the discharge formula has the form,

$$Q = c \tan \frac{1}{2}\theta H^p,$$

the velocity head being negligibly small. The values of p given by different investigators range from 2.47 to 2.5, and c varies from 2.48 to 2.56. For ordinary measurements it is sufficient to adopt

$$Q = 2.5 \tan \frac{1}{2}\theta H^{2.5},$$

which, in the common case of a right-angled notch, gives the easily remembered expression,

$$Q = 2.5 H^{2.5}.$$

Correction for velocity of approach is seldom necessary, but can be made as before.

Trapezoidal Weirs.—The discharge from a trapezoidal weir is the discharge over a rectangular weir of the same length of crest, and having end contractions, plus that over the triangular notch which would be formed by the sloping sides. If, therefore, $\frac{1}{2}\theta$ is the inclination of the sides to the vertical,

$$Q = 3.33(L - 0.2H)H^{3/2} + 2.5 \tan \frac{1}{2}\theta H^{5/2}.$$

By selecting $\frac{1}{2}\theta$ to make $2.5 \tan \frac{1}{2}\theta = .666$, we have

$$Q = 3.33 LH^{3/2}.$$

Cippoletti suggested $\tan \frac{1}{2}\theta = \frac{1}{4}$, and adopted

$$Q = 3.367 LH^{3/2}.$$

Dams.—When an existing dam or broad-crested weir is to be utilised for the measurement of stream flow, the effective length and levels of the crest are measured. The dimensions of the cross section must be ascertained since the coefficient to be used in computing the discharge is derived from published results of experiments on dams of similar form. As with sharp-crested weirs, the head is measured to the surface of the still water behind the dam.

If, at the time of gauging, part of the flow is discharged through or round the dam, the quantity so discharged must be separately measured.

The accuracy of the results is much inferior to that attained with sharp-crested weirs. It depends greatly upon the selection of a suitable value for the discharge coefficient and also upon the condition of the dam, particularly as regards uniformity of crest and apron.

CHEMICAL METHOD OF DISCHARGE MEASUREMENT

This method consists in introducing into the stream, at a uniform and accurately known rate, a fairly concentrated solution of some chemical, and comparing the analyses of the water before and after its introduction. The percentage of the chemical found in the water (or the increased percentage, if the chemical is already present in the untreated water) bears the same ratio to its percentage in the solution as the volume of solution introduced per second bears to the discharge per second of the stream.

Or, let P = percentage of chemical in the solution used,
 p = percentage of chemical in the water samples,
 Q = flow of stream in cub. ft. per sec.,
 q = flow of solution in cub. ft. per sec.

Every second there is added $\frac{Pq}{100}$ cub. ft. of chemical.

Its dilution in the water samples = $\frac{Pq}{100Q} = \frac{p}{100}$,

$$\text{or } Q = \frac{Pq}{p}.$$

The success of the method is dependent upon several factors. The chemical should be one for which a very sensitive reagent is available, and a sufficient quantity must be used that the dilution may not be greater than will permit of the analyses of the water samples being made with the required accuracy. Thorough-mixing of the solution with the stream is essential. On other than very narrow streams it should be introduced at several points across the width, the flow of solution being continuous during the test. Samples of the untreated water are taken at the station where the solution is introduced, and, at some distance downstream, samples of the much diluted solution are collected from all parts of the cross section after a sufficient interval has elapsed for the flow to arrive there. The distance between the stations must be great enough to ensure complete mixing. Conditions favourable to mixing are irregularity of alignment and bed, and the distance between the stations may then be shorter than in the case of a stream with very uniform flow. The method is particularly suitable for turbulent streams with rocky beds.

CONTINUOUS MEASUREMENTS OF STREAM
DISCHARGE

Investigations of stream discharge made in connection with the design of water schemes should be continued over several years to ensure a reasonably accurate estimate of the extremes of flow. The necessary observations are made either by means of weirs or by the area-velocity method.

By Weir.—Continuous records are most easily and accurately secured by the weir method, and simply require the employment of an intelligent and reliable person to read the head daily. A single daily observation is sufficient during periods when the head is changing slowly, but, when it is subject to rapid fluctuations, several readings should be made at intervals throughout the day.* Continuity in the record of head is secured by the use of a self-registering gauge.

A continuous measurement of discharge may be obtained mechanically by means of a weir recorder. In this instrument a float rotates a cylindrical drum on which is cut a spiral groove, the curve of which is deduced from the weir formula. A lever engages with the groove and actuates a pen, which traces out a continuous graph of rate of discharge. The apparatus may also be fitted with an integrating mechanism whereby the total flow up to any time may be exhibited.

By Area-Velocity Method.—In the application of this method, the discharge is measured on several occasions by current meter or floats at times of low, average, and flood stages. With gauge heights as ordinates, and discharges as abscissæ, the observed discharges are plotted, and through the points so obtained a curve, known as the discharge or station rating curve, is drawn. This curve, or a table prepared from it, then serves to give the discharges corresponding to gauge readings taken daily by an observer or recorded by an automatic gauge. The method is dependent upon permanence of the stream bed and stability of the gauge. In streams with shifting beds the discharge curve may not remain applicable throughout a long investigation, but may require modification from time to time in the light of additional gaugings.

* For an example of the comparative results given by one and four daily readings see "Investigations Relating to the Yield of a Catchment Area in Cape Colony," by E. C. Bartlett, M.Inst.C.E., *Min. Proc. Inst. C.E.*, Vol. CLXXXVIII.

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APPENDIX I

NOTES ON SOME RECENT SURVEYING INSTRUMENTS

IN Chapter I we have described the main instruments used in surveying and have given details of basic types. Developments and improvements in instrumental design and manufacture are constantly taking place and here it is proposed to supplement the descriptions already given by others of some later instruments which embody features of special interest.

THEODOLITES

General Trends in Theodolite Design.—The general trends in modern theodolite design may be summarised as follows:—

- (1) Use of very short telescopes with large objectives.
- (2) Application of a special chemical coating to the surfaces of optical components to minimise the effects of internal reflections.
- (3) Use of glass circles combined with an optical micrometer or estimating scale for reading the graduations.
- (4) All readings, not only of horizontal and vertical circles but also of the level bubble and of the tubular compass, if one is fitted, can be taken from the one position—the eyepiece end of the telescope for both face right and face left observations—so that the observer does not have to move around the instrument when taking readings.
- (5) Many instruments are now fitted with built-in optical plummets for centering over a ground mark, thus obviating the necessity for using a plumb bob in stormy weather.
- (6) Some makers now market two alternative models of the same instrument. In one of these the upper and lower parts are not separable but in the other the upper part of the instrument can be separated and lifted out from the centering and levelling base, thus enabling it to be interchanged with special targets for use in the three-tripod system of observing.
- (7) The circles and all optical and mechanical systems are completely enclosed as a protection against dust and damp.
- (8) A large range of accessories can usually be obtained as extras when not supplied as standard fittings. These include internal electrical illumination for the cross hairs and for reading horizontal and vertical circles; diagonal eyepiece for use when the line of sight is steeply inclined to the horizontal; tubular compass; striding level; suncap with prism for illuminating the cross hairs with hand lamp at night or when working underground; wall tripod for setting instrument on a concrete pillar or baulk of timber; chemical dessicator for protecting optical components against fungus; etc. In certain cases a

device for the automatic reduction of stadia readings (Chapter XI) can be obtained as a removable extra fitting (e.g. the Stadia-Altitude usable with the Watts S.T.2 vernier theodolite).

- (9) As an alternative to the ordinary wooden case many instruments are now provided with very light, streamlined metal cases or containers into which the theodolite can be screwed or in which it can be held very firmly.

Wild Pocket Theodolite T.12.—This instrument, specially designed for building work or for cases where great accuracy of reading is not needed, is remarkable not only for its very light weight and small dimensions but also for the telescope being fixed in a vertical position with sights reflected to the telescope objective from a pivoted reflecting prism, the inclination of the prism from its neutral position giving the angle of inclination of the line of sight up to a limit of 36° .

The instrument, which is shown in Fig. 382, weighs only 2·3 lbs., or 3·5 lbs. with sliding leg tripod and plumb bob, and it packs into a cylindrical metal container, 7·5 inches high by 2·8 inches diameter—about two-thirds of the size of a pint thermos flask.

The horizontal circle is made of glass and graduated at degree intervals, readings being taken direct to ten minutes on an estimating scale and by estimation to single minutes. The eyepiece of the scale microscope is placed alongside, and parallel to, the eyepiece of the telescope. Rotation of a chromium plated ring concentric with the telescope controls the tilting of the prism, and a rough vertical reading is obtained from a sector on the right of the telescope, giving 10-degree divisions. Graduations on the chromium plated ring give the final reading direct to 10 minutes and by estimation to single minutes.

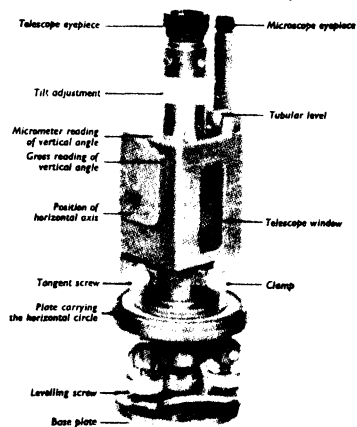


FIG. 382.—WILD "POCKET THEODOLITE" T.12.

(By courtesy of Messrs. Henry Wild & Co. Ltd.)

Wild Compass Theodolite T.O.—The Wild Compass theodolite T.O. consists of a compass built into the body of a theodolite in such a way that, when the compass circle is raised off its pivot, it presses against a rim connected to the base of the instrument so that the

circle corresponds to the horizontal graduated circle of a theodolite. Thus, the instrument can be used either as a compass or as a theodolite.

The compass itself consists of a magnetic plate on which the graduated circle is mounted. This circle is read through two separate lenses according to whether the telescope is in the face right or face left position, but the same optical micrometer is used for both compass and theodolite readings. In this micrometer, by turning a knob and graduated drum, images from opposite sides of the circle are brought into coincidence, when the reading on the circle gives the full degrees and the readings on the graduated drum the minutes. The effect of bringing images of opposite parts of the circle into coincidence is to help to eliminate errors due to circle eccentricity, a point which is of particular importance in an instrument of this kind, since, as the circle has to swing freely on a pin when it is being used as a compass, it cannot be centered as precisely as it could be if it were built in the ordinary way.

The vertical circle is of glass and is read through a microscope placed alongside, and parallel to, the telescope. A double image, representing diametrically opposite graduations of the circle, is seen in the microscope. Degrees and tens of minutes are read and single minutes estimated. Thus, readings on both circles can be taken to the nearest minute.

Fig. 383 shows the instrument with the upper part, the alidade, separated from the lower for adjusting the balance of the circle for local variations in magnetic dip.

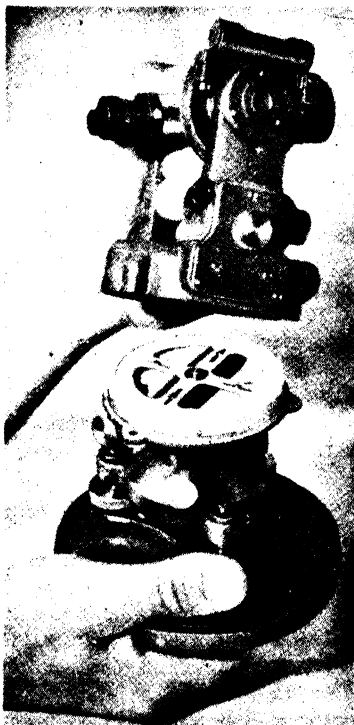


FIG. 383.—WILD COMPASS-THEODOLITE TO.

(By courtesy of Messrs.
Henry Wild & Co. Ltd.)

The Re-Designed Small Tavistock Theodolite.—The small Tavistock theodolite described in pages 78-81 has proved to be very popular and at present it is extensively used in all parts of the world. For some time past, however, the makers have had in view the production of a lighter instrument of similar accuracy and with

a modern "stream-lined" appearance. Hence, they have very recently put on the market a variation of the original model which, packed in its box, represents a saving in weight of 8 lbs. Perhaps the most interesting change to be noted is the abandonment of the "sandwich" method of reading the micrometer as used in the original model, or the method in which the images of graduations from opposite ends of a diameter of the circle are brought to appear equally spaced on either side of an index line, in favour of the "coincidence" method, or the method in which the images of the graduations from opposite ends of a diameter of the circle are made to coincide. Other changes are: (1) the provision of a single micrometer for both circles; (2) the provision of a single circle reading eyepiece placed alongside and parallel to the main telescope, a "throw over" prism determining which circle is used; (3) the incorporation of separate centres, with rotation of the circle by rack and pinion; (4) the provision of a single slow motion screw in azimuth; and (5) the use of a single mirror for illuminating both circles.

The instrument is shown in Fig. 384 and a view of the image in the eyepiece of the micrometer when the horizontal circle reading is $78^{\circ} 56' 27''.5$ is shown in Fig. 385. Here the micrometer setting screw, C in Fig. 384, is turned to bring the nearest upper and lower graduations into coincidence. The position of the coincident graduations relative to the index mark or pointer is noted. If this mark appears to be in line with the coincident upper and lower graduations, the coarse, or approximate, reading in terms of degrees and 10-minute intervals is given by the graduation on the upper scale which is directly under the pointer. If the latter comes midway between two pairs of coincident graduations, 10' must be added to the value of the graduation to the left of the pointer. The circle

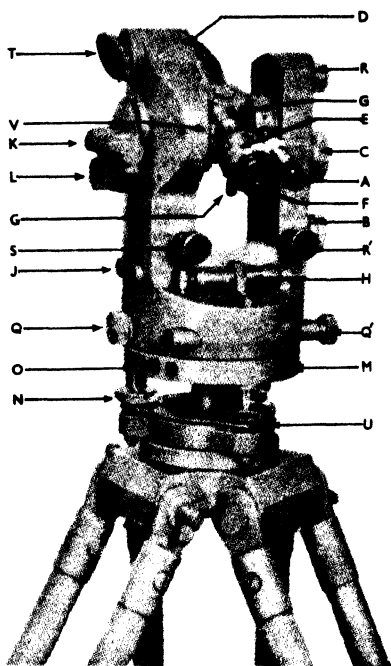


FIG. 384.—THE NEW RE-DESIGNED TAVISTOCK THEODOLITE BY COOKE, TROUGHTON & SIMMS, LTD. NO. V.400.

(By courtesy of Messrs. Cooke, Troughton & Simms Ltd.)

graduations are 20' apart and hence the coarse reading in Fig. 385 is $78^{\circ} + 40' + 10' = 78^{\circ} 50'$. Finally, the single minutes and seconds are read in the window on the right, the reading in this case being $6' 27'' \cdot 5$, and so the complete reading is $78^{\circ} 56' 27'' \cdot 5$.

The passage of the rays for the reading on the horizontal circle is shown in Fig. 386. An image at unit scale of the graduations at one end, D, of a diameter of the circle is formed at D', the other end of the diameter, by light reflected from the mirror A through the right-angled prism C, the prisms C'', E and H and the silvered surface of D. Thence the image is reflected by the silvered surface D' and the prisms J and M to pass through the parallel plate micrometer N* and, after reflection by the prisms O and U, through the mask S to the lenses T, V, W and X which form part of the

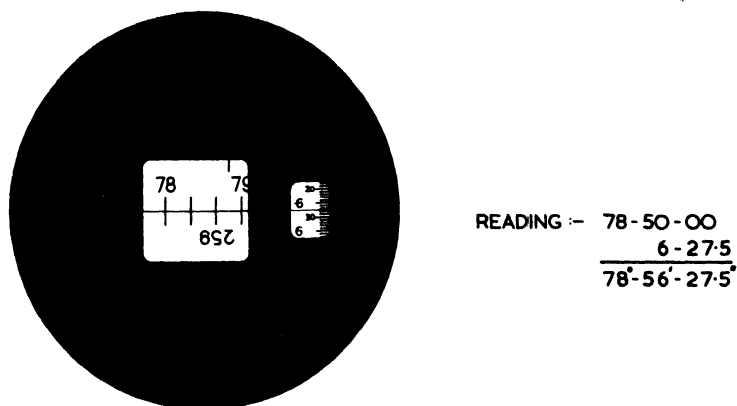


FIG. 385.—VIEW IN THE EYEPIECE OF THE MICROMETER OF THE REDESIGNED SMALL TAVISTOCK THEODOLITE.

(By courtesy of Messrs. Cooke, Troughton & Simms Ltd.)

micrometer eyepiece. At D' the image of the graduations at D does not coincide with the graduations on D' but is separated tangentially from them by a small angle as well as radially. Part of the light reflected at A illuminates the circle at D' and an image of the graduations there is reflected by the prisms J and M to pass through the parallel plate micrometer N' and thence is reflected at O and U to pass through the mask S to the micrometer eyepiece. Thus, an observer looking through the micrometer eyepiece sees in the large window two sets of graduations, one above the other, and tangentially separated by a small interval. He also sees the index

* The parallel plate micrometer is described at some length in Chapter VI of Vol. II. It consists of a sheet of glass with accurately ground flat parallel faces. Rays incident on one face emerge parallel to themselves but displaced laterally, the amount of displacement being a function of the angle of tilt of the plate from the direction of the incident rays.

mark, which is located in the prism O. The images of both sets of graduations can now be brought into coincidence as shown in Fig. 385 by turning the micrometer screw. This screw tilts the two parallel plate micrometers N and N' in opposite directions and so

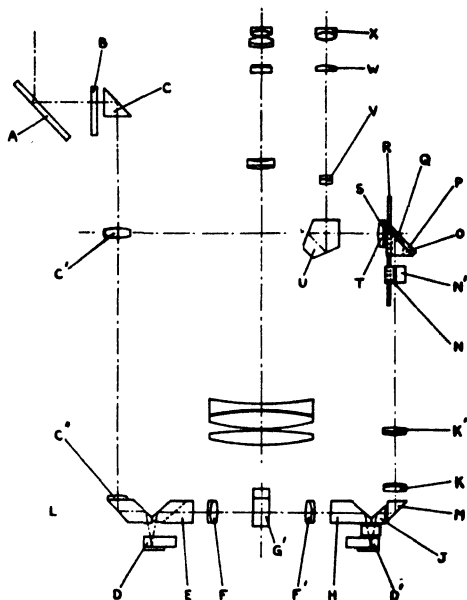


FIG. 386.—DIAGRAM SHOWING THE PASSAGE OF THE RAYS FROM THE HORIZONTAL CIRCLE OF THE NEW SMALL TAVISTOCK THEODOLITE.
(By courtesy of Messrs. Cooke, Troughton & Simms Ltd.)

enables the two images to be brought together. At the same time, the glass plate R carrying the micrometer scale moves past the fine reading aperture in the mask S, and the reading on the scale, which depends on the amount of rotation of the micrometer screw, gives the fine reading in minutes and seconds.

C. T. & S. Optical Micrometer Theodolites V.301-309.—

The Cooke, Troughton & Simms optical micrometer theodolite reading direct to 20" and by estimation to 5" is a somewhat smaller instrument than the standard Tavistock theodolite described on pages 598-601 and is supplied in four forms:—(1) as a general purpose theodolite with centering adjustment on the tripod (V.308); (2) with this adjustment located above the levelling base (V.301); (3) a similar instrument but with a means of disconnecting the body of the instrument from the levelling and centering base for use with the three-tripod system of observing (V.302); and (4) similar to the last but in which the detachable levelling and centering base

is provided with two fine motion adjustments for accurate centering (V.309). Fig. 387 shows the upper and lower parts of the V.309 model and Fig. 388 shows a target for use with the instrument in connection with the three-tripod system of observing. In Fig. 387 the upper part of the instrument is shown provided with a built-in optical plummet for plumbing over a ground mark which is obtainable as an extra, while the target illustrated in Fig. 388 is shown with a detachable bracket, also obtainable as an extra to take a miner's lamp for observations underground or at night.

In this theodolite, the circles are of glass but the micrometer

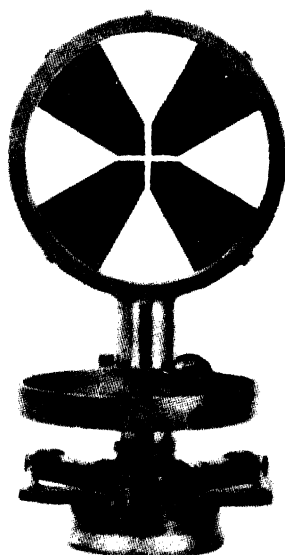
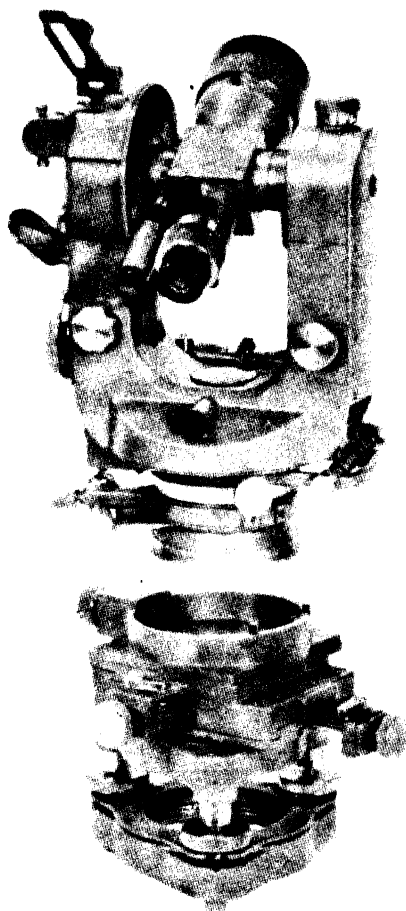


FIG. 387.—C. T. & S. OPTICAL MICRO-METER THEODOLITE V.309, WITH SEPARATE LEVELLING BASE AND SCREW CONTROLLED CENTERING.

FIG. 388.—INTERCHANGEABLE TARGET FOR USE WITH C. T. & S. THEODOLITE V.309, IN CONNECTION WITH THE THREE-TRIPOD SYSTEM OF OBSERVING.

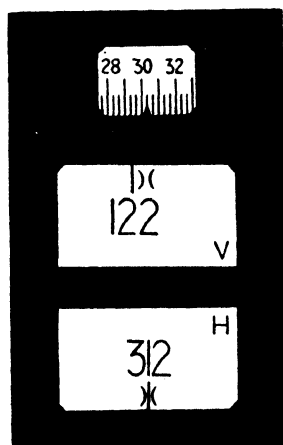
(By courtesy of Messrs. Cooke, Troughton & Simms Ltd.)

gives readings from one end of a diameter only. When readings are derived in this way from only one end of a diameter of the circle, it is very important that there should be no eccentricity in the mounting of the circle and that the centre of the graduations should coincide with the centre of the vertical axis of rotation of the instrument. If the glass used for the circle and the metal of which the instrument is constructed have different coefficients of thermal expansion, there may be a very slight movement of one axis relative to the other as the temperature changes, and this would result in small errors in the readings given on the circle. In earlier models of the C. T. & S. V. 301 theodolite this difficulty was overcome by means of the special kinematic self-centering mount described on pages 596-597 of the last edition of this book but the use of a special nickel-iron alloy in the circle mount, which has the same coefficient of expansion as the glass circle itself, has made it unnecessary to fit the special mount in the later models.

Images of the graduations of the horizontal and vertical circle graduations are brought into the field of view of a microscope which lies parallel to the telescope. Three windows are seen. The lower shows the graduations of the horizontal circle and the middle one the graduations of the vertical circle. The reading is taken by turning a milled head until the nearest degree graduation mark on the circle in use appears to be midway between the curved index lines shown in the illustration, when the minutes and seconds are read direct to 20" against an index mark in the upper window and to 5" by estimation. In the illustration, the setting is for the horizontal circle and the reading is $312^{\circ} 30' 20''$.

FIG. 389.—FIELD OF VIEW IN READING MICROSCOPE IN THE C. T. & S. OPTICAL MICROMETER THEODOLITES V.301-309.

(By courtesy of Messrs. Cooke, Troughton & Simms Ltd.)

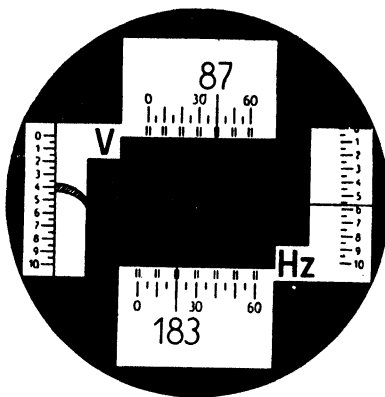


Horizontal circle reading
 $312^{\circ} 30' 20''$

Zeiss-Opton Theodolite Th.3.—The most interesting feature in the Zeiss-Opton Theodolite Th. 3 which differentiates it from ordinary optical scale or optical micrometer theodolites of a similar type is the coupling of the level bubble of the vertical circle to the micrometer for that circle, so that one end of the bubble acts as an

index line in the micrometer, the result being that no correction to the micrometer reading for dislevelment of the vertical circle bubble is needed and it is only necessary to level up this bubble to within $\pm 2'$. Another feature is a special repetition clamp for quick orientation of the scale when angles are being measured by repetition. This is useful for accurate distance measurements with a subtense bar (page 508).

The field of view in the reading microscope is shown in Fig. 390 in which the lower and right hand windows refer to the horizontal circle estimating and micrometer scales respectively and the upper and left hand windows to the vertical circle estimating and micrometer scales. In the left hand window the end of the bubble is seen against the 3.7 graduation of the micrometer scale, the reading of the vertical angle then being $87^{\circ} 43' \cdot 7$. Similarly, the reading of the horizontal circle is $183^{\circ} 25' \cdot 7$. With this instrument three grades of accuracy of reading are available:—



Hz: $183^{\circ} 25' \cdot 7$

V: $87^{\circ} 43' \cdot 7$

FIG. 390.—FIELD OF VIEW IN THE READING MICROSCOPE OF THE ZEISS-OPTON THEODOLITE TH 3.

(By courtesy of
Messrs. Carl Zeiss Ltd.)

- (1) For tacheometry and other similar work where great angular accuracy is not needed: readings on the estimating microscope without micrometer can be estimated to single minutes.
- (2) For more accurate directional readings estimated to tenths of a minute: scale microscope with micrometer.
- (3) For subtense and similar work where values to an accuracy of single seconds are required: micrometer with observations by repetition.

Various accessories for the theodolite are obtainable, including equipment for the three-tripod system of observing; subtense bar interchangeable on the tripods carrying theodolite or targets, front attachment prism and distance staves for optical distance measurements; tubular compass; etc.

Kern Theodolites.—The Swiss firm of Kern & Co. manufacture a series of theodolites provided with glass circles and optical micrometers or scales which embody two unique features in their design. The first is a new vertical axis system which replaces the ordinary vertical axis proper in taking up the weight of the upper part of the instrument and which, it is claimed, is more stable than the ordinary vertical axis, is freer from drag and is less affected by

variations in temperature. This system, which is shown diagrammatically in Fig. 391, consists of a precision ball race set between two

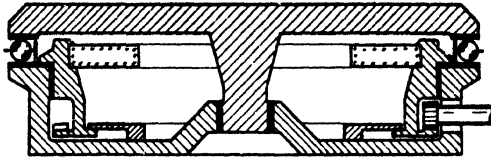
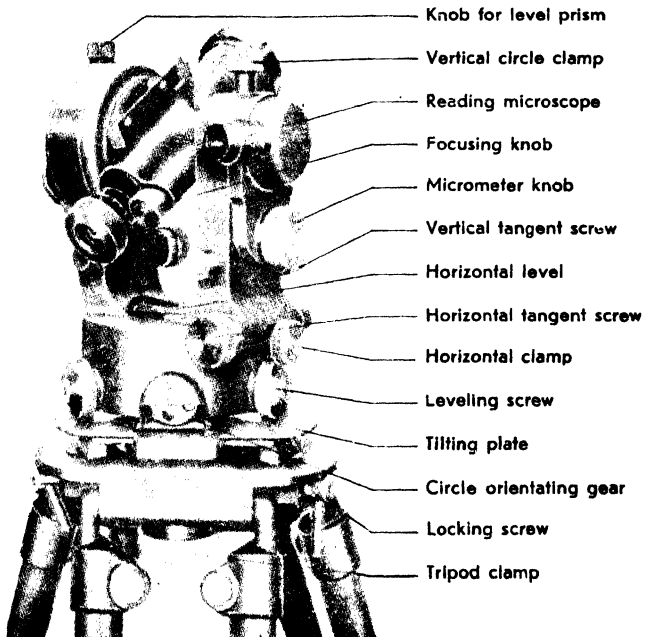


FIG. 391.—CROSS SECTION (SCHEMATIC) OF VERTICAL AXIS SYSTEM OF KERN THEODOLITES.

(By courtesy of Messrs. Kern & Co. Ltd.)

accurately ground bearing surfaces carried by the tribrach and the upper plate respectively.

The second main novelty in the Kern theodolite is the system of



$\frac{1}{3}$ actual size

FIG. 392.—KERN NO. DKMI (SMALL TRIANGULATION) THEODOLITE.
(By courtesy of Messrs. Kern & Co. Ltd.)

levelling. The instrument is fastened to a tilting plate carried on the tripod, and this plate, which is fitted with two small cross levels, admits of rough levelling. Final levelling is then done by three levelling, or foot, screws in the base of the instrument, and these, instead of being vertical as is usual, are horizontal, the actual levelling being affected by cams at the end of the screws. The advantages claimed for this system of levelling are speed in operation, elimination of side play and backlash of the screws, and the reduction of wear to a minimum. The effect also is to lower the centre of gravity of the instrument relative to the top of the tripod, thus making for greater stability.

The different Kern theodolites vary in size from one which reads direct to 10' and by estimation to 0'·5 to one for geodetic triangulation which read direct to 0"·5 and by estimation to 0"·05. The model illustrated in Fig. 392 reads direct to 10" and by estimation to 1". In all cases, the micrometer reading is the arithmetic mean of values at opposite ends of a diameter and a good range of accessories is available for each model.

LEVELS

Perhaps the most interesting development in recent years in the design of levels has been the introduction of models in which the horizontality of the line of collimation is directly controlled by gravity in such a way that final levelling is independent of the indications of a spirit level. Examples of these are the Cowley Builder's level and the Zeiss Automatic Self-Adjusting level Ni.2. In the former, a mirror mounted on a pendulum, and in the latter a swinging mirror system, serve to establish a horizontal line of sight even when the vertical axis of the instrument is not truly vertical. Another development is a growing tendency for makers to offer two models of their standard instruments, one model having a horizontal graduated circle for the observation of horizontal angles and the other being without one. An instrument provided with a horizontal circle and with stadia wires is particularly useful for close and accurate contouring as the observation of a bearing and distance gives the data for plotting the position of a spot level.

The Cowley Automatic Level.—In appearance this level does not look like an ordinary level but resembles a small amateur's cine camera. It is particularly suitable in building and engineering work for setting out foundations, gradients, etc., and, although it can be used for running short lines of levels from point to point, it is not so suitable as an ordinary level for carrying forward long lines of levels. It has the advantage of being very simple and quick to use and it is very cheap considering the degree of accuracy obtainable with it. It also has the advantage of not requiring a skilled surveyor to manipulate it.

The level is used in conjunction with a graduated staff to which is attached an adjustable target at right angles to it which can be slid up and down the staff by the staff holder and clamped at any position in obedience to signals from the observer. With the staff set vertically on any point, the observer looks through the aperture

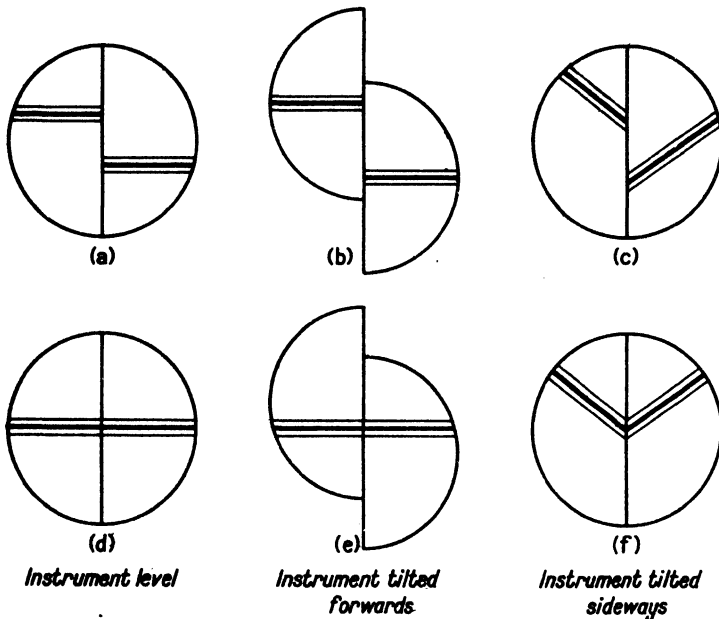
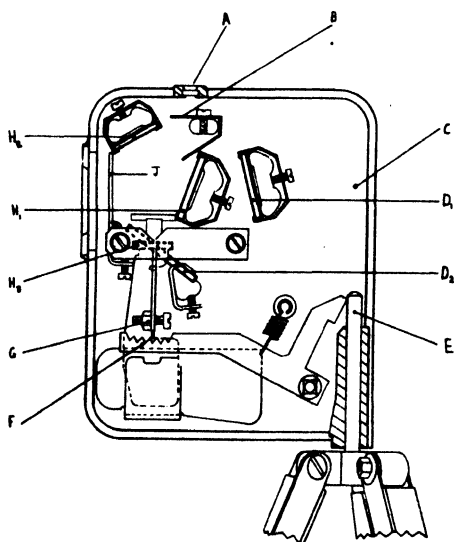


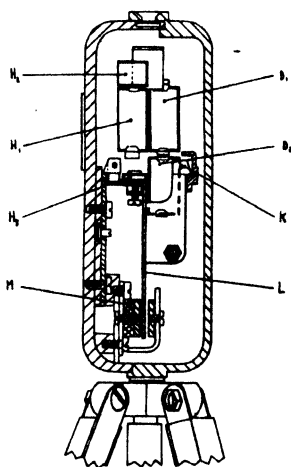
FIG. 393.—IMAGES OF THE TARGET IN THE FIELD OF VIEW OF THE COWLEY AUTOMATIC LEVEL.

on top of the case and sees two images of the target separated by a vertical line. If the black horizontal line marking the centre of the target is not at the same elevation as the optical centre of the instrument, the two images of it will be separated as shown in Figs. 393(a), (b) and (c). If the centre of the line is at the same elevation as the optical centre of the instrument, the two images will coincide as shown in Figs. 393(d), (e) and (f). Hence, the observation consists in sighting on the staff and getting the staff holder to move the target up or down until the two images of the horizontal line at its centre appear to coincide at their edges. The reading of the height of the target above the bottom of the staff gives the amount by which the point on which the staff is held is below a horizontal plane through the optical centre of the level.

Fig. 394 shows sectional side and front views of the instrument. Here there are two mirror systems, one on each side of a vertical plane passing longitudinally through the centre of the front of the instrument. The left hand system consists of the mirrors D_1 and D_2 and the right hand system of the mirrors H_1 , H_2 and H_3 . The mirrors D_1 , D_2 , H_1 and H_2 are fixed relative to the sides of the casing but the mirror H_3 is attached rigidly to a pendulum in such a way that it is always horizontal when the pendulum is vertical. A ray of light passing through the centre of the front aperture to meet D_1 is reflected to the mirror D_2 and thence to the eyepiece A on top of the case. Another ray passing through the centre of the front aperture and meeting the mirror H_1 is reflected to the mirror H_2 and thence to the horizontal mirror H_3 , from which it is reflected upwards to pass through the eyepiece A. If, when the instrument is in proper adjustment, these two rays are in the same horizontal plane when they enter the instrument, they will emerge from the



(a) Side View.



(b) Front View.

- A. Eyepiece
- B. Mask
- C. Optical Centre
- D.1 1st Mirror L.H. System
- D.2 2nd Mirror L.H. System
- E. Tripod Peg
- F. Pendulum Clamp
- G. Pendulum balance adjusting screw

- H.1 1st Mirror R.H. System
- H.2 2nd Mirror R.H. System
- H.3 3rd Mirror R.H. System
- J. Image division bar
- K. Knife edges
- L. Pendulum
- M. Magnet

FIG. 394.—SECTIONAL VIEWS OF THE COWLEY AUTOMATIC LEVEL.
(By courtesy of Messrs. Hilger & Watts Ltd.)

eyepiece A alongside and parallel to one another, so that, if they come from adjoining points on the horizontal central line on the target, these points will appear to lie side by side and the horizontal line on the target will be seen as an unbroken line. If the two rays do not lie in a horizontal plane passing through the optical centre of the instrument when they enter the front aperture, the images of the two halves of the horizontal line on the target will be separated.

In order to damp down the oscillations of the pendulum, and hence of the mirror H_1 , the blade of the pendulum swings between the poles of a powerful magnet M and, as the pendulum swings, induced eddy currents quickly damp down the oscillations.

The level is supported on a long vertical spike E carried by the tripod, and, when it is not attached to the tripod, an arm F lifts the pendulum off the knife edges K so that it is held fixed and is no longer free to swing. When the instrument is placed on the tripod, the spike depresses the arm and the pendulum with the mirror is free to swing freely until the oscillations are damped by the magnet. Thus, in setting up the level, all that is necessary is to set up the tripod with the spike E approximately vertical and then slip the instrument on to the spike.

If the instrument is tilted forwards or backwards, the two halves of the field of view are displaced relative to one another as shown in Figs. 393(b) and (e), and, if it is tilted sideways, the images of the line on the target are inclined to one another as shown in Figs. 393(c) and (f). In all cases, the observation consists in moving the target up and down the staff until the left hand and right hand images coincide as shown in Figs. 393(d), (e) and (f).

When the level is to be set up on a brick course or other place where it is not possible to use the ordinary tripod, a special "brick-layer's stand", provided with a spike similar to that on the tripod, can be employed. This stand is obtainable as an extra.

The adjustment of the instrument can be tested by the two peg test described on pages 124-127 but any lack of adjustment is best corrected by the makers.

The Cowley level weighs only $2\frac{1}{2}$ lbs., the tripod under 2 lbs., and the staff and target $2\frac{1}{2}$ lbs. The accuracy is stated to be about $\frac{1}{4}$ in. at 100 ft., increasing as the distance is decreased and decreasing as the distance is increased.

Zeiss Self-Adjusting Level Ni.2.—Whereas the use of the self-adjusting level described above is best confined to levelling operations of a minor nature, the Zeiss level Ni.2 is an all-round instrument which can be used for all classes of work short of first-order geodetic levelling. The instrument only needs to be levelled approximately with the foot screws to within $\pm 15''$, corresponding to a 2 mm. displacement of the bubble of the small circular level which is attached to it: after that, gravity takes control in such a way that, for any small inclination of the telescope to the horizontal, a hori-

horizontal ray of light passing through the optical centre of the objective system of the telescope automatically passes through the cross hair of the diaphragm. Consequently, a point which appears to coincide with the point of intersection of the cross hairs is a point on a horizontal line passing through the optical centre of the objective.

The principle of the instrument will be understood from Fig. 395. Here a horizontal line of sight XO passing through the optical centre of the objective is reflected by the mirror A to the mirror B, thence to the mirror C and from C to D, the intersection of the cross hairs in the diaphragm. The mirrors A and C are fixed to the barrel of the telescope but the mirror eBf at B is supported at its ends by

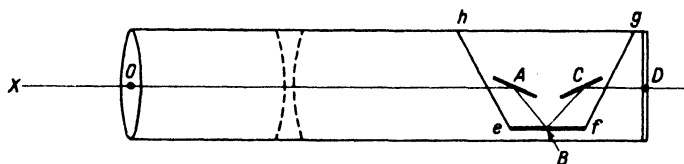


FIG. 395.—DIAGRAM TO ILLUSTRATE THE PRINCIPLE OF THE ZEISS NI. 2 LEVEL.

fine flexible wires eh and fg which are attached to the points h and g on the barrel of the telescope. If the axis OACD of the telescope is tilted through a small angle, the mirror eBf is also tilted through another small angle and, when the instrument is properly adjusted, a horizontal ray entering the telescope and passing through the optical centre is reflected at A, B and C in such a manner that it still passes through the intersection of the cross hairs at D, which is at the focal point of the objective.

In practice, the mirrors A, B and C are not ordinary plane mirrors but are reflecting prisms whose faces are perpendicular to the incident and reflected rays when the axis of the telescope is horizontal. Also, attached to the prism at B and above it is part of a damping device which damps down the oscillations of the prism very quickly. As a result, the centre of gravity G of

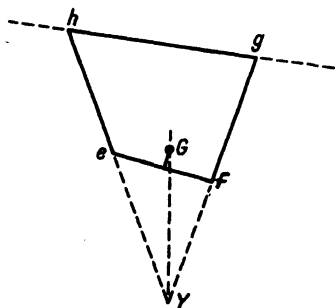


FIG. 396

the swinging system lies somewhat above the reflecting surface as shown in Fig. 396, and it follows from principles of elementary statics that, when the telescope and the line of support, hg , of the wires he and fg are inclined to the horizontal, the system is in equilibrium when the point G lies vertically above Y , the point of intersection of he and gf .* Hence, the angle of tilt of eBf for a given tilt of hg depends on the distance of G from ef and can be altered by altering this distance.

The complete system of fixed and swinging reflecting surfaces is called the "compensator". The wires supporting the lower mirror are very light and flexible and are of rustproof, non-magnetic, material. The instrument is not affected by temperature changes and does not need to be shaded by a large umbrella, not even in bright sunlight. The compensator is not locked after use, so that it does not need to be specially released for observing, and it is unaffected by all ordinary shocks during transit.

Tests with this level have shown that it is much quicker to use than an ordinary level of similar accuracy and it is little affected by earth tremors and vibrations such as those experienced when work is being done in the middle of heavy traffic or heavy construction work.

The level is provided with what is in effect an anallactic lens system so that, when the telescope is tilted, the rotation may be assumed to be about an axis passing through the anallactic point. One result of this is that the compensator still operates when it is necessary to focus for short distances.

TELESCOPIC ALIDADES

Watts Microptic Telescopic Alidade.—An example of a modern telescopic alidade suitable for either small or large scale work with the plane table is provided by the Watts Microptic Telescopic Alidade. The telescope of this alidade is furnished with stadia hairs and the vertical circle is of glass on which are engraved three scales, one giving the angle of elevation or depression, the other two being Beaman Scales (page 488) for reducing stadia readings. The readings of the scales are viewed through a microscope placed alongside and parallel to the telescope, and a reflector above the altitude level throws an image of the bubble in the

* The mirror ef is kept in equilibrium by the tensions in the wires eh and fg acting in the directions eh and fg and by its weight acting vertically downwards through G . The resultant of the tensions acts through Y , their point of intersection, and this resultant must be equal in magnitude and opposite in direction to the vertical force represented by the weight, as otherwise there would be a resultant force causing a linear displacement of the mirror. Also, the two forces must coincide because, if they did not, they would comprise two equal and opposite forces separated by a finite distance, thus forming a couple tending to rotate the mirror. Hence, the point G must lie vertically above the point Y .

direction of the observer as he stands looking through the telescope. Thus, staff, vertical circle scales, and altitude bubble, as well as the cross bubble used for levelling the horizontal axis of the vertical circle, can all be seen from the one position of the observer.

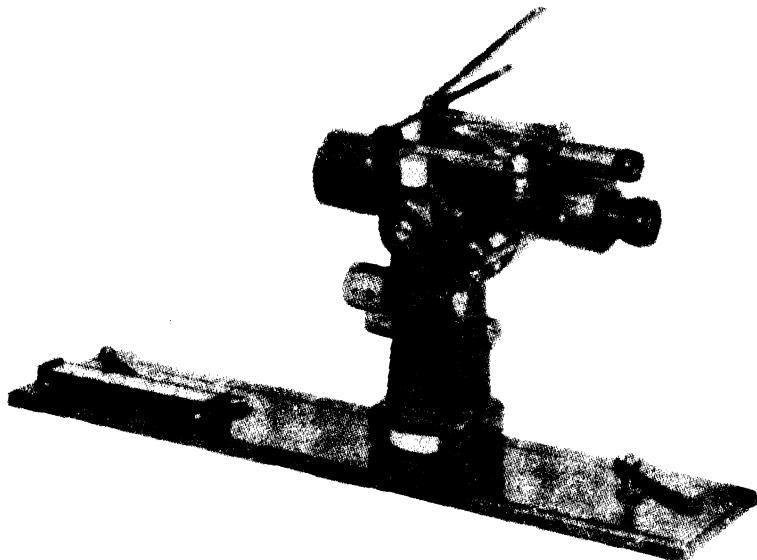


FIG. 397.—WATTS' MICROPTIC TELESCOPIC ALIDADE WITH TROUGH COMPASS AND TELESCOPE BUBBLE.
(By courtesy of Messrs. Hilger & Watts Ltd.)

The vertical circle is carried on a pillar which can be rocked by means of a milled headed screw at its base, the two-minute cross level near the top of the pillar being used to indicate when the horizontal axis is level. The blade is provided with a parallel ruler and a circular level for levelling the table itself.

The field of view in the scale microscope is shown in Fig. 398. The upper scale gives angles of elevation and depression, the degree divisions being divided into ten-minute intervals, single minutes being obtained by estimation. The divisions on the scale

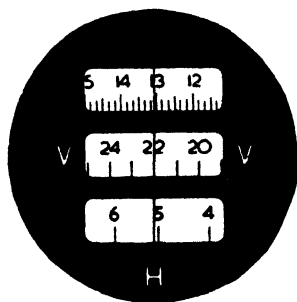


FIG. 398.—FIELD OF VIEW IN SCALE MICROSCOPE OF THE MICROPTIC TELESCOPE ALIDADE.

(By courtesy of Messrs. Hilger & Watts Ltd.)

marked V are in terms of $100 \sin E \cos E$, where E is the vertical angle, plus if an elevation and minus if a depression, and those on the scale marked H are in terms of $100(1 - \cos^2 E)$, these two factors being the correction factors for vertical heights and horizontal distances respectively.

The instrument is used by setting the telescope to read any where on the staff and bringing the altitude bubble to zero by the altitude tangent screw. The telescope is then tilted up or down until the index line coincides exactly with the nearest division on the V scale, after which the readings on the stadia and central hairs are taken. The stadia intercept multiplied by the whole number reading on the V scale gives the difference in elevation between the point on the staff where the middle hair appears to meet it and the horizontal axis of the instrument, while the reduced horizontal distance from staff to instrument is obtained by multiplying the stadia intercept by 100 and then subtracting from this the stadia intercept multiplied by the reading on the H scale. Thus, in Fig. 398, the readings on the V and H scales are 22 and 5 respectively, and, if the staff reading on the middle hair was 9.35 ft. and the stadia intercept 5.20, the difference in elevation between the horizontal axis of the instrument and the point on which the staff was held is $(5.20 \times 22) - 9.35 = 105.05$ ft., while the reduced horizontal distance is $5.20 \times 100 - (5.20 \times 5) = 494$ ft.

Self-Levelling Telescopic Alidade of the U.S. Geological Survey.

The U.S.A. Geological Survey has developed a telescopic alidade in which fine levelling of the alidade bubble is not necessary, as the setting of the zero of the vertical circle is automatically controlled by gravity.

The principle of the instrument is very simple although some of the constructional details present certain difficulties. The vertical circle is of glass and on it is engraved a Beaman scale (page 488). The circle, which is enclosed in a metal casing, is mounted on ball bearings to revolve freely about the horizontal axis of the telescope. The lower half carries a 4-ounce weight which serves as a pendulum to keep the line joining the centre of the arc to the zero graduation on the scale always horizontal. The index mark moves with the telescope, and in the reading microscope an image of it is seen superimposed on an image of the circle graduations. In this way, the appropriate Beaman arc readings for all inclined sights up to $\pm 30^\circ$ are easily obtained. The design includes a suitable damping device so that the pendulum and the attached graduated glass circle come to rest in rather less than 5 seconds.

DOUBLE IMAGE TELEMETERS AND TACHEOMETERS

Distance Wedges or Prism Stadia Telemeters.—The so-called double-image tacheometers are based on an invention by R.

Bosshardt in 1923 in which separate images of two different parts of a horizontal stadia rod are seen simultaneously in the same field of view. In their simplest form, a very acute angled achromatic glass prism or wedge which produces a horizontal deviation in an incident ray of $\tan^{-1} \left(\frac{1}{100} \right)$ ($34' 22'' \cdot 6$) is placed in front of the object glass of the telescope, this wedge only covering about one half of the lens. Hence half of the rays entering the lens form a direct image of the object while the other half pass through the glass wedge and are deviated through an angle of $\tan^{-1} \frac{1}{100}$, corresponding to a stadia constant of 100:1. Thus, when the instrument is directed to a horizontal stadia staff an observer sees a direct image of one part of the staff and on this is superimposed an image of another part of the staff which is formed by rays deflected through the wedge, the actual relative displacement of the two images being $\frac{1}{100}$ of the distance instrument to staff. The lower half of the staff face carries for its whole length a scale of centimeters while the upper half carries at one end a vernier scale. Hence, what one sees in the telescope is an image of the scale of centimeters formed by the undeviated rays and below this is an image of the vernier scale formed by the rays deviated through the wedge (see Fig. 400). The reading of the vernier on the scale gives the inclined distance instrument to staff in metres and decimals of a metre.

For fine measurements the device is sometimes provided with a parallel plate micrometer, the drum of which transmits its movement to a swivelling plane parallel plate of glass and enables the image of the vernier to be moved slightly, so that one of the graduations on it can be brought in the usual way into coincidence with one of the graduations on the main scale, centimeters and millimeters, corresponding to metres and decimeters on the ground being read direct on the scale and vernier on the staff, and tenths and hundredths of millimeters, corresponding to centimeters and millimeters on the ground, being read on the micrometer drum.

Fig. 399 shows the Wild Precision Telemeter DM1 which can be fitted on their Universal or Double Centre theodolites and Fig.

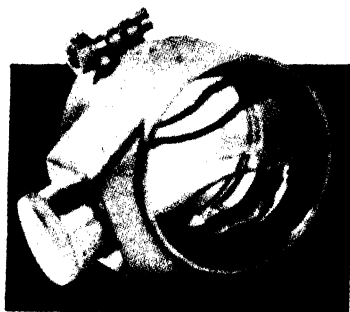


FIG. 399.—WILD PRECISION TELE
METER OR DISTANCE WEDGE DM1

(By courtesy of
Henry Wild & Co. Ltd.)

400 shows the appearance of the staff and vernier scale as seen in the eyepiece of the telescope, with a view of the micrometer drum alongside. The reading on the scale and vernier is 61.5 and on the micrometer drum 8.2 so that the distance is $61.5 + 0.082 = 61.582$ m. A very similar attachment for use with their theodolites is manufactured by Messrs. Hilger and Watts Ltd.

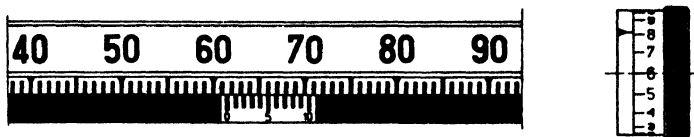


FIG. 400.—VIEW OF STAFF AND VERNIER AS SEEN IN THE WILD PRECISION TELEMETER DM1.

(By courtesy of Messrs. Henry Wild & Co. Ltd.)

Wedge telemeters which can be fitted to their theodolites are also made by Messrs. Kern & Co. Their DM "Stadia Prism Device" is not provided with a micrometer and direct readings can therefore only be taken to give the nearest centimetre in distance. Their DM-M Stadia Prism Outfit is provided with a parallel plate micrometer so that, with it, readings can be taken to give millimeters in distance. All these attachments, like the Wild precision telemeter, only give distances uncorrected for slope but the Kern DR "Reducing Optical Stadia Attachment" gives the true horizontal distance. This attachment is fitted with a pair of "Boscovic" prisms or wedges similar to those used in the Wild RDH Double-Image Self-Reducing Tacheometer which is described on pages 616 621. With it, however, inclining the telescope does not bring these wedges automatically into play as it does in the case of the Wild RDH and other similar self-reducing instruments. Instead, there is a level bubble on the attachment and centering this bubble operates the wedges to give true horizontal distances. Thus, a theodolite fitted with this attachment is not so convenient in use as a double-image self-reducing tacheometer.

All these attachments which fit on to a theodolite are of very appreciable weight and would throw a strain on the instrument if they were not balanced by a counterweight which fits on the eyepiece end of the telescope. They give results which are almost as accurate as those obtainable with the larger self-reducing tacheometers but, as they are not so convenient to use, it is just as well to go to the extra expense of the special instrument if much tacheometric work has to be done. All of the special instruments are provided with horizontal and vertical circles and can be used as ordinary theodolites when required.

Double-Image Self-Reducing Tacheometers by Kern, Zeiss and Wild.—As noted on page 509, the Kern DK-RT and Zeiss Redta tacheometers give the reduced horizontal distance directly, but, in order to get differences of elevation, this distance must be multiplied by the tangent of the angle of elevation or depression, this tangent being obtained from tables in the ordinary way, or read off a scale of tangents carried on the vertical circle in addition to the ordinary angle graduations, as in the case of the Zeiss Redta instrument, or as an alternative to the angle graduations in the case of the Kern instrument. Readings with the Wild RDH tacheometer, on the other hand, can be taken to give directly both horizontal distance and difference of elevation. The basic optical arrangements for obtaining horizontal distances are very similar in each instrument so we shall choose for description the one by Wild which gives reduced differences of elevation as well as reduced horizontal disturbances.

Wild RDH Double-Image Self-Reducing Tacheometer.—With this instrument the horizontal graduated staff, Fig. 401, is supported on a vertical graduated rod to which it can be clamped at a height corresponding to the height of the instrument. The vertical rod is supported by two stays which can be clamped at any suitable height. The bottom half of the staff carries a scale of centimetres with the graduation marks upside down so that they appear right side up in the telescope, the graduation numbers to the right of the zero being in red and the others in black. The top half of the staff carries three verniers, one at the centre marked 0 to 2 in black lettering, one at the right hand end marked 70 to 72, also in black lettering, and one at the left hand end marked 30 to 32 with the graduation numbers in red. In the field of the telescope the ends of the staff are reversed so that the 70 to 72 vernier appears on the left and the 30 to 32 vernier on the right.

When the instrument is set up, the graduated scale is seen above

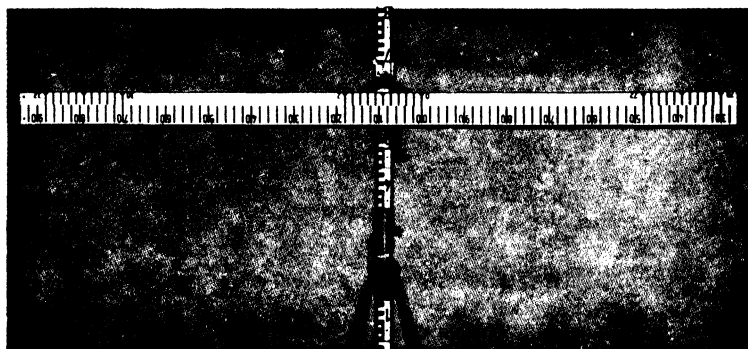


FIG. 401.—STAFF USED IN CONJUNCTION WITH THE WILD RDH DOUBLE-IMAGE SELF-REDUCING TACHEOMETER.

(By courtesy of Henry Wild & Co. Ltd.)

and the vernier scale below, each with the graduation numbers right side up, the zero of the vernier being displaced from the zero of the scale by an amount depending on the distance between instrument and staff. The interval between graduations is 2 cms. and the multiplying constant is 100, so that this interval represents an amount of 2m. difference in distance between instrument and staff. By turning the micrometer knob on the telescope, a graduation of the vernier can be brought into alignment with a

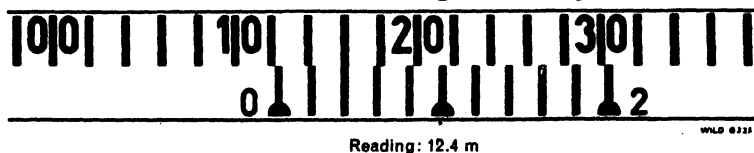


FIG. 402.—EXAMPLE OF READING ON HORIZONTAL STAFF FOR DISTANCES UNDER 70M. READING: 12.4M.

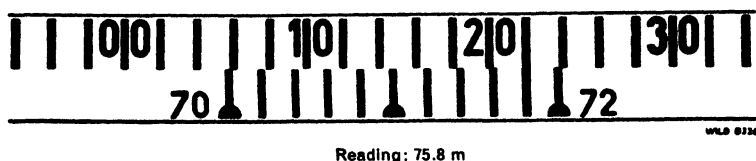


FIG. 403.—EXAMPLE OF READING ON HORIZONTAL STAFF FOR DISTANCES OVER 70M. READING: 75.8M.

graduation on the scale. The graduations to the left of the zero of the vernier give the metres, the reading on the vernier gives the decimetres, and the reading on the micrometer the centimetres, and, by estimation, millimetres. Thus, in Fig. 402, if the reading on the micrometer for horizontal distance measurement was 6.5, the complete distance would be:—

Main graduation	12.000
Vernier reading	0.400
Micrometer reading	0.065
Full Reading	12.465

If the distance between instrument and staff is more than 70m., the image of the central vernier comes outside the staff and in this case the left hand vernier as viewed in the telescope is used and 70 metres added to the reading on the upper scale. Thus, in Fig. 403, had the micrometer reading been 7.3, the full reading would have been 75.873 metres.

A change over knob on top of the right hand standard of the instrument determines whether horizontal distances or differences of elevation are being measured. When the horizontal line on the knob appears, readings give distances, but, when the vertical line on the knob appears, the instrument is set for giving differences of elevation.

When the instrument is set to give distances and the line of

sight is horizontal, staff and vernier readings give the true horizontal distance, but, when the line of sight is inclined, there is a small correction, always negative, of $74 (1 - \cos \beta)$ mm. This correction is read direct on a graduated ring at the right between the telescope and the standard.

When the instrument is set for reading differences of elevation, readings are taken in the same way as in distance measurement. If the angle of sight is an angle of depression, the red graduations on the staff must be used and 100 subtracted from the reading giving a negative value. If the altitude difference exceeds 70 m., the vernier marked 30 to 32 with red figures must be used. A red graduation means that 100 metres must be subtracted from the final reading so that if *both* the main and the vernier figures are red the difference in elevation is 200 minus the reading, again a negative figure. A small correction of $74 \sin \beta$ mm. is applicable to all inclined sights, this correction having the same sign, positive or negative, as β . The correction itself can be read off the same ring as is used for obtaining the distance correction. When the change over knob is turned from distance to elevation readings, the correction ring is rotated through 90° to prevent the two scales from overlapping. The elevation correction graduations and the corresponding numbers are coloured red.

The instrument itself is shown in Fig. 404. For use as an ordinary theodolite for reading horizontal and vertical angles the

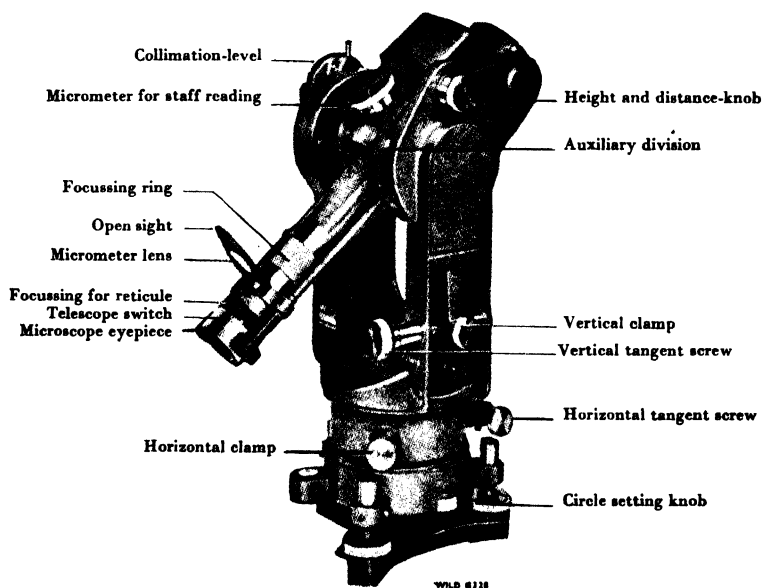
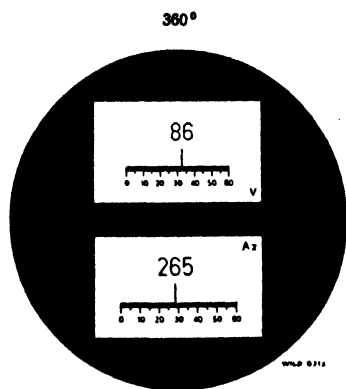


FIG. 404.—THE WILD RDH DOUBLE-IMAGE SELF-REDUCING TACHEOMETER.
(By courtesy of Messrs. Henry Wild & Co. Ltd.)

knurled eyepiece sleeve of the telescope is turned to the right until it reaches a stop. This brings into view in the telescope a non-deflected image formed by rays entering the telescope through the rhomboid prism shown in Fig. 406. Angles are read in an optical scale micrometer, the eyepiece of the micrometer microscope being situated parallel to, and alongside, the telescope. The circles are read direct to single minutes and by estimation to one-tenth of a minute, or 6". Fig. 405 shows the field of view in the optical scale microscope, the horizontal circle reading being $265^{\circ} 28' 30''$ and the vertical circle reading $86^{\circ} 32' 30'' = +3^{\circ} 27' 30''$.



Vertical circle $86^{\circ} 32' 30'' = +3^{\circ} 27' 30''$
Horizontal circle $265^{\circ} 28' 30''$

FIG. 405.—FIELD OF VIEW IN MICRO-METER MICROSCOPE OF THE WILD RDH SELF-REDUCING TACHEOMETER.

(By courtesy of
Messrs. Henry Wild & Co. Ltd.)

Fig. 406 shows diagrammatically a vertical cross section of the instrument with the telescope horizontal and set for the reading of differences of elevation. Parallel rays entering the rhomboid prism 4 on the left of the diagram emerge parallel to themselves to fall on the upper half of the telescope objective 5 whence they are refracted to form an image in the ordinary way. Other rays entering the lower half of the objective do so through a slightly wedge-shaped glass cover 1 and then pass through two similar achromatic glass wedges 2 and 3 which can be rotated through 90° in opposite directions in planes perpendicular to the optical axis of the telescope. When this axis is horizontal and the wedges

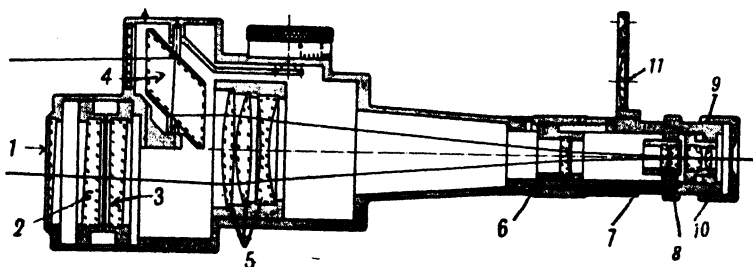


FIG. 406.—DIAGRAMMATICAL SECTION OF THE TELESCOPE OF THE RDH DOUBLE-IMAGE SELF-REDUCING TACHEOMETER.

(By courtesy of Messrs. Henry Wild & Co. Ltd.)

2 and 3 are set, as shown in the Figure, so that a deflection of the rays by the first wedge is cancelled out by an equal and opposite deflection by the other, then rays pass through the combination without any deflection before falling on the lower part of the objective 5. This is the position when the telescope is set for the measurement of differences of elevation. As the telescope is rotated to give an angle of elevation or depression of β , the wedges are rotated in opposite directions by means of a suitable gear train to give a horizontal deflection proportional to $\sin \beta$, which is the factor by which the distance of the instrument to the staff must be multiplied to give differences of elevation, the vertical components of the deflections of the two wedges cancelling out. Similarly, when the instrument is set to measure horizontal distances, the wedges are turned as a unit through 90° from the zero position so that the maximum deflection of $\tan^{-1} 0.01$, corresponding to a stadia constant of $1/100$, occurs when the telescope is horizontal. As the telescope is rotated about its horizontal axis the wedges also are rotated in opposite directions in such a way that the horizontal component of the deflection is decreased proportionately to $\cos \beta$, the factor by which the inclined distance of instrument to staff must be multiplied to give horizontal distances, the vertical components of the deflections again cancelling out. After passing through the wedges, rays of light fall on the lower half of the objective 5, while the rays from the rhomboid prism 4 pass through the upper half. Then, after passing through the internal focussing lens 6, each set of rays falls on, and passes through, the special prism 7 set in the focal plane of the objective system, which, with the stop 10, produces two images separated by the horizontal line formed by the apex of the prism 7.

Fig. 407 shows diagrammatically the passage of the rays from objective to the stop. Rays entering the objective from the rhomboid prism fall on the lower half of the prism P and are

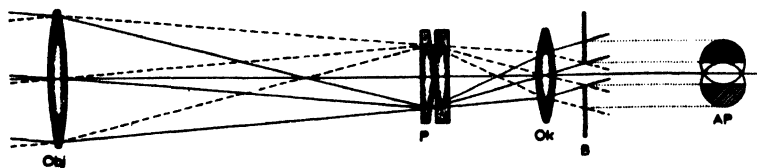


FIG. 407.—DIAGRAM SHOWING THE PASSAGE OF RAYS FROM THE OBJECTIVE TO THE SLOT DIAPHRAGM IN THE WILD RDH DOUBLE-IMAGE SELF-REDUCING TACHEOMETER.

deviated upwards to the eyepiece Ok. It will, however, be seen that any which fall on the lower half of the objective are cut off after emersion from the eyepiece by the stop B while those which fall on the upper half of the objective are transmitted. Similarly,

rays entering the objective from the wedges fall on the upper half of the prism P, and are deviated downwards to the eyepiece Ok. Any rays which enter the upper half of the objective are intercepted by the stop B, while those entering the lower half of the objective pass through the stop. The exit pupils for both sets of rays are shown at the right in Fig. 407. The lower part of the upper circle only contains rays which enter the objective through the rhomboid prism, while the upper part of the lower circle only contains rays which enter the objective after passage through the wedges. All other rays are stopped by the stop B. Hence, one sees in the telescope two halves of images separated by a horizontal line represented by the apex of the prism P. In observing, this horizontal line is set to view the middle of the staff. The main divisions of the staff, the image of which is formed by rays passing through the rotatable wedges, appear above the line, while the vernier, the image of which is formed by rays passing through the rhomboid prism, appears below the line. The rhomboid prism is mounted to rotate by a small amount about a vertical axis, this rotation being affected by turning the graduated micrometer drum shown on top of the telescope in Fig. 404. Rotation of the micrometer drum thus produces a small deviation of the rays entering the rhomboid prism and this, in turn, produces a small horizontal shift of the image of the vernier on the staff, the amount of this shift, as measured on the drum, giving the centimetres (and by estimation the millimetres) in the distance.

Watts-Ewing Stadia Altimeter.—This device, designed by Mr. Alister Ewing, for use with a vertical staff, converts an ordinary theodolite simply and easily to a Direct-Reading Tacheometer without interfering with its normal functions in any way. It can easily be used with the Watts Microptic Theodolite No. 1 and Watts Standard Vernier Theodolites, and in some cases it can be fitted to theodolites of other makes by means of special adaptors which must be fitted in the first place by a skilled instrument mechanic. Once fitted, it can easily be removed from the theodolite or attached to it without readjustment. In the case of the Watts Standard Vernier Series of theodolites, it is necessary to make a minor alteration to the theodolite to take the scale unit.

The attachment consists of two parts—a “Cylindrical Scale Unit” or rotatable drum mounted on one of the theodolite standards and an “Optical Reader” mounted on the horizontal or transit axis. The cylindrical scale unit embodies two separately coloured sets of curves, called “Intercept Lines,” one set giving the difference in elevation, $100s \cdot \sin \theta \cos \theta$, and the other set the correction, $100s(1 - \cos^2 \theta)$, to the slant distance to give the reduced horizontal distance. The index of the optical reader is a bright pinpoint of light which falls on the drum. After the instrument has been properly set and the staff intercept read, the cylinder of the scale

unit is rotated until the appropriate curve equal to 100s for difference of elevation, or slant distance correction, as the case may be, coincides with the reader index, when the difference in elevation, or the slant distance correction, is read on an external graduated circular head fitted on the axis of the cylinder of the Scale Unit.

APPENDIX II

ADDITIONAL NOTES ON THE THEORY OF INSTRUMENTAL ADJUSTMENT

THE operations necessary in the ordinary adjustments of the theodolite are described on pages 92-101 and those of the level on pages 123-128, but in a few cases the theory of these adjustments, although often indicated in the figures, has not been fully explained. Accordingly, the following supplemental notes may be helpful in enabling the reader to understand the reason for every operation.

THEORY OF ADJUSTMENT OF TRANSIT THEODOLITE

(1) **Adjustment of the Plate Levels.**—If the altitude level is not perpendicular to the vertical axis, the latter will not be truly vertical when the bubble is at the centre of its run. Let the bubble AB (Fig. 394) make angle $ACD = 90^\circ - e$ with the vertical axis CD. Then, when AB is horizontal, CD makes an angle e with the vertical CE. On rotating the instrument through 180° about CD, the line AB takes up the position A'B' such that angle $A'CD = ACD = 90^\circ - e$. Hence, $BCA' = 180^\circ - 2(90^\circ - e) = 2e$. Thus, the dislevelment of the bubble tube caused by the rotation through 180° about CD is $2e$, and, if half of this is corrected by the levelling screws, the axis CD will be made vertical. The remainder of the adjustment then consists in

making the bubble perpendicular to the vertical axis, and this, in the case of the plate levels, is done by turning the plate level screws, or, in the case of the long altitude bubble, by turning the tangent screw or clip screws until the bubble is at the centre of its run.

This adjustment, of course, is an example of the principle of reversal (page 10).

(2) **Adjustment of the Line of Sight or Collimation Adjustment.**—(i) *Horizontal Hair.* In Fig. 80, if the instrument is not in adjustment, the line of sight h_1, ob_1 , in the first position will make an angle e with the horizon when the vertical axis is truly vertical. Transiting the telescope and setting it to read the same vertical angle as before will cause the line of sight to lie along the direction b_1, oh_1 and swing-

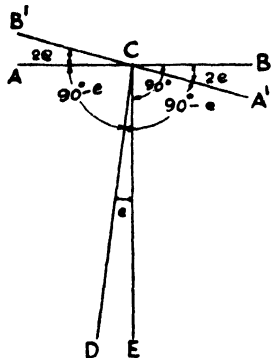


FIG. 408.

ing the instrument through 180° will cause this line to lie along $h_1 ob_1$. From this it can be seen that angle $b_1 ob_1 = 2e$. Hence, bringing the line of sight to read midway between b_1 and b_2 will cause it to be horizontal and hence perpendicular to the vertical axis.

(ii) *Vertical Hair.* If, in Fig. 81, the line of sight makes angle $90^\circ - e$ with the horizontal axis, the latter is not perpendicular to the line OA, and, after the telescope has been transited, the line of sight will lie in the direction OB, as shown in the top diagram, such that angle $AOB = 2(90^\circ - e)$. Hence angle $BOA' = 180^\circ - AOB = 2e$. Similarly, on swinging through 180° and sighting A without transiting, the angle moved through by the line of sight OB is $180^\circ + BOA' = 180^\circ + 2e$, the direction of the horizontal axis of the instrument then making angle $2e$ with its original direction. On transiting it will be seen that the line of sight comes to the position OC such that $A'OC = A'OB = 2e$. Hence, by moving the line of sight to D, where $CD = \frac{1}{2}CB$, the error e is removed and the line of sight is made perpendicular to the horizontal axis. On again transiting, it will be seen that A will no longer be sighted, the line of sight making angle e with OA, but, on turning the instrument through this angle, the horizontal axis will be perpendicular to OA and A and A' will now lie on the line of sight.

(3) *Adjustment of the Horizontal Axis.*—Let XX' , Fig. 409(a), be the first position of the trunnion axis when the line of sight is directed towards the point A, the end X being higher than the end X'. Let O be the centre of the axis and xx' the projection of XX' on a horizontal plane through O. Let θ be the inclination of the axis to the horizontal so that angle $XOx = X'Ox' = \theta$. Let OAD'D be a vertical plane through O and A meeting the horizontal plane through O in the line OD'. Through A draw a vertical plane APD'P' perpendicular to OD' and meeting the horizontal plane through O in the line PP'.

Now let the telescope be depressed so that it sights the peg B immediately below A. As the line of sight has already been made perpendicular to the trunnion axis XX' , this line as the telescope is depressed will sweep out a plane OAB'B which will intersect the plane APD'P' in the line AB'B. This plane will be perpendicular to XX' and will meet the horizontal line PP' at B', where OB' is perpendicular to OX.

Through O draw the horizontal line QOQ' parallel to PD'P' and let OZ be the vertical at O. Then ZQOQ' is a vertical plane through O parallel to the vertical plane APD'P'. The line OD', which is perpendicular to the plane APD'P', is also perpendicular to QOQ'. Also, the line OB' is perpendicular to XX' and to OZ. Hence, it is perpendicular to the vertical plane XZX'. But xOx' , being the horizontal projection of the line XOX' , lies in the vertical plane XZX' and consequently OB' is perpendicular to xOx' . Thus, the lines OD' and OB', which lie in the horizontal plane

through O, are perpendicular to the lines QOQ' and xOx' , which also lie in the same plane. It follows, therefore, that the angle $xOQ = \text{angle } B'OD' = \beta$.

When the telescope is transited and sighted on A, the trunnion axis assumes the position YOY' , the end Y being above the horizontal plane through O and the end Y' below it. When the telescope is depressed to sight on C the line of sight traces out the line $AC'C$ on the vertical plane $APD P$, and, by the principle of

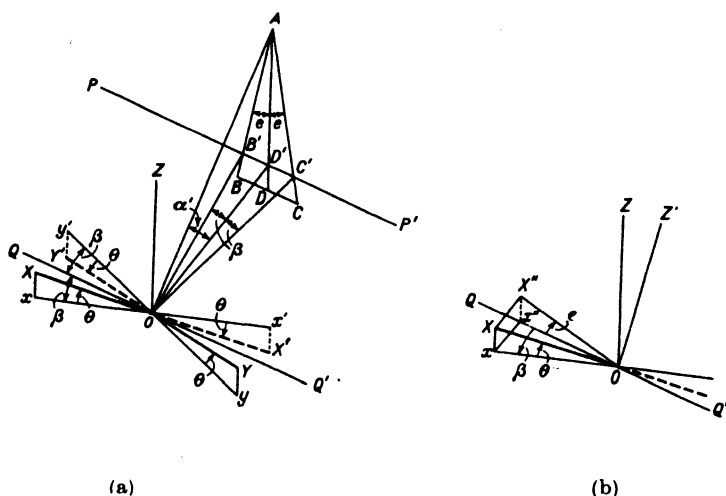


FIG. 409

symmetry, the projection, yOy' , of YOY' on the horizontal plane through O makes angle $yOQ = xOQ = \beta$ with the line QOQ' , and the angle COD , where C is the point where $AC'C$ intersects the horizontal line $PD P$, is equal to the angle $B'OD' = \beta$. Also, D is the point which bisects the line BC and it is on the vertical $AD'D$ from A.

Suppose the instrument is turned without transiting the telescope so that the line of sight lies along the direction OD. The horizontal projection of the trunnion axis will then coincide

with the line QOQ' , and, if the upper end of it is lowered and the lower end raised so that the axis coincides with QOQ' , and the line of sight still intersects D , as the telescope is raised this line will trace out the line $DD'A$ on the plane APP' , and will thus intersect A .

It will be seen from the above explanation that, when the line of sight is perpendicular to the trunnion axis but the latter is not perpendicular to the vertical axis, and the lower circle is kept clamped so as to record directions, face right and face left readings on this circle will differ by the angle 2β when sights are taken to an elevated point. In this case the angle β represents the error in the observation on a single face and this error cancels out when, the vertical axis remaining vertical, the mean of the readings on both faces is taken. It will also be seen that, when the vertical angle is an angle of elevation and the error is calculated and applied as a correction to a direction measured on a horizontal circle graduated clockwise from zero, the correction is additive when the left hand side of the trunnion axis as the observer faces the point observed is higher than the right hand side, and subtractive when the right hand side of the axis is higher than the left hand side. With vertical angles of depression, the signs are reversed. (See Vol. II, page 226.)

Calculation of the Error in an Observed Direction due to Dislevelment of the Trunnion Axis.—On page 225 we have given a formula for calculating the error in the observed direction of an elevated point when the trunnion axis is not perpendicular to the vertical axis. This formula may easily be deduced from Fig. 409 as follows.

Through O in Fig. 409(a) draw OZ' Fig. 409(b) perpendicular to XX' — in the vertical plane through XX' . Then, OZ being perpendicular to the horizontal plane through O , the angle ZOZ' is equal to θ , the angle of tilt of XX' , and this angle lies in the vertical plane $XZZ'X'$. The angle e in Fig. 409(a) lies in the vertical plane $APP'A$, which is parallel to the vertical plane QZQ' . Hence, it will be seen that e is equal to the angle between OZ and the projection of OZ' on the vertical plane QZQ' . That is, it is equal to the projection on the latter plane of the angle θ .

Through X draw a horizontal line XX'' (Fig. 409(b)) perpendicular to the vertical plane QZQ' and through x draw the horizontal line xx'' perpendicular to this plane. Then X'' and x'' are the projections on the vertical plane QZQ' of the points X and x , and the angle $X'Ox''$ is equal to the angle e in Fig. 409 (a).

From Fig. 409 (b),

$$\tan \theta = \frac{Xx}{Ox},$$

$$\tan e = \frac{X'x''}{Ox''}$$

But $Ox'' = Ox \cos \beta$ and $Xx = X''x''$.

$$\therefore \tan e = \frac{\tan \theta}{\cos \beta}$$

Now in Fig. 409(a), if α is the angle of elevation of the point A, $\alpha = AOD'$ and

$$\tan \alpha = \frac{AD'}{OD'}$$

Also, from the same Figure,

$$\tan e = \frac{B'D'}{AD'}$$

$$\tan \beta = \frac{B'D'}{OD'}$$

$$\begin{aligned} \therefore \tan \beta &= \tan e \tan \alpha \\ &= \frac{\tan \theta}{\cos \beta} \tan \alpha \end{aligned}$$

$$\therefore \sin \beta = \tan \theta \tan \alpha,$$

or, since β and θ are small angles,

$$\beta = \theta \tan \alpha$$

β and θ being expressed in the same units, minutes or seconds, but usually seconds. This last is the form in which the expression is normally used. In practice, however, it is seldom large enough to be applicable in ordinary work, but in geodetic work, and more particularly in astronomical work, it has to be applied as a correction to the observed directions of elevated points or of stars (See Vol. II, fourth edition, pages 101 and 226).

APPENDIX III

NOTE ON THE CHECKING OF TRAVERSE COMPUTATIONS

IN Chap. V, pages 262-264 and 270-271, we have described the use of auxiliary bearings, that is ordinary bearings plus 45° , for checking the latitudes and departures of a traverse. A simplification of this method has recently been proposed by B. Goussinsky, of the Survey of Palestine, by which a check may be obtained with rather less labour than by the method already described.* We have (page 263):

$$\begin{aligned}\Delta X &= S + C; \quad \Delta Y = S - C. \\ \therefore \Delta X + \Delta Y &= 2S; \quad \Delta X - \Delta Y = 2C.\end{aligned}$$

Inserting the values for S and C, we obtain:

$$\begin{aligned}\Delta X + \Delta Y &= \sqrt{2} l \sin (\alpha + 45^\circ) \\ \Delta X - \Delta Y &= \sqrt{2} l \cos (\alpha + 45^\circ).\end{aligned}$$

Hence, the check merely consists in taking either the sum or the difference of the latitude and departure and seeing if it gives either 2S or 2C respectively. Consequently, it is not necessary to work out both $\sqrt{2} l \sin (\alpha + 45^\circ)$ and $\sqrt{2} l \cos (\alpha + 45^\circ)$ as one alone will do.

For a whole traverse section, the check can also be simplified by working out the sums of the latitudes and departures (i.e., the total latitude and total departure of a section of traverse) and taking either the sum or the difference of the resulting summations, which sum or difference should then equal $\sqrt{2}$ times the sum of the quantities $l \sin (\alpha + 45^\circ)$ or $l \cos (\alpha + 45^\circ)$. Thus:

$$(\Delta X_1 + \Delta X_2 + \Delta X_3 + \dots + \Delta X_n) + (\Delta Y_1 + \Delta Y_2 + \Delta Y_3 + \dots + \Delta Y_n) = \sqrt{2} [l_1 \sin (\alpha_1 + 45^\circ) + l_2 \sin (\alpha_2 + 45^\circ) + l_3 \sin (\alpha_3 + 45^\circ) + \dots + l_n \sin (\alpha_n + 45^\circ)]$$

or,

$$(\Delta X_1 + \Delta X_2 + \Delta X_3 + \dots + \Delta X_n) - (\Delta Y_1 + \Delta Y_2 + \Delta Y_3 + \dots + \Delta Y_n) = \sqrt{2} [l_1 \cos (\alpha_1 + 45^\circ) + l_2 \cos (\alpha_2 + 45^\circ) + l_3 \cos (\alpha_3 + 45^\circ) + \dots + l_n \cos (\alpha_n + 45^\circ)]$$

In theory, therefore, this summation method gives a check which avoids having to multiply each of the quantities $l \sin (\alpha + 45^\circ)$ or $l \cos (\alpha + 45^\circ)$ by $\sqrt{2}$, as the multiplication can be applied to their sum. The disadvantage, however, is that, although the result shows whether or not an error exists, it does not indicate the exact place at which it occurs if one does exist so that it then becomes

* See "Checking Traverse Computations," by B. Goussinsky. *Empire Survey Review*, Vol. VII, No. 49, July 1933.

necessary to examine each individual leg, and this means multiplying each of the sine or cosine functions already found by $\sqrt{2}$ until the error is located. Consequently, it is normally just as well to proceed with these single multiplications in the first place, the work being simplified if it is done on a printed form and the numerical value of $\sqrt{2}$, or its logarithm, is printed in all places where it is used, exactly as is done in the case of $\log \frac{1}{\sqrt{2}}$ in Traverse Form B, page 270.

Goussinsky has also proposed another very simple method of checking the latitudes and departures which does not involve the use of auxiliary bearings but which, however, is best used with a calculating machine. Since

$$\Delta X = l \cos \alpha; \quad \Delta Y = l \sin \alpha,$$

then,

$$\begin{aligned} \Delta X - \Delta Y &= l \cos \alpha - l \sin \alpha \\ &= l (\cos \alpha - \sin \alpha). \end{aligned}$$

Hence the difference between the latitude and the departure of a leg is equal to l times the difference between the natural cosine and the natural sine of the real bearing of that leg, and, for summations, we have :

$$\begin{aligned} (\Delta X_1 + \Delta X_2 + \Delta X_3 + \dots + \Delta X_n) - (\Delta Y_1 + \Delta Y_2 + \Delta Y_3 + \dots + \Delta Y_n) &= l_1 (\cos \alpha_1 - \sin \alpha_1) + l_2 (\cos \alpha_2 - \sin \alpha_2) + \\ &\quad l_3 (\cos \alpha_3 - \sin \alpha_3) + \dots + l_n (\cos \alpha_n - \sin \alpha_n) \end{aligned}$$

Goussinsky has accordingly prepared, and the Survey Department of Palestine has produced, special six-figure tables for use with this method. These tables give, at intervals of one minute of arc, the difference between the natural cosine and the natural sine of every angle from 0° to 90° . By means of such tables, especially when used in conjunction with a calculating machine, it is easy to find out if an error has been made, and, by examination of individual legs, to locate it when it becomes apparent that one exists.

APPENDIX IV

FURTHER NOTES ON TRANSITION CURVES

The Ideal Transition Curve or Euler's Spiral as a Road Transition Curve.—As noted in Chapter X, page 457, when the Euler Spiral is used as a transition curve for roads the deflection angle ϕ_1 is often large and the approximations usually employed in the case of railway transition curves, where the deflection angle is small, can no longer be justified and more accurate formulae and methods must be used. For setting out, when the value of R , the radius of the circular arc, has been decided, these formulae may be summarised as follows:

$$L = \frac{3.155 V^3}{aR} \quad (1)$$

or, if Shortt's value of $a = 1 \text{ ft./sec}^2$. is accepted, we have

$$L = \frac{3.155 V^3}{R} \quad (1a)$$

Also

$$\phi \text{ (radians)} = \frac{l^2}{2RL}; \quad \phi_1 \text{ (radians)} = \frac{L}{2R} \quad (2) \text{ and } (2a)$$

$$\phi \text{ (minutes)} = 1718.8734 \frac{l^2}{RL}; \quad \phi_1 \text{ (minutes)} = 1718.8734 \frac{L}{R} \quad (3) \text{ and } (3a)$$

$$s = L \left\{ \frac{\phi_1}{12} - \frac{\phi_1^3}{336} + \frac{\phi_1^5}{15,840} - \frac{\phi_1^7}{1,209,600} + \dots \right\} \quad (4)$$

$$c = L \left\{ \frac{1}{2} - \frac{\phi_1^2}{60} + \frac{\phi_1^4}{2,160} - \frac{\phi_1^6}{131,040} + \frac{\phi_1^8}{12,377,920} \dots \right\} \quad (5)$$

$$T = (R + s) \tan \frac{1}{2} I + c \quad (6)$$

Note that, just as the equation $R = v^2/\mu g$ gives a minimum permissible radius of the circular curve on an unbanked road, so does equation (1a) above give the minimum permissible length of the transition curve, and that, just as we can use values of R greater than the minimum, so, if the ground permits, it is advisable to use a transition curve of somewhat greater length than the minimum given by the formula. In practice, curves for high-speed traffic are almost invariably banked, and in that case R may be computed from the formula given on page 455, viz.

$$f = \frac{v^2}{gR} = 0.066804 \frac{V^2}{R} = j + \beta,$$

where j varies from about 0.16 for low speeds to about 0.10 for high speeds and β , the cant, does not exceed 0.1. Also, some

other value, such as $a = 1.75 - 0.0075V$ which decreases with the velocity, may be chosen for a in determining the minimum length of the curve. In any event a good plan is to take L as a whole multiple of 25 ft., if 25 ft. is to be the length of the standard chord length used for setting out, this whole number of chords giving a length somewhat greater than the theoretical minimum length. Thus, if $V = 60$ m.p.h. and $R = 1200$ ft., then minimum $L = 570$ ft. and the length of the transition used could be 575 ft. or 600 ft., giving 23 whole 25-ft. chord lengths in the one case and 24 in the other.

Having determined T , the tangent length, CT, can be taped out from C and the beginning of the transition fixed. The curve can then be set out by deflection angles and chords using the formula

$$\phi = \frac{l^2}{2RL} \quad (7)$$

and

$$\alpha = \frac{\phi}{3} - \frac{8\phi^3}{2835} - \frac{32\phi^5}{467,775} - \frac{2944\phi^7}{1,915,538,625} - \dots \quad (8)$$

in which α and ϕ are in radians. The terms after the first on the right of (8) only amount to 5" when ϕ is approximately 12° , to $10''$ when ϕ is approximately 15° , to $20''$ when ϕ is approximately 19° and to $1'$ when ϕ is approximately 25° . The last term in the series is $7\frac{1}{2}''$ when $\phi = 90^\circ$ and $10''$ when $\phi = 92^\circ 49'$.

If L is divided into a whole number, say n , of equal chord lengths, each length being a convenient one for setting out, then ϕ_p for p chord lengths is given by

$$\phi_p = \left(\frac{pL}{n} \right)^2 = \left(\frac{p}{n} \right)^2 \frac{L}{2R} \quad (9)$$

or

$$\phi_p = \frac{p^2}{n^2} \phi_1 \quad (10)$$

Thus, once ϕ_1 has been computed, intermediate values of ϕ are easily obtained and the corresponding values of α obtained from (8).

Without Tables, all this involves a good deal of computation but the work can be much simplified by using a Table such as Table I on pages 638-640. When L and ϕ_1 have been calculated as above, s and c are obtained by multiplying the factors s/L and c/L in columns 2 and 4 by L , using ϕ_1 as argument. T can then be calculated from (7) and values of ϕ for different values of l calculated from (10). From these values, the α 's can be found by subtracting from $\frac{1}{3}\phi$ the correction γ , given in column 6 of the Table.

If rectangular co-ordinates of intermediate points are required, they can be computed from formulae (4) and (7) on page 447 viz.

$$x = l \left(1 - \frac{\phi^2}{10} + \frac{\phi^4}{216} - \frac{\phi^6}{720,720} + \dots \right), \quad (11)$$

$$y = l \left(\frac{\phi}{3} - \frac{\phi^3}{42} + \frac{\phi^5}{1320} - \frac{\phi^7}{609,840} + \dots \right). \quad (12)$$

or, better still, if Tables such as Table I are available, the long and short subtangents, a and b , at any point distant l from the start of the transition curve can be obtained by multiplying the factors a/l and b/l in the 8th and 10th columns of the Table by l . Then,

$$x = a + b \cos \phi, \quad (13)$$

$$y = b \sin \phi, \quad (14)$$

and, if a_1 and b_1 are the values of a and b for $\phi = \phi_1$ and $l = L$,

$$X = a_1 + b_1 \cos \phi_1, \quad (13a)$$

$$Y = b_1 \sin \phi_1. \quad (14a)$$

or, if Tables are not available,

$$X = R \sin \phi_1 + c, \quad (13b)$$

$$Y = R \text{ versin } \phi_1 + s. \quad (14b)$$

Checks on the calculations for setting out can be made as follows:—

In Fig. 306 produce VS to meet OC at P and let SP = w , PQ = e , CQ = E , TV = a_1 , VS = b_1 . Calculate w from

$$w = R \tan \left(\frac{1}{2} I - \phi_1 \right)$$

and then E and e from one or other of the following expressions

$$\begin{aligned} E &= (R + s) \sec \frac{1}{2} I - R \\ &= (R + s) \text{exsec } \frac{1}{2} I + s \\ &= (R + s) \tan \frac{1}{2} I \tan \frac{1}{2} I + s, \\ e &= R \sec \left(\frac{1}{2} I - \phi_1 \right) - R \\ &= R \text{exsec } \left(\frac{1}{2} I - \phi_1 \right) \\ &= w \tan \frac{1}{2} \left(\frac{1}{2} I - \phi_1 \right), \end{aligned} \quad (15)$$

where, in the second expressions for E and e , $\text{exsec} ()$ has been written for the external secant, $[\sec () - 1]$, which can be obtained at once from Tables of natural secants by subtracting 1 from the tabulated secant. a_1 and b_1 are then obtained either direct from Table 1, pages 638–640, or else computed from

$$a_1 = (X - Y \cot \phi_1), \quad b_1 = Y \text{cosec } \phi_1, \quad (16)$$

X and Y having been calculated from (13(b)) and (14(b)).

Finally, by solving the triangle VPC by the sine rule, we have:

$$(T - a_1) \sec \left(\frac{1}{2} I - \phi_1 \right) = (E - e) \text{cosec } \phi_1 = (w + b_1) \sec \frac{1}{2} I \quad (17)$$

Each of these terms can be calculated in turn and all should give the same answer.

Of the expressions for E and e given above, the second one for E is useful when a calculating machine is available, because, having set up $(R + s)$, multiplication in turn by $\tan \frac{1}{2} I$ and $\text{exsec } \frac{1}{2} I$ gives the first terms in the expressions for T and E , while the third expression is useful if logarithms are used since the logarithm of the first term is obtained by adding $\log \tan \frac{1}{2} I$ to the $\log (R + s) \tan \frac{1}{2} I$

already obtained in calculating T . Similarly, both the second and third expressions for e are useful for machine computation and the third for computation by logarithms as $\log w$ has already been found and $\log \tan \frac{1}{2}(I - \phi_1)$ can be obtained directly from Tables.

As a check on the field work, the whole curve, both circular and end transitions, can be laid out in two halves, one half starting at one tangent point T , and the other half at the other tangent point T' , when the two halves should meet at the mid-point of the circular curve within the limits of accuracy of ordinary measurement. A further check is to tape the external distance E and compare the measured and computed values. Still another check is to establish the point of intersection P of the tangents at the ends of the circular arc and then to measure the external distance, e , of the circle itself.

Difference between the Total Length of a Series of Equal Chords and the Length of the Corresponding Arc of an Euler Spiral.—

Suppose that the curve is laid out with a series of p equal chords and let d_1 be the length of one of these chords and l_p the length of the arc of the curve to the end of the p .th. chord. Then it can easily be shown that

$$l_p = p.d_1 + \frac{p(2p-1)(2p+1)}{3} \cdot \frac{d_1^5}{96R^2L^2} + \dots\dots\dots$$

in which the second term on the right is usually very small and the omitted higher terms negligible.

If, in the whole length L of the curve, there are p equal chords of length d_1 and then a short end chord of length d_2 , where $d_2 < d_1$, then

$$L = p.d_1 + d_2 + \frac{p(2p-1)(2p+1)}{3} \cdot \frac{d_1^5}{96R^2L^2} + \frac{d_2^3}{24} \frac{(pd_1 + \frac{1}{2}d_2)^2}{R^2L^2} + \dots\dots$$

the last term being quite negligible in all ordinary cases.

As an example, take an Euler Spiral in which $R = 1,000$ ft., and $L = 650$ ft., and which is to be laid out with a succession of equal 25ft. chords. Then $p = 26$ and the length of arc corresponding to 26 chord lengths each of 25 ft., is given by

$$\begin{aligned} L &= 25.26 + \frac{26.51.53}{3} \cdot \frac{(25)^5}{96 \times (1000)^2 \cdot (692)^2} + \dots\dots \\ &= 650 + 0.006 + \dots\dots, \end{aligned}$$

so that the difference between the length of the whole arc and 26 chord lengths is only 0.006 ft. If the chords were 50 ft. long, p would be 13 and the difference between the arc length and the length of 13 chords of 50 ft. each would be only 0.023 ft.—approximately 1/30,000. Thus, in nearly all cases in practice the difference between the sum of the lengths of the chords used in laying out a curve and the true length of the arc of the curve is very small.

'Versine' Method of Establishing Intermediate Points on the Euler Spiral Transition Curve.—Suppose that, instead of using chords of 25 ft. in the last example, we used chords of 50 ft. Intermediate points on the curve at 25 ft. intervals can then be established by setting out offsets from the centre of each chord, these offsets being calculated by the formula

$$o_1 = \frac{C^2}{8r}$$

given on page 436 (Fig. 295) for laying out circular curves, the value of r used in their case being given by

$$r = \frac{RL}{l_c}$$

where l_c is the length of the curve from its origin to the point where the extra point is required. Since o_1 in Fig. 295 is equal to $r(1 - \cos \delta) = r \text{ versin } \delta$, where r is the radius of the circle and δ half the angle subtended at the centre by the chord AB, this method of laying out intermediate points is sometimes called the "versine" method. It is often used in "string-lining" curves,—that is, smoothing out or rectifying existing curves—and for restoring the odd peg which has been displaced or lost during, or prior to, construction. The assumption, of course, is that the arc considered is so short that it will not differ appreciably in shape from an arc of equal length on a circle whose radius is the radius of curvature of the spiral at the middle point of the arc.

Proof of Formulae for the Lemniscate.—The lemniscate is defined as a curve whose polar equation is

$$\rho = C\sqrt{\sin 2\alpha}$$

where ρ denotes the polar ray, α the polar deflection angle and C is a constant.

In Fig. 410 P is a point on the curve, OP the polar ray ρ , and angle POX the polar deflection angle α . Let Q be another point on the curve very close to P. Then, drawing PR perpendicular to OQ, we have angle QOP = $\delta\alpha$, OR = ρ , RQ = $\delta\rho$. Also, as Q approaches nearer and nearer to P the line QPV, which cuts the axis OX in V, becomes the tangent at P and the angle RQP is very approximately equal to the angle OPV = θ . Hence, in the limit,

$$\begin{aligned} \tan \theta &= \tan RQP \\ &= \rho \frac{d\alpha}{d\rho} \end{aligned}$$

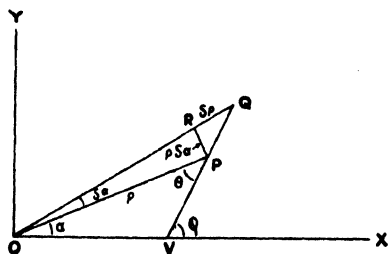


Fig. 410

But, from the equation of the curve,

$$\frac{d\rho}{da} = \frac{C \cos 2a}{\sqrt{\sin 2a}}$$

$$\therefore \tan \theta = C \sqrt{\sin 2a} \sqrt{\sin 2a} / C \cos 2a \\ = \tan 2a$$

$$\therefore \theta = 2a,$$

and

$$\phi = a + \theta \\ = 3a,$$

a relation which is thus rigorously true for the lemniscate but is only approximately true for the Euler spiral, cubic spiral and cubic parabola.

To find the radius of curvature at any point, we have, by the usual formula for polar co-ordinates:

$$r = \frac{\left[\rho^2 + \left(\frac{d\rho}{da} \right)^2 \right]^{\frac{3}{2}}}{\left[\rho^2 + 2 \left(\frac{d\rho}{da} \right)^2 - \rho \frac{d^2\rho}{da^2} \right]}$$

Here

$$\frac{d\rho}{da} = \frac{C \cos 2a}{\sqrt{\sin 2a}}.$$

$$\therefore \frac{d^2\rho}{da^2} = - \frac{C}{(\sin 2a)^{\frac{3}{2}}} \left[1 + \sin^2 2a \right]$$

$$\therefore r = \frac{C}{3\sqrt{\sin 2a}} = \frac{\rho}{2 \sin 2a}.$$

$$\therefore C = \sqrt{3\rho r}$$

Again,

$$\frac{dl}{d\phi} = r = \frac{C}{3\sqrt{\sin 2a}}$$

But

$$\frac{dl}{d\phi} = \frac{dl}{da} \cdot \frac{da}{d\phi} = \frac{1}{3} \frac{dl}{da}$$

$$\therefore \frac{dl}{3da} = r = \frac{C}{3\sqrt{\sin 2a}}.$$

$$\therefore l = C \int \frac{da}{\sqrt{\sin 2a}} = C \int \sqrt{\operatorname{cosec} 2a} da$$

Now

$$\sqrt{\operatorname{cosec} 2a} = \frac{1}{\sqrt{2a}} \left(1 + \frac{2}{3}a^2 + \frac{14}{45}a^4 + \frac{124}{945}a^6 + \frac{254}{4725}a^8 + \dots \right)^{\frac{1}{2}}$$

Whence, by applying the binomial theorem to the terms in brackets,

$$\sqrt{\operatorname{cosec} 2a} = \frac{1}{\sqrt{2a}} \left(1 + \frac{1}{3}a^2 + \frac{1}{10}a^4 + \frac{61}{1890}a^6 + \frac{1261}{113,400}a^8 + \dots \right).$$

and therefore by integration

$$l = C\sqrt{2a} \left(1 + \frac{1}{15}a^2 + \frac{1}{90}a^4 + \frac{61}{24570}a^6 + \frac{1261}{1,927,800}a^8 + \dots \right),$$

the constant of integration vanishing since $l = 0$ when $a = 0$.

TABLES

The following two Tables are useful for working out problems in transition curves. The first, which relates to the *Euler Spiral*, gives certain selected general spiral functions as coefficients of L or l with ϕ , the deflection angle, as argument at intervals of 1° , and is an abridgment from larger 7-figure Tables with ϕ as argument at intervals of 10 minutes of arc which have been designed and computed by Mr. A. G. Tyson, who has kindly given permission for the reproduction here of the abridgment. In this Table, the factors in the second and fourth columns when multiplied by L , the total length of the transition, give the shift, s , and shift point distance, c ; the sixth column gives the correction, γ , to $\frac{1}{3}\phi$ to obtain a ; and the eighth and tenth columns give factors which, when multiplied by l , the length of the curve at any point, give the long and short sub-tangents, a and b , at that point.

The second Table, which also is an abridgment from a larger Table designed and computed by Mr. Tyson, relates to the *Lemniscate* transition curve and gives a factor, k , for use with the formula

$$l \text{ (unit-chords)} = k\sqrt{a} \text{ (in minutes)}$$

for determining the length of the curve in unit-chords (pages 458-461) for different values of a at 1° intervals between 0° and 50° .

It will be noticed that, in Table II, the first differences, which are tabulated in the third and seventh columns, are not constant but differ by amounts—the second differences—which are shown in the fourth and eighth columns. Also, the first differences for some of the quantities tabulated in Table I are not constant but differ by more than one or two units in the last place. In such cases, straight linear interpolation to determine values of the functions for intermediate values of ϕ or a will not give last figure accuracy, and, for full accuracy, it will be necessary to use the rules for interpolation by successive differences given in Chapter I of Vol. II. Thus, using Table II to find k for $a = 27^\circ 36'$, we have by the Bessel formula for interpolation,

$$f_n = f_0 + n\Delta'_{\frac{1}{2}} + \frac{n(n-1)}{4}(\Delta''_0 + \Delta''_1) + \dots,$$

$$\begin{aligned} n &= \frac{36}{60} = 0.6, \quad (n-1) = -0.4, \quad f_0 = k \text{ for } 27^\circ = 0.253845, \\ \Delta'_{\frac{1}{2}} &= +0.000302, \quad \Delta''_0 = +0.000012, \quad \Delta''_1 = +0.000014. \\ k_n &= 0.253845 + 0.6 \times 302 \times 10^{-8} + \frac{(0.6) \times (-0.4)}{4} \\ &\quad (+12 + 14) \times 10^{-8} + \dots \\ &= 0.253845 + 0.0001812 - 0.0000016 \\ &= 0.254025. \end{aligned}$$

Similarly, to find $\frac{a}{l}$ from Table I for $\phi = 87^\circ 24'$,

$$\begin{aligned} n &= \frac{24}{60} = 0.4, \quad (n-1) = -0.6, \quad f_0 = 0.77031, \quad \Delta'_{\frac{1}{2}} = +0.00312, \\ \Delta''_0 &= +0.00008, \quad \Delta''_1 = +0.00007, \quad \Delta''_0 + \Delta''_1 = +15 \times 10^{-8} \\ \therefore \frac{a}{l} &= 0.77031 + 0.4 \times 312 \times 10^{-8} + \frac{(0.4) \times (-0.6)}{4} \times 15 \times 10^{-8} \\ &= 0.77031 + 0.001248 - 0.000009 \\ &= 0.77155. \end{aligned}$$

Alternatively, in the last example, since second differences are not tabulated in Table I, it might be more convenient to work with first differences only by using $\Delta'_{\frac{1}{2}} - \Delta'_{-\frac{1}{2}}$ instead of $\Delta''_0 + \Delta''_1$ in the third term of the Bessel formula. Thus, in the example,

$$\begin{aligned} \Delta'_{\frac{1}{2}} &= +0.00319, \quad \Delta'_{-\frac{1}{2}} = +0.00304 \\ \therefore \Delta'_{\frac{1}{2}} - \Delta'_{-\frac{1}{2}} &= +15 \times 10^{-8} = \Delta''_0 + \Delta''_1 \text{ as before.} \end{aligned}$$

Straight linear interpolation would give $k_n = 0.254026$ in the first example and $\frac{a}{l} = 0.77156$ in the second, so that taking into account second differences has very little effect, and it should normally be sufficient to ignore the third term and proceed by simple linear interpolation. The coefficient $\frac{1}{4}n(n-1)$ of this term is a maximum when $n = 0.5$, when its value is -0.0625 .

APPENDIX IV

TABLE I

FACTORS FOR THE LAYOUT OF THE EULER SPIRAL
TRANSITION CURVE

ϕ	Shift s/L	First Diff. Δ'	Shift- Point Distance c/L	First Diff. Δ'	Deflection Angle Correction $\gamma = \frac{1}{3}\phi - \alpha$	First Diff. Δ'	Long Sub- tangent a/l	First Diff. Δ'	Short Sub- tangent b/l	First Diff. Δ'
0°	0-00000		0-50000		00' 00"		0-66667		0-33333	
1	00145	145	49999	-1	00 00	-	66668	1	33334	1
2	00291	146	49998	-1	00 00	-	66671	3	33337	3
3	00436	145	49995	-3	00 00	-	66676	5	33342	5
4	00582	146	49992	-3	00 00	-	66684	8	33349	7
		145		-5	00 00	-		9		9
5°	0-00727		0-49987		00' 00"		0-66693		0-33358	
6	00872	145	49982	-5	00 01	1	66705	12	33368	10
7	01018	146	49975	-7	00 01	-	66719	14	33381	13
8	01163	145	49968	-7	00 02	1	66735	16	33395	14
9	01308	145	49959	-9	00 02	-	66753	18	33412	17
		145		-10				20		18
10°	0-01453		0-49949		00' 03"		0-66773		0-33430	
11	01598	145	49939	-10	00 04	1	66796	23	33451	21
12	01743	145	49927	-12	00 05	1	66820	24	33473	22
13	01887	144	49914	-13	00 07	2	66847	27	33498	25
14	02032	145	49901	-13	00 09	2	66876	29	33524	26
		144		-15		1		32		28
15°	0-02176		0-49886		00' 10"		0-66908		0-33552	
16	02321	145	49870	-16	00 13	3	66941	33	33583	31
17	02465	144	49854	-16	00 15	2	66978	37	33615	32
18	02609	144	49836	-18	00 18	3	67015	37	33650	35
19	02753	144	49817	-19	00 21	3	67055	40	33686	36
		143		-19		4		42		39
20°	0-02896		0-47998		00' 25"		0-67097		0-33725	
21	03040	144	49777	-21	00 29	4	67142	45	33766	41
22	03183	143	49755	-22	00 33	4	67189	47	33809	43
23	03326	143	49733	-22	00 38	5	67238	49	33854	45
24	03469	143	49709	-24	00 43	5	67290	52	33901	47
		142		-25		6		54		49
25°	0-03611		0-49684		00' 49"		0-67344		0-33950	
26	03754	143	49659	-25	00 55	6	67400	56	34001	51
27	03896	142	49632	-27	01 01	6	67459	59	34055	54
28	04038	142	49605	-27	01 08	7	67520	61	34111	56
29	04180	142	49576	-29	01 16	8	67584	64	34169	58
		141		-29		8		66		60
30°	0-04321		0-49547		01' 24"		0-67650		0-34229	
31	04462	141	49516	-31	01 33	9	67718	68	34292	63
32	04603	141	49485	-31	01 42	9	67790	72	34356	64
33	04743	140	49452	-33	01 52	10	67863	73	34424	68
34	04883	140	49419	-33	02 03	11	67939	76	34493	69
		140		-35		11		79		72
35°	0-05023		0-49384		02' 14"		0-68018		0-34565	
36	05163	140	49349	-35	02 26	12	68100	82	34640	75
37	05302	139	49313	-36	02 38	12	68184	84	34717	77
38	05441	139	49276	-37	02 52	14	68270	86	34796	79
39	05579	138	49238	-38	02 06	14	68360	90	34878	82
		139		-39		14		92		84

APPENDIX IV

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TABLE I—continued

ϕ	Shift s/L	First Diff. Δ'	Shift- Point Distance c/L	First Diff. Δ'	Deflection Angle Correction $\gamma = \frac{1}{2}\phi - \alpha$	First Diff. Δ'	Long Sub- tangent a/l	First Diff. Δ'	Short Sub- tangent b/l	First Diff. Δ'
40°	0.05718	137	0.49199	-40	03' 20"	16	0.68452	95	0.34962	87
41	05855	138	49159	-41	03 36	16	68547	98	35049	90
42	05993	137	49118	-42	03 52	17	68645	101	35139	92
43	06130	136	49076	-43	04 09	18	68746	104	35231	96
44	06266	137	49033	-44	04 27	19	68850	106	35327	97
45°	0.06403	136	0.48989	-44	04' 46"	20	0.68956	110	0.35424	101
46	06539	135	48945	-46	05 06	21	69066	113	35525	104
47	06674	135	48899	-46	05 27	21	69179	116	35629	106
48	06809	134	48853	-48	05 48	23	69295	119	35735	109
49	06943	135	48805	-48	06 11	23	69414	122	35844	113
50°	0.07078	133	0.48757	-49	06' 34"	25	0.69536	125	0.35957	115
51	07211	133	48708	-50	06 59	25	69661	129	36072	118
52	07344	133	48658	-51	07 24	26	69790	132	36190	122
53	07477	132	48607	-51	07 50	28	69922	136	36312	125
54	07609	132	48556	-53	08 18	29	70058	139	36437	128
55°	0.07741	132	0.48503	-54	08' 47"	29	0.70197	143	0.36565	132
56	07873	130	48449	-54	09 16	31	70340	146	36697	135
57	08003	131	48395	-55	09 47	32	70486	150	36832	138
58	08134	129	48340	-56	10 19	33	70636	154	36970	142
59	08263	130	48284	-57	10 52	35	70790	158	37112	146
60°	0.08393	128	0.48227	-58	11' 27"	35	0.70948	162	0.37258	149
61	08521	129	48169	-58	12 02	37	71110	165	37407	153
62	08650	127	48111	-60	12 39	38	71275	170	37560	157
63	08777	127	48051	-60	13 17	39	71445	174	37717	161
64	08904	127	47991	-61	13 56	41	71619	178	37878	165
65°	0.09031	126	0.47930	-62	14' 37"	42	0.71797	182	0.38043	169
66	09157	125	47868	-62	15 19	44	71979	187	38212	173
67	09282	125	47806	-64	16 03	44	72166	192	38385	178
68	09407	125	47742	-64	16 47	47	72358	196	38563	182
69	09532	123	47678	-65	17 34	47	72554	201	38745	186
70°	0.09655	123	0.47613	-66	18' 21"	49	0.72755	206	0.38931	191
71	09778	123	47547	-66	19 10	51	72961	211	39122	196
72	09901	122	47481	-68	20 01	52	73172	216	39318	201
73	10023	121	47413	-68	20 53	54	73388	221	39519	206
74	10144	120	47345	-69	21 47	55	73609	227	39725	211
75°	0.10264	120	0.47276	-69	22' 42"	57	0.73836	232	0.39936	217
76	10384	120	47207	-71	23 39	58	74068	238	40153	221
77	10504	119	47136	-71	24 37	60	74306	243	40374	228
78	10623	118	47065	-71	25 37	62	74549	250	40602	233
79	10741	117	46994	-73	26 39	64	74799	256	40835	239
80°	0.10858	117	0.46921	-73	27' 43"	65	0.75055	262	0.41074	245
81	10975	116	46848	-74	28 48	67	75317	268	41319	251
82	11091	115	46774	-75	29 55	69	75585	275	41570	257
83	11206	115	46699	-75	31 04	71	75860	282	41827	264
84	11321	114	46624	-76	32 15	72	76142	289	42091	271

TABLE I—continued

ϕ	Shift s/L	First Diff. Δ'	Shift- Point Distance c/L	First Diff. Δ'	Deflection Angle Correction $\gamma = \frac{1}{2}\phi - \alpha$	First Diff. Δ'	Long Sub- tangent a/l	First Diff. Δ'	Short Sub- tangent b/l	First Diff. Δ'
85°	0.11435	113	0.46548	-76	33' 27"	74	0.76431	296	0.42362	278
86	11548	113	46472	-78	34 41	77	76727	304	42640	285
87	11661	112	46394	-78	35 58	78	77031	312	42925	293
88	11773	111	46316	-78	37 16	80	77343	319	43218	300
89	11884	111	46238	-80	38 36	83	77662	327	43518	308
90°	0.11995	110	0.46158	-80	39' 59"	84	0.77989	336	0.43826	316
91	12105	109	46078	-80	41 23	86	78325	345	44142	325
92	12214	108	45998	-81	42 49	89	78670	353	44467	333
93	12322	108	45917	-82	44 18	91	79023	363	44800	342
94	12430	107	45835	-82	45 49	92	79386	373	45142	352
95°	0.12537	106	0.45753	-83	47' 21"	96	0.79759	382	0.45494	361
96	12643	106	45670	-84	48 57	97	80141	392	45855	371
97	12749	104	45586	-84	50 34	99	80533	403	46226	382
98	12853	104	45502	-85	52 13	103	80936	414	46608	392
99	12957	103	45417	-85	53 55	105	81350	426	47000	403
100°	0.13060		0.45332		55' 40"		0.81776		0.47403	

APPENDIX IV

TABLE II

FACTOR k FOR DETERMINING THE LENGTH OF AN ARC OF THE LEMNISCATE GIVEN THE POLAR DEFLECTION ANGLE α

Formula — l (in unit-chords) = $k\sqrt{\alpha}$ (in minutes of arc)

α	k	Δ'	Δ''	α	k	Δ'	Δ''
0°	0.250000			25°	0.253278		
1	0005	5	10	26	3555	277	13
2	0020	15	10	27	3845	290	12
3	0045	25	11	28	4147	302	14
4	0081	36	10	29	4463	316	13
		46				329	
5°	0.250127			30°	0.254792		
6	0183	56	10	31	5134	342	13
7	0249	66	10	32	5489	355	13
8	0326	77	11	33	5859	370	15
9	0413	87	10	34	5859	384	14
		97			6243	399	15
10°	0.250510			35°	0.256642		
11	0618	108	11	36	7055	413	14
12	0736	118	10	37	7484	429	16
13	0865	129	11	38	7928	444	15
14	1005	140	11	39	8388	460	16
		150	10			477	17
15°	0.251155			40°	0.258865		
16	1317	162	12	41	9359	494	17
17	1489	172	10	42	9870	511	17
18	1672	183	11	43	260399	529	18
19	1867	195	12	44	0946	547	18
		206	11			566	19
20°	0.252073			45°	0.261512		
21	2290	217	11	46	2098	586	20
22	2519	229	12	47	2703	605	19
23	2760	241	12	48	3330	627	22
24	3013	253	12	49	3978	648	21
		265				670	22
25°	0.253278			50°	0.264648		
			12				

APPENDIX V

EXAMPLE OF ANALYTICAL SOLUTION OF THE THREE-POINT PROBLEM

THE Three-Point Problem, described on pages 563-565, is not only used in hydrographical work for the location of the position of soundings but is also very extensively employed in minor triangulation and in fixing points required for ordinary surveying purposes. Accordingly, as an analytical solution, of which some form is used when moderate accuracy is needed, involves a fairly complicated computation, it seems desirable to give here an example of such a computation using the method described on page 565.

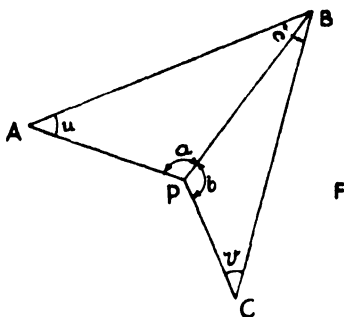


FIG. 410.

In Fig. 410 let A, B and C be the three fixed points and P the point to be fixed, the angles observed at P being $APB = a = 122^\circ 24' 23''$ and $BPC = b = 109^\circ 03' 05''$. Also, let the co-ordinates of A, B and C be :

Point		X	Y
A	31,252.4	14,641.3
B	39,761.8	43,338.6
C	14,297.1	30,771.8

It is required to find the co-ordinates of the point P.

In this case the bearings and distances of the lines AB and BC and the angle c have not been given and they must consequently be computed from the given co-ordinates. Hence, we proceed as follows :

Bearing and Distance BA

	X	Y		
A	31,252.4	14,641.3	$\log \Delta Y$	= 4.457 8410
			$\log \Delta X$	= 3.929 8989
B	39,761.8	43,338.6	$\log \tan BA$	= 0.527 9421
Δ	- 8,509.4	- 28,697.3	Bearing BA	= $253^\circ 29' 01''.4$
$\log \Delta X$	= 3.929 8989		$\log \Delta Y$	= 4.457 8410
$\log \cos BA$	= 9.453 7582		$\log \sin BA$	= 9.981 7003
$\log AB$	= 4.476 1407		$\log AB$	= 4.476 1407

Bearing and Distance BC

	X	Y	$\log \Delta Y$	$= 4.099\ 2247$
C	14,297.1	30,771.8	$\log \Delta X$	$= 4.405\ 9386$
B	39,761.8	43,338.6	$\log \tan BC$	$= 9.693\ 2861$
Δ	<u>25,464.7</u>	<u>12,566.8</u>	Bearing BC	$= 206^\circ\ 15'\ 58''.6$
$\log \Delta X$	$= 4.405\ 9386$		$\log \Delta Y$	$= 4.099\ 2247$
$\log \cos BC$	$= 9.952\ 6700$		$\log \sin BC$	$= 9.645\ 9560$
$\log BC$	$= 4.453\ 2686$	\longleftrightarrow	$\log BC$	$= 4.453\ 2687$

Calculation of θ , u and v

Bg.BA	$= 253^\circ\ 29'\ 01''.4$	$\log BC$	$= 4.453\ 2686$
Bg.BC	$= 206^\circ\ 15'\ 58''.6$	$\log \sin a$	$= 9.926\ 4804$
c	<u>$= 47^\circ\ 13'\ 02''.8$</u>	$\log \sin s$	<u>$= 9.995\ 0026$</u>
a	$= 122^\circ\ 24'\ 23''$	$\log BC \sin a \sin s$	$= 4.374\ 7516$
b	<u>$= 109^\circ\ 03'\ 05''$</u>		
$360^\circ - s$	$= 278^\circ\ 40'\ 30''.8$	$\log AB$	$4.476\ 1407$
s	$= 81^\circ\ 19'\ 29''.2$	$\log \sin b$	<u>$= 9.975\ 5357$</u>
		$\log AB \sin b$	$= 4.451\ 6764$
		$\log BC \sin a \sin s$	<u>$= 4.374\ 7516$</u>
θ	$= 39^\circ\ 57'\ 07''.3$	$\log \cot \theta$	$= 0.076\ 9248$
s	<u>$= 81^\circ\ 19'\ 29''.2$</u>		
$(\theta + s)$	$= 121^\circ\ 16'\ 36''.5$		
$\log \sin (\theta + s)$	$= 9.931\ 7980$		
$\log \sin \theta$	$= 9.807\ 6338$		
	<u>$0.124\ 1642$</u>		
$\log \sin s$	$= 9.995\ 0026$		
$\log \cot u$	<u>$= 0.129\ 1616$</u>		
u	$36^\circ\ 36'\ 10''.3$		
s	<u>$81^\circ\ 19'\ 29''.2$</u>		
v	$44^\circ\ 43'\ 18''.9$		

Solution of Triangles APB and CPB

u	$= 36^{\circ} 36' 10'' \cdot 3$	v	$= 44^{\circ} 43' 18'' \cdot 9$
a	$= 122^{\circ} 24' 23''$	b	$= 109^{\circ} 03' 05''$
$180^{\circ}-ABP$	$= 159^{\circ} 00' 33'' \cdot 3$	$180^{\circ}-CBP$	$= 153^{\circ} 46' 23'' \cdot 9$
ABP	$= 20^{\circ} 59' 26'' \cdot 7$	CBP	$= 26^{\circ} 13' 36'' \cdot 1$
$\log PB$	$= 4 \cdot 325 \ 0996$	$\log PB$	$= 4 \cdot 325 \ 0998$
$\log \sin u$	$= 9 \cdot 775 \ 4393$	$\log \sin v$	$= 9 \cdot 847 \ 3669$
$\log AB$	$= 4 \cdot 476 \ 1407$	$\log BC$	$= 4 \cdot 453 \ 2686$
$\log \operatorname{cosec} a$	$= 0 \cdot 073 \ 5196$	$\log \operatorname{cosec} b$	$= 0 \cdot 024 \ 4643$
$\log \sin ABP$	$= 9 \cdot 554 \ 1464$	$\log \sin CBP$	$= 9 \cdot 645 \ 3474$
$\log PA$	$= 4 \cdot 103 \ 8067$	$\log PC$	$= 4 \cdot 123 \ 0803$

Co-ordinates of P from A and C

Bg.AB	$= 73^{\circ} 29' 01'' \cdot 4$	Bg.CB	$= 26^{\circ} 15' 58'' \cdot 6$
u	$= 36^{\circ} 36' 10'' \cdot 3$	v	$= 44^{\circ} 43' 18'' \cdot 9$
Bg.AP	$= 110^{\circ} 05' 11'' \cdot 7$	Bg.CP	$= 341^{\circ} 32' 39'' \cdot 7$
$\log \triangle X$	$= 3 \cdot 639 \ 6573$	$\log \triangle X$	$= 4 \cdot 100 \ 1493$
$\log \cos AP$	$= 9 \cdot 535 \ 8506$	$\log \cos CP$	$= 9 \cdot 977 \ 0690$
$\log AP$	$= 4 \cdot 103 \ 8067$	$\log CP$	$= 4 \cdot 123 \ 0803$
$\log \sin AP$	$= 9 \cdot 972 \ 7464$	$\log \sin CP$	$= 9 \cdot 500 \ 4702$
$\log \triangle Y$	$= 4 \cdot 076 \ 5531$	$\log \triangle Y$	$= 3 \cdot 623 \ 5505$

	X	Y		X	Y
A	$= 31,252 \cdot 4$	$14,641 \cdot 3$	C	$= 14,297 \cdot 1$	$30,771 \cdot 8$
\triangle	$= -4,361 \cdot 7$	$+ 11,927 \cdot 6$	\triangle	$= + 12,593 \cdot 6$	$- 4,202 \cdot 9$
P	$= 26,890 \cdot 7$	$26,568 \cdot 9$	P	$= 26,890 \cdot 7$	$26,568 \cdot 9$

Co-ordinates of P: $X = 26,890 \cdot 7$; $Y = 26,568 \cdot 9$.

Note in the above the following checks :

- (1) Two values are obtained for each of the distances AB and BC.
- (2) Two values are obtained for the distance PB by solving both of the triangles PAB and PBC.
- (3) Two sets of values are obtained for the co-ordinates of P by computing them from A and C respectively. A third set could, of course, also be obtained by computing from B, but this is hardly necessary.

If no check bearing is available, the work can be finally checked by using the co-ordinates of P to compute the bearings PA, PB and

PC (preferably by using the method of auxiliary bearings as described on page 265), and, from these bearings, the angles a and b can be computed and compared with the observed values. In practical work, however, it is always advisable, whenever this is possible, to observe the angle to a fourth fixed point. The bearing of this point from P can be computed from the co-ordinates of both points. Also, the bearings PA, PB and PC are known or can be found from the solution and hence the measured angle from any of these lines to the fourth point enables the bearing to the latter to be calculated and compared with the value computed from the co-ordinates. The additional observation to the fourth point therefore checks both the original observations and the computations.

It should be noted that this example has been computed to seven-figure accuracy to show the full working and the operation of the checks. For secondary and minor work, however, a solution with six-, or even five-, figure logarithms is quite sufficient in most cases.

SOLUTION BY ASSUMED APPROXIMATE VALUE OF u OR v

If it is possible to obtain a good approximate value for either of the angles u or v (Fig. 410), correct to a degree or so, the three point problem can be solved fairly quickly and easily by a method recently described in the *Empire Survey Review*.*. For instance, if the magnetic bearing of the line PA is observed with a compass attached to the theodolite, and the deviation of the zero of the compass from grid north is known, an approximate value of u , correct to within a degree, can be computed from the difference of bearing of AP and AB.

In Fig. 410 the bearings of BA and BC are known, or can be computed from the data, and hence the angle c is known. Also, the angles a and b have been observed and are therefore known. As on page 565, we have

$$u + v = s = 360^\circ - (a + b + c), \quad (1)$$

and hence s is a known quantity.

Again, the lengths of AB and BC are known from the data, or can be computed, and hence by solving the triangles ABP and CBP for BP, we have as on page 565

$$\frac{AB \sin u}{\sin a} = \frac{BC \sin v}{\sin b}$$

$$\therefore \log \sin u - \log \sin v = \log BC - \log AB + \log \sin a - \log \sin b \quad (2)$$

The right hand sides of equations (1) and (2) can both be calculated from quantities already known so that what we now want to

* See "A Resection Method" by S. E. Evans, *Empire Survey Review*, Vol. XII, No. 89, July, 1953. The method appears to have been described originally by E. Sidlaczek in 1875. (For its early history see letter from Prof. A. Tarczy-Hornock in the *Empire Survey Review*, Vol. XIII, No. 99, January, 1956.)

find are two angles, u and v , such that their sum and the difference of their log sines are known quantities.

Let u_1 be an approximate assumed value of u . Then calculate an approximate value v_1 , of v from

$$u_1 + v_1 = s = u + v \quad (3)$$

$$\text{Let } u = u_1 + \Delta u \quad (4)$$

$$v = v_1 + \Delta v$$

Then, from (3),

$$\begin{aligned} u_1 + v_1 &= u_1 + \Delta u + v_1 + \Delta v \\ \therefore \Delta v &= -\Delta u \end{aligned} \quad (5)$$

Again, let d_1 and d_2 be the tabulated differences in the log sines of the angles u_1 and v_1 for 1" difference in angle so that $\log \sin (u_1 + 1") - \log \sin u_1 = d_1$, $\log \sin (v_1 + 1") - \log \sin v_1 = d_2$. These differences are to be found tabulated in Shortrede's and similar tables on the same page as the log sine for each angle, and so are easily found. Then, assuming u_1 and v_1 to be so close to u and v that d_1 will be the same for all angles between u_1 and u , and d_2 the same for all angles between v_1 and v ,

$$\log \sin u = \log \sin u_1 + \Delta u \cdot d_1$$

$$\log \sin v = \log \sin v_1 + \Delta v \cdot d_2$$

$$\therefore \log \sin u - \log \sin v = \log \sin u_1 + \Delta u \cdot d_1 - \log \sin v_1 - \Delta v \cdot d_2$$

$$\therefore \Delta u \cdot d_1 - \Delta v \cdot d_2 = (\log \sin u - \log \sin v) - (\log \sin u_1 - \log \sin v_1).$$

or, using (5),

$$\Delta u = \{(\log \sin u - \log \sin v) - (\log \sin u_1 - \log \sin v_1)\} / (d_1 + d_2) \quad (6)$$

The first term in brackets on the right can be computed from (2) and the second term in brackets, and the denominator of the fraction, found from the assumed values of u_1 and v_1 , so that Δu can be computed, and having found Δu , we have $\Delta v = -\Delta u$.

The solution so far depends on the assumption that d_1 and d_2 are constants throughout the intervals $(u - u_1)$ and $(v - v_1)$. d_1 and d_2 , however, vary with the angle so that, unless $(u - u_1)$ and $(v - v_1)$ are very small, d_1 for u_1 will not be the same as the d_1 for u or the d_2 for v_1 the same as the d_2 for v . Hence, a first solution will usually not satisfy (2) exactly and in that case it will be necessary to proceed to a second, or even a third approximation, using as new approximate values the u and v found from the preliminary solution.

Example. As an example take the data from the previous example (page 642), viz.—

$$a = 122^\circ 24' 23'';$$

$$b = 109^\circ 03' 05''.$$

$$\log AB = 4.476 \ 1407;$$

$$\text{Bg. BA} = 253 \ 29 \ 01.4.$$

$$\log BC = 4.453 \ 2686;$$

$$\text{Bg. BC} = 206 \ 15 \ 58.6.$$

and assume that, after plotting an assumed approximate position of

the point P on a plan on which A, B and C have already been plotted, we measure the angle u with a protractor and find it to be 39° . Then,

$c = 47^\circ 13' 02''.8$	$\log BC$	$= 4.453\ 2686$
$a = 122\ 24\ 23$	$\log BA$	$= 4.476\ 1407$
$b = 109\ 03\ 05$		<hr/> 9.977 1279
$a + b + c = 278\ 40\ 30.8$	$\log \sin a$	$= 9.926\ 4804$
$360\ 00\ 00$		<hr/> 9.903 6083
$s = 81\ 19\ 29.2$	$\log \sin b$	$= 9.975\ 5357$
$u_1 = 39\ 00\ 00$	$(\log \sin u - \log \sin v)$	$= 9.928\ 0726$
$v_1 = 42\ 19\ 29.2$		
$\log \sin u_1 = 9.798\ 8718$	$d_1 = + 26.0$	
$\log \sin v_1 = 9.828\ 2294$	$d_2 = + 23.1$	
$(\log \sin u_1 - \log \sin v_1) = 9.970\ 6424$	$d_1 + d_2 = + 49.1$	
$(\log \sin u - \log \sin v) = 9.928\ 0726$		
$[(\log \sin u - \log \sin v) - (\log \sin u_1 - \log \sin v_1)]$	$\Delta u = \frac{(-42\ 5698)}{(+49.1)}$	
	$= -8670''.4$	
	$= -2^\circ 24' 30''.4$	
$\Delta u = -2^\circ 24' 30''.4$	$\Delta v = +2^\circ 24' 30''.4$	
$u_1 = 39\ 00\ 00$	$v_1 = 42\ 19\ 29.2$	
$u(2\text{nd. approx.}) = 36\ 35\ 29.6$	$v(2\text{nd. approx.}) = 44\ 43\ 59.6$	
Hence, the second approximations are $u = 36^\circ 35' 29''.6$, $v = 44^\circ 43' 59''.6$. Take these as the new u_1 and v_1 .		
$\log \sin u_1 = 9.775\ 3239$	$d_1 = + 28.3$	
$\log \sin v_1 = 9.847\ 4530$	$d_2 = + 21.3$	
$(\log \sin u_1 - \log \sin v_1) = 9.927\ 8709$	$d_1 + d_2 = + 49.6$	
$(\log \sin u - \log \sin v) = 9.928\ 0726$		
$[(\log \sin u - \log \sin v) + 2017]$	$\Delta u = \frac{(+2017)}{(+49.6)} = +40''.7$	
$-(\log \sin u_1 - \log \sin v_1)]$	$\Delta v = -40''.7$	
$\Delta u = +00^\circ 00' 40''.7$	$\Delta v = -00^\circ 00' 40''.7$	
$u_1 = 36\ 35\ 29.6$	$v_1 = 44\ 43\ 59.6$	
$u(3\text{rd. approx.}) = 36\ 36\ 10.3$	$v(3\text{rd. approx.}) = 44\ 43\ 18.9$	

For a fourth approximation take $u_1 = 36^\circ 36' 10''.3$, $v_1 = 44^\circ 43' 18''.9$.

$\log \sin u_1 = 9.775.4392$	$d_1 = + 28.4$
$\log \sin v_1 = 9.847.3669$	$d_2 = + 21.2$
<hr/> $(\log \sin u_1 - \log \sin v_1) = 9.928.0723$	<hr/> $d_1 + d_2 = + 49.6$
$(\log \sin u - \log \sin v) = 9.928.0726$	
<hr/>	
	$+3$

The difference is now only 3 in the seventh place of decimals, which, for purposes of trigonometrical resection, is negligible. Thus, the solution is $u = 36^\circ 36' 10''.3$, $v = 44^\circ 43' 18''.9$. Having found u and v , the problem is completed as before.

In the example, the initial assumed value of u differs by almost $2\frac{1}{4}^\circ$ from the final value and the labour of computation is very little, if any, less than what it is by the method already used. It is when the initial assumed approximate value of u is reasonably close to the true value, as when it is obtained by reliable magnetic observations, that the method comes into its own. Hence, if much trigonometrical resection is being done, a compass fitted to the theodolite is a very useful accessory to have.

Again, as before, this example has been worked out with seven-figure logarithms and with certain angles taken to tenths of a second but in practice the accuracy to be expected from a trigonometrical resection does not justify the use of more than six figures in the logarithms or single seconds in an angle.

ANSWERS TO EXAMPLES

CHAPTER II (page 174)

1. 1,332.6 ft.
2. 99.642 ft. [Note here that the divisor in the solution is the apparent length, 1,206, and not the true length, 1,201. This can be seen as follows: Let x = error of tape, so that $100 + x$ = true length of tape, and let n be the number of times that the tape has got to be applied. Then $1,201.44 = n(100 + x)$. Also, apparent length of line = $n \times 100 = 1,205.75$, so that $n = \frac{1,205.75}{100}$.
3. 2,362.09 ft.: 44.3° F.
4. 635.89 ft.
5. 0.024 ft.
6. 299.981 ft.: 88.0° F.
7. 2,698.34 ft.
8. 76.37 ft.
9. 76.40 ft.
10. 6,472.27 ft..
11. 6,472.87 ft., the correction being made up of 0.092 ft. due to elasticity and 0.594 ft. due to the change in the sag correction. [Note that in this problem the difference in pull is too large for the correction to be worked out by the formula:

Change in Sag Correction due to Change δF in Pull = $\frac{2C}{F} \delta F$,

which is given on page 169, as this formula only holds when δF is small. The full expression when δF is fairly large is:

Change in Sag Correction =

$$-\frac{2C}{F} \delta F + \frac{3C}{F^2} \delta F^2 - \frac{4C}{F^3} \delta F^3 + \frac{5C}{F^4} \delta F^4 - \dots$$

where C is the sag correction for pull F , but the solution can easily be obtained by working out the sag correction for both 15 lbs. and 17 lbs. and using the difference.]

12. $w = 12.8$ ozs. per 100 ft.; $C = 0.0119$ ft.
13. 0.0007 ft.

CHAPTER V (page 294)

1. $BC, 16^\circ 53'$; $CD, 339^\circ 10'$.
2. $283^\circ 32' 55''$, $275^\circ 49' 30''$, $192^\circ 28' 35''$, $230^\circ 21' 00''$, $131^\circ 31' 15''$.
3. 1,632 ft., $244^\circ 1'$.
4. 2.6 ft.
5. Total co-ordinates: $B, +706.6, +212.9$; $C, +1,206.4, -39.7$; $D, +2,099.1, -61.8$ ft.
6. 858.9 ft. S., 177.0 ft. W.
7. 1.7 ft.
8. $57^\circ 17'$, $95^\circ 30'$, 673.7 ft.
9. 3,756.9 ft.
10. 636.4 ft.
11. 1,204 ft., $359^\circ 10'$.
12. 1,662.1 ft.
13. $1^\circ 59'$.
14. $86^\circ 41' 18''$.
15. 927.4 ft.
16. (a) 1,507, 128; (b) 1,295.7, $258^\circ 33' 36''$; (c) $40^\circ 9' 36''$.
17. 254.8 ft., $249^\circ 12'$.
18. 538.5 ft., $44^\circ 26'$.
19. 374.1 ft., 180° .
20. $13^\circ 34' 40''$.
21. Latitude of EF should be 363.8 ; 1.5 ft.

22. Line DE should be 100 ft. longer.
23. $101^{\circ} 44'$.
24. 79.3 ft. towards G; 1,070.7 ft.
25. $279^{\circ} 57' 49''$, 1,521.5 ft.
26. 223 ft. and 216 ft. from A and J respectively.
27. 347.9 ft.
28. 1,565 ft., $150^{\circ} 23'$.
29. 8.5 ft.
30. B, 23,786 ft. N., 39,845 ft. W.; C, 22,756 ft. N., 40,862 ft. W.
31. 3.4 ft.
32. 407.81 chn., 2.316 chn.
33. 4.7 ft.
34. 841 ft. N., 13,911 ft. E.
35. 529.4 ft.
36. 6,957 ft.
37. $114^{\circ} 16'$.
38. 2,907 ft.
39. AB, $85^{\circ} 24'$; BC, $161^{\circ} 37'$; CD, $262^{\circ} 44'$; DE, $348^{\circ} 28'$; EA, $6^{\circ} 29'$.
40. Corrected min. and sec. of lines in given order: $22^{\circ} 40''$, $36^{\circ} 00''$, $14^{\circ} 10''$, $49^{\circ} 40''$, $43^{\circ} 00''$, $51^{\circ} 00''$, $10^{\circ} 00''$, $6^{\circ} 30''$, $56^{\circ} 20''$, $35^{\circ} 30''$, $17^{\circ} 00''$, $6^{\circ} 30''$, $47^{\circ} 40''$, $26^{\circ} 10''$, $5^{\circ} 40''$, $57^{\circ} 00''$, $43^{\circ} 00''$, $36^{\circ} 00''$, $12^{\circ} 20''$, $22^{\circ} 40''$.
41. 1/17,900.
42. Adjusted X's: 32,044.12, 34,823.20, 35,722.15, 36,639.98, 36,388.27. Adjusted Y's: 21,036.85, 22,941.44, 23,322.48, 24,117.29, 26,113.46. Corrected Bearings: $339^{\circ} 28' 58''$, $34^{\circ} 25' 27''$, $22^{\circ} 58' 14''$, $40^{\circ} 53' 30''$, $97^{\circ} 11' 13''$, $105^{\circ} 40' 03''$. Corrected Distances: 1,728.50, 3,369.08, 976.38, 1,214.13, 2,011.98, 1,308.27.
43. Bearing BA = $0^{\circ} 14' 09.3''$, Distance = 5,457.41.
44. (I) On line X = 44,000, Y = 33,642.36; X = 42,000, Y = 30,738.37; Y = 27,000, X = 39,425.35; Y = 30,000, X = 41,491.48.
(II) On line X = 44,000, Y = 28,635.06; X = 42,000, Y = 31,814.59; Y = 27,000, X = 45,028.49; Y = 30,000, X = 43,141.42.
45. X = 30,632.83, Y = 20,945.56.
46. Old co-ordinates of D: X = 10,646.43, Y = 24,796.85. New co-ordinates of D: X =

- 10,646.93, Y = 24,796.23. Old co-ordinates of G: X = 10,959.53, Y = 28,847.31. New co-ordinates of G: X = 10,962.45, Y = 28,846.12.
47. X = 20,847.3, Y = 16,741.0.

CHAPTER VI (page 330)

1. Reduced levels: 75.40, 68.75, 71.41, 76.54, 75.79, 80.01, 84.50, 81.94.
2. .06 ft.
3. .05 ft.
4. A, 116.75; B, 115.77.
5. .05 ft.
6. 13.82 ft.
7. .01 ft., .03 ft.
8. $6' 2\frac{1}{2}''$, $5' 1\frac{1}{2}''$, $5' 0\frac{1}{2}''$, $6' 7''$, $5' 11\frac{1}{2}''$, $7' 5\frac{1}{2}''$, $5' 8\frac{1}{2}''$.
9. Pegs too low by .06, .02, .08, .03, .01, 0, .04, 0, 0, .02 ft.
10. 7.23 ft., 2,115.08 chn.
11. 6.16 ft. at 500 ft. in excavation; 5.87 ft. at 1,000 ft. in embankment.
12. Instrument is not in adjustment; 100.00.

CHAPTER IX (page 411)

1. 7 ac. 1 rd. 6 pl.
2. 175 cub. ft. per sec.
3. 17.7, .9964.
4. 9 ac. 2 rd. $2\frac{1}{2}$ pl.
5. 488 sq. ft.
6. 38.8 ft., 28.7 ft.; 593 sq. ft.
7. 2 ac. 2 rd. 5 pl.
8. 12 ac. 3 rd. 38 pl.
9. BX = 195.3 ft., CY = 166.4 ft.
10. DG = 113.5 ft.
11. 2,294 cub. yd.
12. 2,281 cub. yd.
13. 313 sq. ft.; 1,170 cub. yd.
14. 12,701 cub. yd.
15. 141 cub. yd.
16. 1 in 4.47; 70.4 cub. yd.
17. 330 cub. yd.; 267 cub. yd.
18. -31.6 cub. yd.
19. 4,069 cub. yd.
20. (a) 784 million gal., (b) 787 million gal., (c) 486.9.

CHAPTER X (page 473)

1. $38^{\circ} 49'$.
2. $318 + 91.2$, $340 + 82.2$.

3. 1,053 ft.
5. (a) 6-85, (b) 1-14, (c) 13-21, (d) 313+45, 326+66, 320+05.
6. Total length 15,012.2 ft.; chainages, 4,696.3, 5,046.1, 11,008.5, 11,368.5.
7. 133° 6' 30", 132° 9' 0", 131° 12' 0", 130° 14' 30", 130° 0' 0".
8. 11.59 chn.; 32' 40", 1° 15' 40", 1° 58' 40".
9. 382.14 chn., 394.26 chn.; 13.7 links.
10. 680.47, 691.08, 710.56 chn.
11. 36' 10", 1° 12' 10", 1° 48' 20", 2° 24' 20".
12. 2½°, 490.4 ft., 969 ft.
13. 4,082 ft.
14. 66.2 ft., 120.7 ft., 1,863 ft.
15. 3.47 chn., 5.18 chn.
16. 21.33 chn., 252+22.
17. 19.64 chn., 11.34 chn., 159.43 chn.
18. 48.2 chn.
19. (a) 8.16 chn., (b) 529.45 chn., (c) 12.53 chn.
20. 380.4 ft., 219.6 and 380.4 ft. from B and C respectively.
21. 40.82 chn., 23.6 links.
22. 891.7 ft., 15.1 ft. from A away from C, 27.1 ft. from B towards C.
23. 145.6 ft. towards A.
24. 13,340.5 ft., 126.5 ft.
25. 2,000 ft., 110.1 ft. towards C.
26. 2 links towards C, 1.26 chn. away from E.
27. 0° 16', 1.97 chn.
28. 9.02 chn.
29. (a) 5° 48', (b) 352.34, (c) 3.48 chn.
30. 19.1 ft.
31. 2° 7' 40", 6.99 chn.
32. 16.3 links towards centres.
33. 57.26 chn., 100.11 chn.
34. 11.51 chn., 16.85 chn.
35. 17.63 chn., 14.57 chn.
36. 480 ft.
37. 6.88 chn., 2.70 chn., 2.32 chn.
39. .82 of theoretical cant.
40. 4 in., 145 ft., 5.3 in.
41. 207 ft. for radial acceleration of 1 ft. per sec.², 6½ in.
42. 236.42, 235.78, 235.46, 235.45, 235.77, 236.40, 237.35.
43. Minimum R = 327.32 ft.; D = 61.33 ft.; Maximum ρ = 411.46 ft.; Maximum α = 12° 33' 11"; 2θ = 10° 40' 54"; Length of tangent point, OT = 524.09 ft.; Maximum super-elevation = 1/10; Polar deflection angles = 0° 16', 1° 04', 2° 24', 4° 16', 6° 40', 9° 36', 12° 23' 11"; Lengths of corresponding polar chords = 61.33, 122.64, 183.87, 244.85, 305.25, 364.52, 411.46 ft.; Corresponding chord lengths = 61.33, 61.33, 61.34, 61.37, 61.46, 61.68, 50.58 ft.
44. Maximum ρ = 134.68 ft.; Maximum α = 1° 17' 11.5"; 2θ = 77° 16' 51"; OT = 984.35 ft.; Maximum super-elevation = 0.0327 = 1/30.5; Polar deflection angles of transition curve = 0° 01', 0° 04', 0° 09', 0° 16', 0° 25', 0° 36', 0° 49', 1° 04', 1° 17' 11"; Corresponding polar chord lengths = 15.33, 30.66, 46.00, 61.33, 76.66, 91.99, 107.31, 122.64, 134.68 ft.; chord lengths = 15.33, 15.33, 15.34, 15.33, 15.33, 15.33, 15.33, 15.34, 12.04 ft.
45. Minimum R = 541.08 ft.; D = 89.41 ft.; Maximum ρ = 529.02 ft.; Maximum α = 9° 30' 36.6"; 2θ = 2° 56' 20.4"; OT = 588.24 ft.; Maximum super-elevation = 1/10; Polar deflection angles = 0° 16', 1° 04', 2° 24', 4° 16', 6° 40', 9° 30' 37"; Corresponding polar chords = 89.41, 178.79, 268.06, 356.96, 445.01, 529.02 ft.; chord lengths = 89.41, 89.41, 89.47, 89.62, 87.40 ft.
47. D = 104.71 ft., which is too long for setting out purposes. Use ½D = 52.36 ft. as working unit chord length. Hence, first polar deflection angle = $\frac{16}{4} = 4^\circ$. Working out the larger deflection angles first in order to see when to approximate, we find that, when $l = 4$ unit-chords, $\alpha = 4^\circ 15' 53''$, which only differs from $4^\circ 16'$ by 7". Consequently, the polar deflection angles, for working unit-chords up to 8, may be taken equal to $4^\circ \times 1^2$, $4^\circ \times 2^2$, $4^\circ \times 3^2$, ..., $4^\circ \times 8^2$. The polar deflection angles are therefore 0° 04', 0° 16', 0° 36', 1° 04',

1° 40', 2° 24', 3° 16', 4° 16', 5° 23' 46", 6° 39' 34", 8° 03' 15", 8° 23' 20". The chord length to be used in setting out with these angles is, therefore, 52.36 ft. for the first 11 chords, and, as $L = 587.80$ ft., 11.84 ft. for the last chord. Minimum $R = 668.00$ ft. Angle ϕ at end of transition curve is $25^\circ 12' 28''$, so that the angle between the polar ray and the tangent at the beginning of the circular curve is $16^\circ 49' 08''$.

CHAPTER XI (page 518)

1. 99.5 and 1.5, 484 ft.
2. 454 sq. ft.

3. 531 ft., level of axis 169.3 or 1.0 ft.
4. 3° 18'.
5. 104.3.
6. 260.3, 258.9, 279.6 ft.
7. 1,407 ft., 223.3.
8. 171.7, 643.4 ft.
9. 521.6.
10. 173.1.
11. 1 in 39.3.
12. 1, 299.7 ft., 606.8; 2, 384.0 ft., 635.2; 3, 484.6 ft., 729.1.
13. 286.1.
14. 1,022 ft.
15. $282^\circ 9'$, 732.6 ft.
16. 99.1.
17. 13.421.
18. 303 ft.
19. 202.8, 593 ft.
20. 245.0.
21. 474 ft., 233.3.
22. 1,910 ft., 0.3 ft.

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